Circling Infinity: String Art with Rational Numbers

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Abstract

Numbers play a crucial role in human culture, from simple daily tasks to complex scientific achievements. Rational numbers have been utilized for thousands of years and nowadays, we tend to represent them as decimals or fractions. These representations reveal intriguing patterns that have captivated both mathematicians and artists. Converting numbers into an image can be done in many ways, as for example through string art. We propose a workshop where participants are invited to create string art designs derived from decimal representations of rational numbers. By dividing a circle into 10 equal parts numbered from 0 to 9, infinitely repeating decimals can be transformed into visual paths defined by the sequence of digits. The resulting images have an artistic appeal reinforced by an unexpected common vertical reflection symmetry.

Introduction

The adage “A picture is worth a thousand words” applies to many contexts, including data visualization techniques used in science. Converting numbers into visual representations opens a universe of possibilities not only to scientists but also to artists and teachers.

String (or thread) art is the arrangement of threads strung between points to form geometric patterns or representational designs. Most commonly, the thread is tensioned between pins distributed on a frame. However, in some forms of string art, the string is tensioned using different techniques, such as curve stitching, where the thread is stitched through holes. In all cases, the patterns are made of straight lines which may produce curves since, as Julius Plücker (1801-1868) noticed, a curve need not be regarded as a set of points; it can just as well be described as a set of tangent lines [13].

The first person known for forming curves out of straight lines is Mary Everest Boole (1832-1916), the wife of the well-known logician George Boole (1815-1864). She used curve stitching to help teach children mathematics in the late 19th century [4]. At present, string art is very popular in the craft world. However, it also spikes the interest of artists, scientists, and teachers. Over the past few years, computer-assisted artists have expanded this art form through both digital images and real works based on digital models. The idea was boosted by Petros Vrellis in 2016 when he developed a specially designed computer algorithm to generate patterns for his series of artwork “A New Way to Knit” [17]. Vrellis and other artists after him used this art form to physically recreate portraits made by famous painters such as El Greco, Botticelli, Leonardo da Vinci, and Vermeer (see, for instance [3]). Digital string art portraits also attract artists and computer-scientists, as can be seen for instance in [2] and [4].

String art has also been used to help teach various mathematical topics at all levels, especially with regard to the envelopes produced by the straight lines. Topics that naturally arise include tangency, circles, conics, and epicycloids ([10][1][13][16]). Simpler topics are also of interest. Attractive designs can be produced by using simple modular addition and multiplication tables depicted on a circle divided into equal parts. Addition tables give rise to polygrams (see for instance [18] for an educational activity) and multiplication tables give rise to a wide variety of designs, including epicycloids (see for instance [12]). Hans-Peter Stricker noticed that multiplication graphs, created by drawing lines from a number x to $k \times x \pmod{n}$ with numbers arranged regularly on a circle, may exhibit different cycles which can be highlighted by color and line width ([14][15]). These graphs are attractive both from an artistic and a mathematical point of view and can be used as teaching resources at an elementary level.

Inspired by these applications, we decided to create string art designs based on the decimal representations of rational numbers, as proposed in [5]. In this workshop, participants are challenged to use the same technique to create a set of designs of their choice, using a paper template and colored pencils. The following section describes the technique and presents several examples. The workshop plan is then detailed, followed by some didactical considerations and concluding remarks.

Visual Representation of Decimals Through String Art

Any real number may be expressed as a decimal, in a base 10 system, and this type of representation is the one most used. Integer numbers and part of the fractional numbers may be written as terminating decimals, but irrational numbers and some rational numbers may only be written as non-terminating decimals. Non-terminating decimals that represent rational numbers are necessarily periodic (repeating), whereas those representing irrational numbers are non-periodic.

