Mathematical Magic: Connecting Wonder, Creativity and Learning

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Abstract

This workshop invites participants to learn a variety of mathematical tricks, including geometric paradoxes, card tricks, and numeric illusions. Beyond mastering these tricks, participants will gain insight into the underlying mathematical principles and discover how to use them to engage students in the math classroom.

Introduction

The relationship between mathematics and magic has been a subject of fascination for centuries, captivating the minds of both mathematicians and magicians alike. This intriguing intersection of logic and illusion offers a unique perspective on how mathematical principles can be applied to create seemingly impossible feats of magic. From geometric paradoxes that challenge our visual perception to card tricks that exploit numerical properties, and numeric illusions that manipulate our understanding of basic arithmetic, mathematical magic serves as a bridge between the abstract world of numbers and the world of entertainment.

The connection between mathematics and magic dates back centuries, with Luca Pacioli describing the first known math-related card trick in his manuscript "De Viribus Quantitatis" five hundred years ago [4]. Since then, a vast body of work has emerged, applying mathematical concepts to create diverse magic tricks and integrating mathematical magic into educational contexts.

Looking at the literature, we notice that notable figures have contributed significantly to this field. Júlio Cesar de Mello e Souza (1895-1974), writing under the pseudonym Malba Tahan, published over 100 books, many of which focused on mathematics. His book "Matemática Divertida e Curiosa," first published in 1934 and now in its 38th edition [19], is a pioneering work in recreational mathematics, featuring enigmas, puzzles, and mathematical magic tricks.

Martin Gardner (1914-2010) deserves special mention for bridging the worlds of magic and mathematics. As a science disseminator, he published more than 100 books, including "Mathematics, Magic, and Mystery" [7], which has captivated magicians, mathematicians, and educators alike ever since.

Inspired by Gardner, the American mathematician Colm Mulcahy (b. 1958) created "Card Colm," a bimonthly column dedicated to math-based card tricks, on the American Mathematical Association's website between 2004 and 2014 [16]. He later compiled many of these tricks in a book [17].

Fernando Blasco (b. 1968), from Spain, is another disseminator of mathematical magic, authoring books like "Matemagias" [3] and others listed in his website [2].

While numerous authors contributed to the field, we only mention a few prominent figures. To conclude, we refer to a book written by Diaconis and Graham [6], which stands out for the mathematical depth of their magic tricks, making it most suitable for university students.

In recent years, mathematical magic shows have emerged as an engaging way to disseminate mathematics to both general audiences and students. Fernando Blasco from Spain is one of those performers and stated that:

Though seemingly different at first glance, magic and mathematics can become complementary. In magic tricks, the necessary elements for scientific creation emerge: amazement, enchantment, and leaving the audience wondering how it all happens [3].

Similar initiatives exist in other countries, such as the “Mathematical Circus” project [5] in Portugal, which combines magic tricks with light-hearted clown interventions to create a vibrant atmosphere. This project, launched by the Ludus Association in 2011, performs shows primarily for students under 18, in schools, science centers, and universities, fostering active participation and interaction [10][13].

Recognizing the need to make mathematics education more stimulating and relevant to today’s students [12], the authors regularly incorporated mathematical magic in their classrooms or in talks to school students, to engage and motivate learners. Each trick actively involves at least one volunteer, fostering interaction between the teacher and the class. Through this approach, the authors have successfully increased student motivation, captured attention, and stimulated interest in mathematics through playful learning [8][9]. In a recent research experiment [11], the authors integrated various active learning methodologies into a mathematics course for pre-service teachers. A survey revealed that mathematical magic was the most popular resource, surpassing even the well-liked game-based learning platform Kahoot!, rated second.

A rich collection of mathematical magic tricks, tailored to specific school curriculum topics, has been compiled in a series of master's theses supervised by the second author [1][14][15][18]. These theses were designed to provide teachers with engaging classroom resources. Performing a mathematical magic trick and revealing its underlying principles creates a unique environment for reasoning and exploration, enhanced by the element of surprise.

The Workshop

In this workshop, we will start by presenting a selection of mathematical magic tricks, involving the participants as volunteers. After each trick, we shall thoroughly explain the underlying mathematical concepts, invite the participants to perform the trick by themselves, and discuss how it can be used in the classroom. A selection of four illustrative effects is provided below as a sample of the workshop's content.

