Rectified Cubic Honeycomb using Origami

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Abstract

In this workshop, we will build a handful of octahedrons and cuboctahedrons. By connecting them, we will see how they tile space. This process will allow us to observe the geometric relationships and symmetries that emerge when these polyhedrons are combined.

Introduction

The cuboctahedron and the octahedron can tile space, forming the rectified cubic honeycomb (Figure 1). The rectified cubic honeycomb comes from a cubic honeycomb (a space filled with cubes) and truncating the cubes until it is "rectified." The result of this truncation are cuboctahedrons, and the combination of the excess is what makes up the octahedrons.

Origami Construction Steps

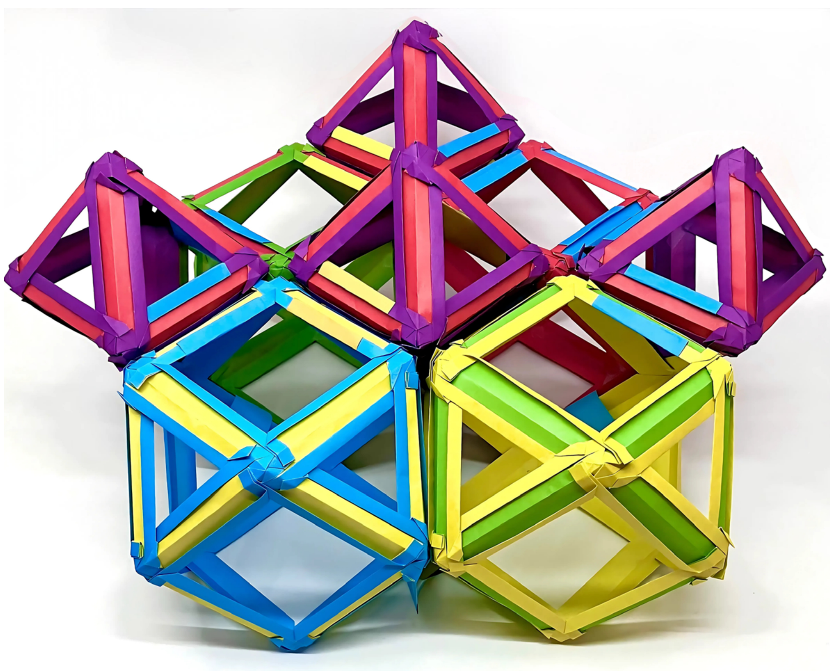
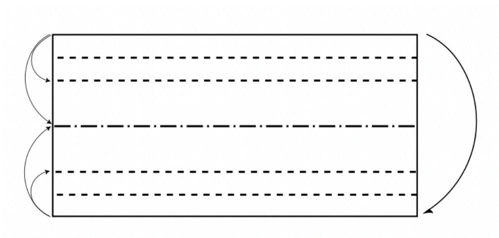
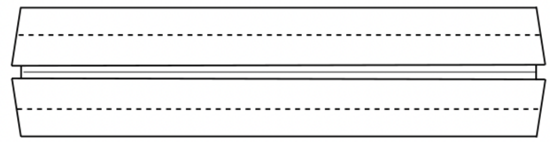


Figure 1: *Rectified Cubic Honeycomb, using origami.*

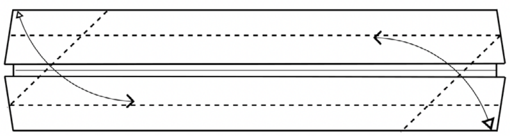
Each cuboctahedron requires twenty-four units and each octahedron requires twelve units to construct. Modules are folded using 2 x 4 inch paper. 2 x 3 inch pieces of paper is used to connect the models together, so the honeycomb can have the final shape. In Figure 1, there are four cuboctahedrons and 4 octahedrons. These modules are similar to modules used in a previous paper[2]. An adhesive was used to secure each model, but is not necessary.



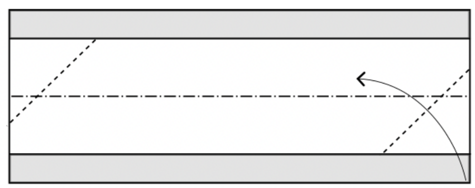
1. Mountain fold in half, then crease into fourths and eights.



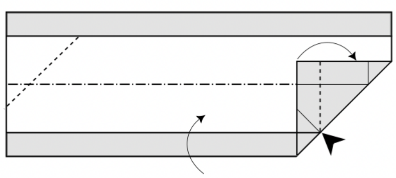
2. Fold into fourths.