Rational numbers may also be represented by fractions and using some results from number theory, it is possible to predict several characteristics of the decimal representation by inspection of the fraction. In summary [6, pp. 143–144]: *Let* n/q *be an irreducible fraction* ($n, q \in \mathbb{N}$)*.*

- *If* $q = 2^{\alpha}5^{\beta}$*, and* $max(\alpha, \beta) = \mu$*, then the decimal terminates after* μ *digits.*
- *If* $q = 2^{\alpha}5^{\beta}Q$*, where* $Q > 1, (Q, 10) = 1$*, and* $\nu = ord_Q 10$*, where* $(Q, 10)$ *is the greatest common divisor of* Q *and 10, and* $ord_Q 10$ *is the order of 10* $(mod\ Q)$*, i.e., the least positive integer k such that* $10^k \equiv 1 (mod\ Q)$*, then the decimal contains* μ *non-repeating and* ν *repeating digits.*

Note that this result implies that all decimals of (irreducible) fractions with the same denominator have the same number of non-repeating digits and the same repetend length. It also implies that the only fractions with terminating decimals are those whose denominators have no other prime factor apart from 2 or 5. Furthermore, if the denominator is coprime with 10 then the decimal is pure repeating.

Finding the number of distinct repetends for a given denominator is not straightforward, but for prime denominators, q (coprime with 10), the number of distinct repetends of the decimals of irreducible fractions with denominator q is simply $(q-1)/\nu$, where $\nu = ord_q 10$ [11, p. 25].

We further refer to Midy's theorem [8, p. 2] which allows us to predict, in some situations, the second half of the repetend given the first half. Given an irreducible proper fraction, p/q, where q is a prime (coprime with 10), and the corresponding repetend has even length $2m$, $n/q = 0.\overline{a_1a_2a_3 \dots\ a_na_{n+1} \dots\ a_{2m}}$, then, for $1 \le i \le m$, $a_i + a_{m+i} = 9$. As an example, consider the repetend of $1/19 = 0.\overline{052631578947368421}$. The first half, 052631578, is the complement to 9, digit by digit, of the second half, 947368421.

Bearing these results in mind, we traced the decimal expansions of all fractions with denominators coprime with 10, up to 51. To obtain them we used an Excel spreadsheet. Because Excel only provides up to 16 decimal places, we used the above-mentioned results to obtain the complete repetends for all cases exceeding this limit. We then converted these expansions into visual paths as follows.

Divide a circle into 10 equal parts numbered from 0 to 9. Create a string art design by following the path defined by the sequence of digits of a particular repeating decimal, starting after the decimal point. On the same circle trace all paths of the decimal expansions of irreducible fractions with the same denominator.

Let's look at some particular examples. Figure 1 shows the paths of all irreducible factions with denominators 19, 13 and 11. These decimal expansions are all pure repeating. Note that there is one common path (cycle) for all fractions with $q = 19$ (this means the repetends are the cyclic permutations of one only repetend); there are two distinct cycles for $q = 13$ ($\overline{076923}$ and $\overline{153846}$; half of the expansions share one of the repetends) and five cycles for $q = 11$ ($\overline{09}$, $\overline{18}$, $\overline{27}$, $\overline{36}$ and $\overline{45}$). Note also that in the latter case the paths reduce to line segments since the repetend length is 2.

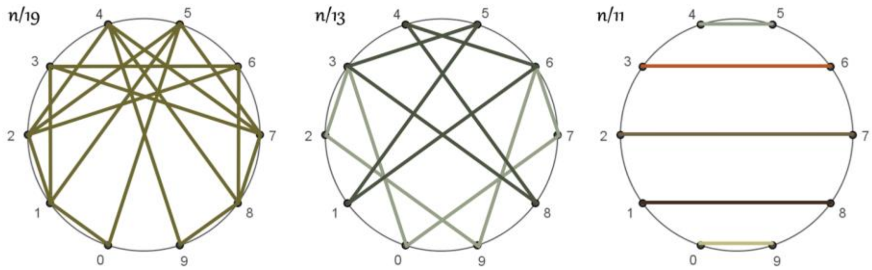


Figure 1: *Decimal trajectories of irreducible fractions with denominators 19, 13 and 11.*

Looking at Figure 1, we see that all paths have vertical reflection symmetry. This is not necessarily true for other decimal expansions as can be seen in Figure 2 where we trace the decimal expansions of fractions with denominators 9 (repetends $\overline{1}$, $\overline{2}$, $\overline{4}$, $\overline{5}$, $\overline{7}$ and $\overline{8}$), 21 (repetends $\overline{047619}$ and $\overline{095238}$) and 41 (repetends $\overline{02439}$, $\overline{04878}$, $\overline{07317}$, $\overline{09756}$, $\overline{12195}$, $\overline{14634}$, $\overline{26829}$ and $\overline{36585}$).

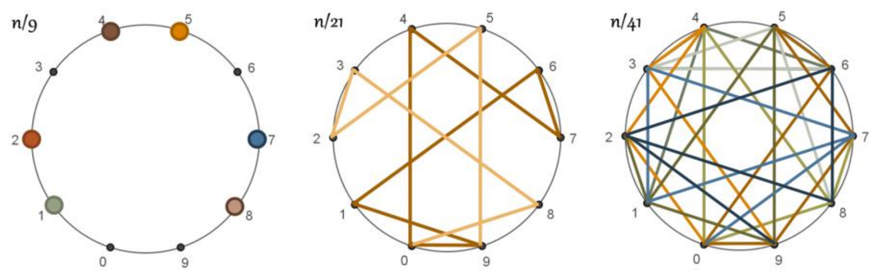
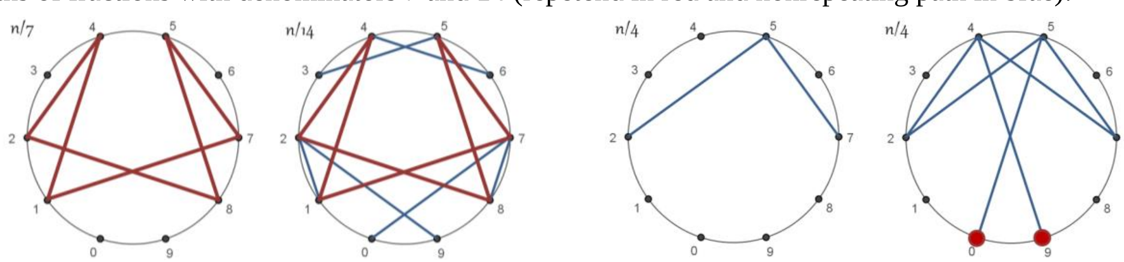


Figure 2: Decimal trajectories of irreducible fractions with denominators 9, 21 and 41.

Nevertheless, there is always a vertical symmetry in the complete set of trajectories with the same denominator (a proof of this property can be seen in [5]). Note that in Figure 2, the paths from fraction with denominator 9 degenerate into isolated points since the repetend length is only 1. Overall, the paths may be points, lines or polygons (frequently self-intersecting).

The previous examples were all pure repeating decimals. Not all repeating decimals are like that. Some have non-repeating digits preceding the repetend. Nevertheless, converting them into visual paths as described before also gives rise to symmetrical string art designs as illustrated in Figure 3(a) and (b) with the paths of fractions with denominators 7 and 14 (repetend in red and nonrepeating path in blue).



(a) (b) (c) (d)

Figure 3: Decimal trajectories of irreducible fractions with denominators 7 and 14 ((a) and (b)), and 4 (finite trajectory (c) and repeating trajectories (d)).

When tracing the trajectories of terminating decimals, we find that the symmetry is lost and the visual result can be rather upsetting as can be seen in Figure 3(c) depicting the paths of decimals corresponding to quarters. However, every terminating decimal has one repeating equivalent, adding infinitely many zeros to the right. Furthermore, it has another repeating equivalent, using infinitely many nines to the right. For example, $0.25 = 0.25\overline{0} = 0.24\overline{9}$. In general, any terminating decimal, $0.a_1a_2\ldots a_n$ with $a_n > 0$, has two repeating equivalents given by $0.a_1a_2\ldots a_m = 0.a_1a_2\ldots a_n\overline{0} = 0.a_1a_2\ldots a_{m*}\overline{9}$, where $a_{m*} = a_m - 1$.

By representing each terminating decimal through its two repeating equivalents, the symmetry of the string art designs is recovered as illustrated in Figure 3(d) for decimals corresponding to quarters.

We next present two digital artworks produced by the first author of this paper and based on the previously described paths. Each artwork contains 20 circular string art designs, arranged in a 4x5 grid. Non-digital versions of the artworks, created using embroidered canvas, can be seen in [5].