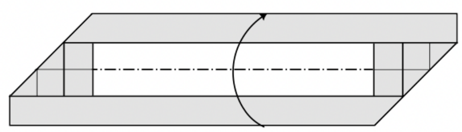
3. Fold corners, then unfold.



4. Fold in the eights. Then fold in the previous fold.

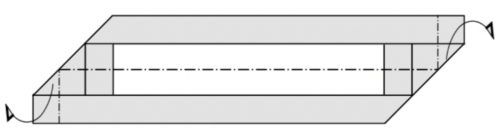


5. Fold in the previous step, then fold the vertical line as shown above, resulting in the bottom folding upwards and collapsing along the intersection. Repeat on both sides.

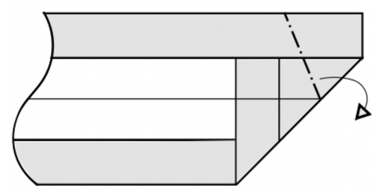


6. Reinforce the mountain fold along the middle.

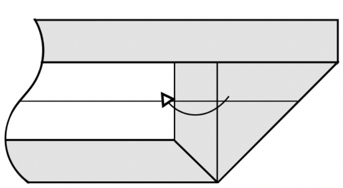
Steps 7-9 show different ending steps for modules. These steps are independent of each other.



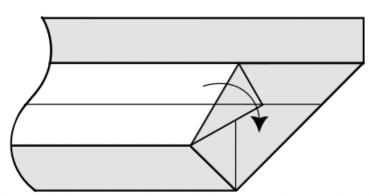
7. Mountain fold on both sides as indicated above.



8. Mountain fold so that it bisects the angle formed by the center and the diagonal line.

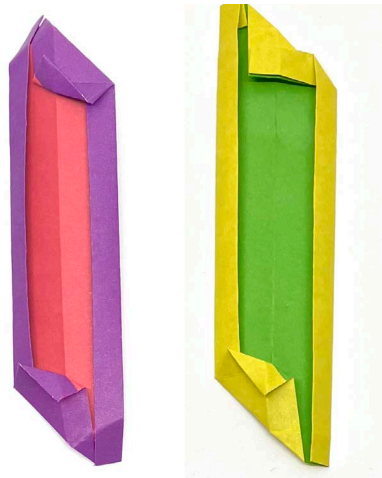


9. Unfold as indicated above.



9b. Fold as shown above, so the upper corner intersects with the central line.

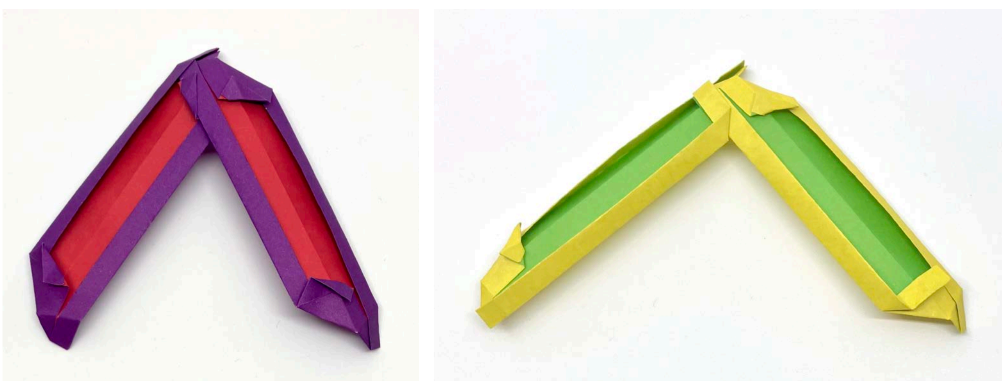
Figure 2(a) shows an image of the triangle-type module used for the octahedron, with both sides using the flap and pocket made from Steps 8 and 9. Figure 2(b) shows the module used for the cuboctahedron with the bottom side with the pocket and flap made from Steps 7 and 9, and the top flap made from Step 8.



(a) (b)

Figure 2: *(a) Module with "Triangle-type" flaps and pockets, (b) Module with a mix of types of flaps and pockets.*

The octahedron requires 12 of the units shown on Figure 2(a). To connect two modules together, insert the flap of one unit into the pocket of the second unit until the crease on the flap overlaps with the central mountain fold from the second unit (Figure 3(a)). Reinforce the central mountain fold to keep the two modules in place. Tuck the end of the flap behind the pocket (Figure 3(b)).