The first artwork contains the paths of the fractional parts of the decimals of all irreducible fractions with denominators coprime with 10, up to 51. These are all pure repeating converting into closed paths. Figure 4 shows the artwork and Table 2 contains all the involved repetends. For each denominator distinct cycles have different colors. Cycles of length one, which degenerate into isolated points, are represented with a circle over the repeating digit (see denominators 3 and 9). Whenever a digit finitely repeats within a cycle, a small circle is placed on that digit (see denominators 29, 31, 43, 47 and 49).

The second artwork contains the paths of the fractional parts of the decimals of all fractions with denominators from 1 to 20. Whenever a decimal is terminating, we replace it with its repeating equivalents, terminating in $\overline{0}$ and $\overline{9}$. Therefore, for each denominator, different types of paths may coexist. For instance, for denominator 6 we have: $1/6 = 0.1\overline{6}$; $2/6 = 0.\overline{3}$; $3/6 = 0.5\overline{0} = 0.4\overline{9}$; $4/6 = 0.\overline{6}$ $5/6 = 0.8\overline{3}$; $6/6 = 1.\overline{0} = 0.\overline{9}$. In the artwork, repeating cycles are colored dark red, except for a green cycle in denominator 13 and cycles of length 1, which are all bright red. Non-repeating paths are all the same blue. Figure 5 shows the result and Table 3 contains all the paths excluding the ones already present in Table 2.

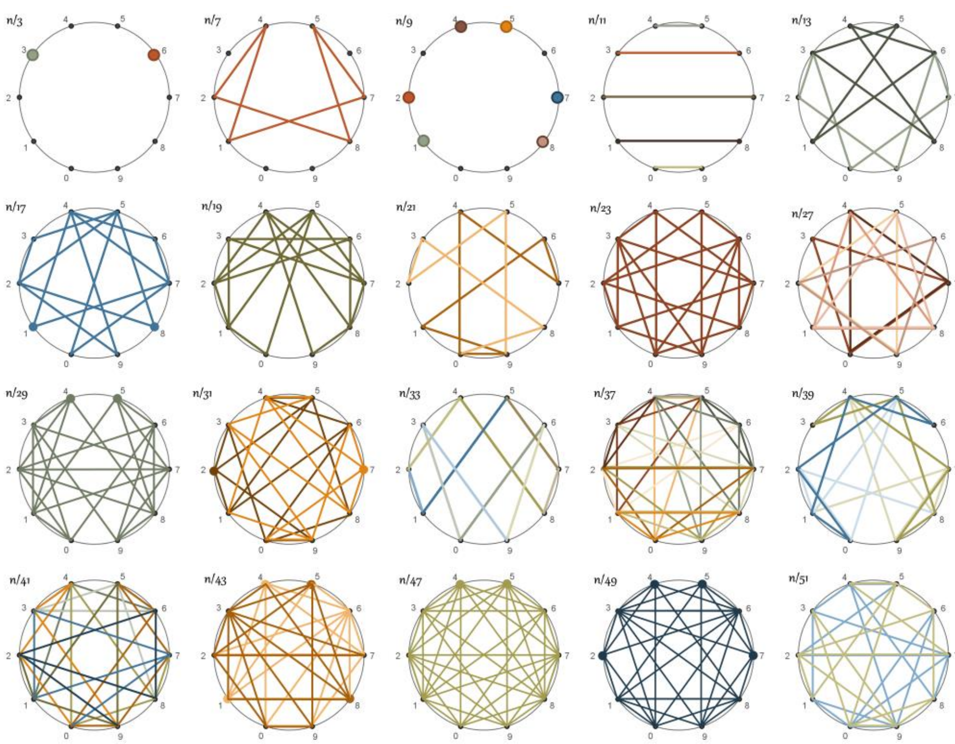


Figure 4: *Digital Dancing Decimals I by the first author, 2022. Decimal trajectories of irreducible fractions with denominators coprime with 10, up to 51, digital string art.*