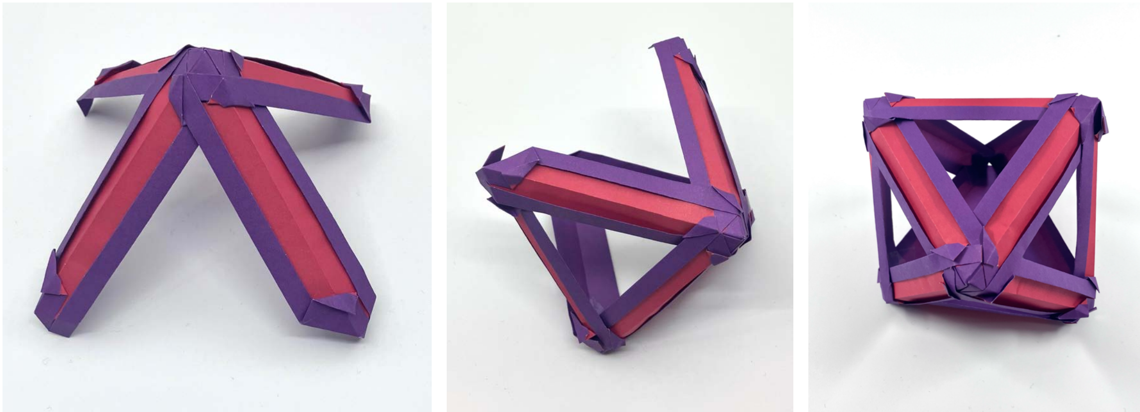


(a) (b)

Figure 3: *(a) Connection of two "Triangle-type" modules, (b) Connection of two "Square-type" modules.*

The cuboctahedron requires twenty-four modules. The process of building one follows the same type of connection as in Figure 3(a), but introduces a second type of interlocking, as shown in Figure 3(b). Place the rectangular flap into the rectangular pocket until the crease on the flap overlaps with the central mountain fold. Reinforce the mountain fold.

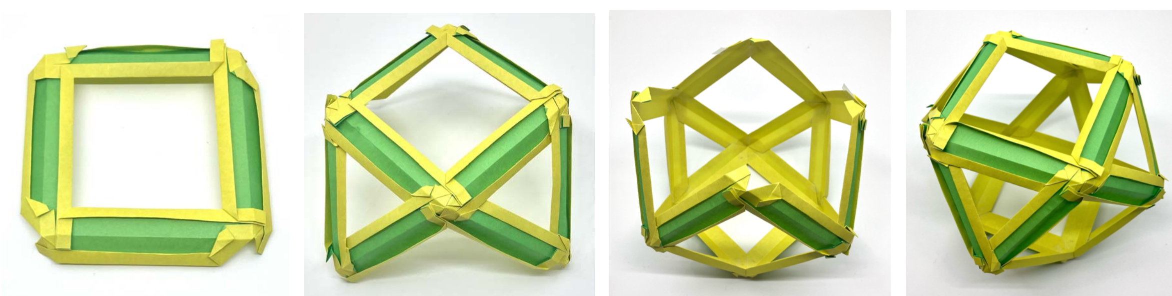
To assemble the octahedron, gather four modules and connect them together in a loop (Figure 4(a)). Continue assembling each point of the octahedron, a group of four modules (Figure 4(b)). After using all 12 modules, the octahedron is complete (Figure 4(c)).



(a) (b) (c)

Figure 4: *(a) 4 modules connected together, (b) Additional modules connected, (c) Completed octahedron.*

The process of building the cuboctahedron begins by constructing a square using four of the modules (Figure 5(a)). Use the triangular flaps and pockets on the unused sides of the modules in a square, to build a triangle along each edge of the square (Figure 5(b)). Connect the sides of each triangle to form two edges of a square. Continue by completing each square (Figure 5(c)). Complete the model by adding the final modules to build triangles between each square, and connect the unused corners to create the final face of the cuboctahedron (Figure 5(d)).



(a) (b) (c) (d)

Figure 5: *(a) 4 modules connected together, (b) Triangular pieces added to each side of the square, (c) More modules added to create squares between triangles, (d) Completed cuboctahedron.*

The Honeycomb

The honeycomb can tile a space infinitely, so make as many cuboctahedrons and octahedrons as you wish! To bind an octahedron with a cuboctahedron, tuck a 2 x 3 inch piece of paper and wrap around the center of the hole of the intersecting faces to tightly connect two touching edges. It is recommended to tape the cuboctahedron and octahedron together during this process.

Acknowledgements

The diagrams were created using the Inkscape software application [1].

References

[1] Inkscape. https://inkscape.org/

[2] N. Shrestha. “Creating Square Koch Surfaces Using Origami.” *Bridges Conference Proceedings*, Richmond, Virginia, Aug. 1–5, 2024, pp. 567–572.