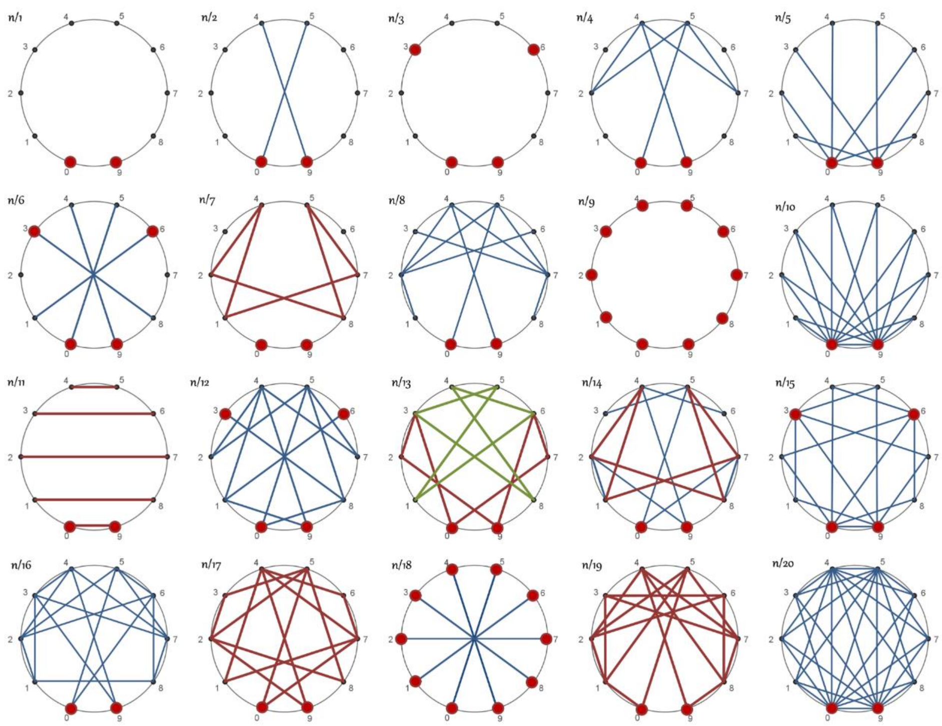


Figure 5: *Digital Dancing Decimals II by the first author, 2022. Decimal trajectories of all fractions with denominators from 1 to 20, digital string art.*

Workshop Plan

This workshop unfolds in three stages.

Stage One (approximately 30 minutes): Participants are introduced to the theoretical aspects outlined in the previous section, along with several illustrative examples of decimal paths. Animated versions of some of these paths – created by the second author of this paper – are presented during this stage. While string art designs are inherently static, the animations offer real-time simulations of the paths determined by decimal expansions. Participants are encouraged to use these animations in future replications of the workshop, via the links provided in Table 3 [5]. The interactive features of the animations allow users to adjust the playback speed, pause, and restart at any point. In some cases, users can also choose which fractions to animate from within a set of fractions sharing the same denominator.

Stage Two (approximately 30 minutes): Participants explore the decimal paths of fractions based on one or more denominators of their own choosing, within the set of numbers covered in Tables 1 and 2. To determine these paths, they are encouraged to use the calculators on their mobile phones and combine the outputs with the information presented in stage one. A summary of this information is provided in a worksheet distributed during the session, which is also available for future replications in [6]. Participants may verify their findings using Tables 1 and 2, also included in the worksheet. Using these tables, all the designs shown in Figures 4 and 5 can be reproduced.

Stage Three (approximately 30 minutes): Each participant receives a paper template, colored pencils, and a ruler to draw three string art designs of their choice. They are encouraged to use the paths they discovered in stage two and may consult Tables 1 and 2 to assist in completing the task. The paper templates, also included in the worksheet [6], feature pre-printed supporting circles with ten evenly spaced points, numbered 0 to 9.

Workshop facilitators should adjust the level of challenge based on participants' mathematical backgrounds. Those with less experience may rely primarily on the tables provided in the worksheet to create their designs, while more advanced participants are encouraged to derive the paths independently, using the tables for confirmation only.

Table 1: *Cycles of pure repeating decimals of irreducible fractions with denominator q, q coprime with 10 (λ is the number of distinct cycles and ν is the cycle length).*

q	λ	ν	Cycles
3	2	1	$\overline{3}$ $\overline{6}$
7	1	6	$\overline{142857}$
9	6	1	$\overline{1}$ $\overline{2}$ $\overline{4}$ $\overline{5}$ $\overline{7}$ $\overline{8}$
11	5	2	$\overline{09}$ $\overline{18}$ $\overline{27}$ $\overline{36}$ $\overline{45}$
13	2	6	$\overline{076923}$ $\overline{153846}$
17	1	16	$\overline{0588235294117647}$
19	1	18	$\overline{052631578947368421}$
21	2	6	$\overline{047619}$ $\overline{095238}$
23	1	22	$\overline{0434782608695652173913}$
27	6	3	$\overline{037}$ $\overline{074}$ $\overline{148}$ $\overline{185}$ $\overline{259}$ $\overline{296}$
29	1	28	$\overline{0344827586206896551724137931}$
31	2	15	$\overline{032258064516129}$ $\overline{096774193548387}$
33	10	2	$\overline{03}$ $\overline{06}$ $\overline{12}$ $\overline{15}$ $\overline{24}$ $\overline{39}$ $\overline{48}$ $\overline{57}$ $\overline{69}$ $\overline{78}$
37	12	3	$\overline{027}$ $\overline{054}$ $\overline{081}$ $\overline{135}$ $\overline{162}$ $\overline{189}$ $\overline{243}$ $\overline{297}$ $\overline{378}$ $\overline{459}$ $\overline{486}$ $\overline{567}$
39	4	6	$\overline{025641}$ $\overline{051282}$ $\overline{179487}$ $\overline{358974}$
41	8	5	$\overline{02439}$ $\overline{04878}$ $\overline{07317}$ $\overline{09756}$ $\overline{12195}$ $\overline{14634}$ $\overline{26829}$ $\overline{36585}$
43	2	21	$\overline{023255813953488372093}$ $\overline{046511627906976744186}$
47	1	46	$\overline{0212765957446808510638297872340425531914893617}$
49	1	42	$\overline{020408163265306122448979591836734693877551}$
51	2	16	$\overline{0196078431372549}$ $\overline{0392156862745098}$

Table 2: *Paths of all decimals of fractions with denominator q.*

q	Paths
1	$\overline{0} \equiv \overline{9}$
2	$5\overline{0} \equiv 4\overline{9}$ $\overline{0} \equiv \overline{9}$
4	$25\overline{0} \equiv 24\overline{9}$ $5\overline{0} \equiv 4\overline{9}$ $75\overline{0} \equiv 74\overline{9}$ $\overline{0} \equiv \overline{9}$
5	$2\overline{0} \equiv 1\overline{9}$ $4\overline{0} \equiv 3\overline{9}$ $6\overline{0} \equiv 6\overline{9}$ $8\overline{0} \equiv 7\overline{9}$ $\overline{0} \equiv \overline{9}$
6	$1\overline{6}$ $\overline{3}$ $5\overline{0} \equiv 4\overline{9}$ $\overline{6}$ $8\overline{3}$ $\overline{0} \equiv \overline{9}$
8	$125\overline{0} \equiv 124\overline{9}$ $25\overline{0} \equiv 24\overline{9}$ $375\overline{0} \equiv 374\overline{9}$ $5\overline{0} \equiv 4\overline{9}$ $625\overline{0} \equiv 624\overline{9}$ $75\overline{0} \equiv 74\overline{9}$ $875\overline{0} \equiv 874\overline{9}$ $\overline{0} \equiv \overline{9}$
10	$1\overline{0} \equiv 0\overline{9}$ $2\overline{0} \equiv 1\overline{9}$ $3\overline{0} \equiv 2\overline{9}$ … $8\overline{0} \equiv 7\overline{9}$ $9\overline{0} \equiv 8\overline{9}$ $\overline{0} \equiv \overline{9}$
12	$8\overline{3}$ $1\overline{6}$ $25\overline{0} \equiv 24\overline{9}$ $\overline{3}$ $41\overline{6}$ $5\overline{0} \equiv 4\overline{9}$ … $\overline{0} \equiv \overline{9}$
14	$0\overline{714285}$ $\overline{142857}$ $2\overline{142857}$ $\overline{285714}$ $3\overline{571428}$ $\overline{428571}$ $5\overline{0} \equiv 4\overline{9}$ … $\overline{0} \equiv \overline{9}$
15	$0\overline{6}$ $1\overline{3}$ $2\overline{0} \equiv 1\overline{9}$ $2\overline{6}$ $\overline{3}$ $4\overline{0} \equiv 3\overline{9}$ $4\overline{6}$ $5\overline{3}$ $6\overline{0} \equiv 5\overline{9}$ … $\overline{0} \equiv \overline{9}$
16	$0625\overline{0} \equiv 0624\overline{9}$ $125\overline{0} \equiv 124\overline{9}$ $1875\overline{0} \equiv 1874\overline{9}$ $25\overline{0} \equiv 24\overline{9}$ … $\overline{0} \equiv \overline{9}$
18	$0\overline{5}$ $\overline{1}$ $1\overline{6}$ $\overline{2}$ $2\overline{7}$ $\overline{3}$ $3\overline{8}$ $\overline{4}$ $5\overline{0} \equiv 4\overline{9}$ $\overline{5}$ $6\overline{1}$ $\overline{6}$ … $\overline{0} \equiv \overline{9}$
20	$05\overline{0} \equiv 04\overline{9}$ $1\overline{0} \equiv 0\overline{9}$ $15\overline{0} \equiv 14\overline{9}$ $2\overline{0} \equiv 1\overline{9}$ $25\overline{0} \equiv 24\overline{9}$ $3\overline{0} \equiv 2\overline{9}$ … $\overline{0} \equiv \overline{9}$

Table 3: *Links to interactive GeoGebra animations*

Denominator	Link
3	https://www.geogebra.org/m/tsy9w37c
4	https://www.geogebra.org/m/bxawsj7n
7	https://www.geogebra.org/m/nnusvw2n
10	https://www.geogebra.org/m/gadm749z
11	https://www.geogebra.org/m/uqywgzyj
13	https://www.geogebra.org/m/fxmva8ey
14	https://www.geogebra.org/m/rsvzsg3n
18	https://www.geogebra.org/m/gdbhrkad
19	https://www.geogebra.org/m/ktdvk5pb
21	https://www.geogebra.org/m/bjq82r75

Didactical Considerations

The concept of infinity has long intrigued humankind, posing challenges for both teaching and learning. One of the earliest encounters with an infinite process occurs when students attempt to express 1/3 as a decimal and discover its infinitely repeating nature. While rational numbers are neatly expressed as fractions, their decimal representations can be infinitely repeating, introducing difficulties in computation and approximation.

The materials proposed in this workshop serve as valuable pedagogical tools, allowing teachers to introduce students to the concept of infinity in a more creative and intuitive manner. The visual representation of repeating decimals helps students develop a deeper understanding of rational numbers, their periodic nature, and other properties.

We have implemented these designs in pre-service teacher education, encouraging students to explore them through hands-on activities, as outlined in this workshop. This process fostered curiosity and discovery, prompting students to identify key number concepts such as pre-period digits, repetends, repetend length, the number of distinct repetends, and circular permutations of repetends. In this implementation, since students had more time than the duration proposed for the workshop, each was further challenged to complete at least one design using decimal expansion paths of fractions with denominators not included in Tables 1 and 2. Overall, by creating these patterns, students were able to observe the relationships between different fractions and their decimal representations in a more concrete way. They enhanced their number sense and developed a deeper appreciation for the intrinsic beauty of mathematical structures.

Summary and Conclusions

In this workshop, we explore an innovative approach to representing rational numbers through string art proposed by [5], transforming repeating decimals into visually appealing symmetrical designs. These artistic and mathematical representations may be used as a pedagogical tool. They facilitate conceptual understanding by bridging the abstract nature of decimal expansions with concrete, visual experiences.

By blending mathematics with art, we offer an alternative and enriching pathway for exploring numerical properties, making abstract concepts more accessible and fostering a deeper appreciation for the elegance of mathematical patterns.

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