The missing number

Start by providing a volunteer with a calculator and a deck of cards. Draw a 3x3 grid on the blackboard, numbering the cells 1 through 9 as is shown in Figure 1:

1	2	3
4	5	6
7	8	9

Figure 1: *Initial board*

Blindfold yourself and ask the volunteer to choose a row, column, or diagonal and form a 3-digit number using the corresponding digits, in any order. Have them enter this number into the calculator. Then, ask them to choose another row, column, or diagonal and form a second 3-digit number. They should multiply this second number by the first. Next, ask the volunteer to select a numerically matching card from the deck for each digit of the product, treating Ace as the number 1 and discarding any zeros. Finally, have them hide one of the chosen cards and display the remaining ones in random order. Remove your blindfold. After a quick glance at the displayed cards, you can guess the hidden card.

The trick works because any 3-digit number formed from a row, column, or diagonal of the grid is a multiple of 3, and therefore the product of two such cumbers is necessarily a multiple of 9. According to the divisibility rule of 9, the sum of the digits of any multiple of 9 is also a multiple of 9. To determine the hidden card, sum the displayed cards and find the difference between this sum and the nearest multiple of 9 that is greater than the sum. This difference is the value of the hidden card.

As a didactical activity, you can discuss with the students why each row, column, and diagonal sums to a multiple of 3 and review the divisibility rules.

Five cards

This mathematical card trick was created by William Fitch Cheney Jr. and was initially published by Wallace Lee in his book "Math Miracles" in 1950. It requires audience members, the magician, and an assistant. The magician's assistant invites members of the audience to select five cards from a standard 52-card deck. The audience has complete freedom in their selection. These five cards are then handed to the assistant. The assistant shows four of these cards to the magician while keeping one card hidden—known only to the audience and the assistant. After examining the four visible cards, the magician successfully identifies and announces the fifth secret card.

How does this trick work? First we apply the pigeonhole principle to obtain very important information. There are only 4 suits (clubs ♣, diamonds ♦, hearts ♥, and spades ♠), while there are 5 cards chosen. This means that at least 2 cards must be of the same suit and one of these will be the hidden card. The assistant and the magician have an agreement about the ordering of cards and a code on how to use the cards. Since there is a card with the same suit as the hidden one, this is placed in an agreed position. So, the magician directly knows the suit of the hidden card by looking at this card. To obtain the value of the hidden card, the assistant must code in base two the difference between the shown card and the hidden one. The remaining three cards are used to make the coding. To make the method clear, we will show it with the example of Figure 2.

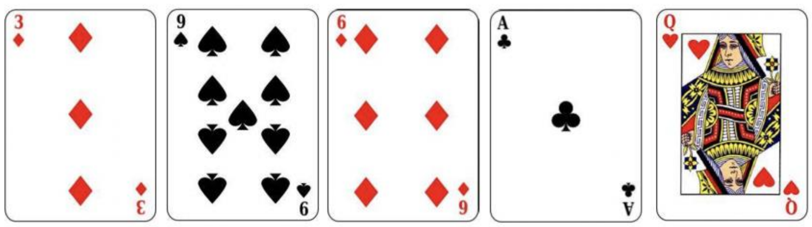

Figure 2: *A possible configuration of the cards selected by the audience*

The two cards of the same suit are the 3 and the 6 of diamonds. In the order described in Figure 3 (clockwise), the shortest distance between the two diamond cards is from the 3 to the 6, and that distance is 3. Note that in this circular order, the shortest distance between any two cards is always less than eight and therefore 3 digits are enough for encoding the difference in base 2.

On this path, the "highest" card is the 6, and so the magician's assistant hands to the audience the 6 of diamonds to be the hidden card (Figure 4). In a previously agreed position, for instance the 2nd position (from left to right), the magician's assistant places the other card of the same suit as the hidden card.

The magician thus knows the suit of the hidden card easily. The three cards in the 1st, 3rd, and 4th positions encode a base two number. Using the code 0 = face down, 1 = face up we have $011_{(2)} = 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3$. So, the magician's assistant is informing the magician that he needs to "move up" 3 units from the card in 2nd position to find the hidden card. In this case, and following the circular scheme in Figure 3, the magician knows that the hidden card is the 6 of diamonds.

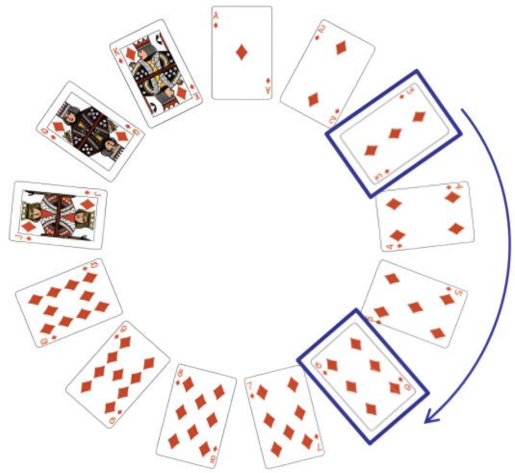

Figure 3: *Circular order*

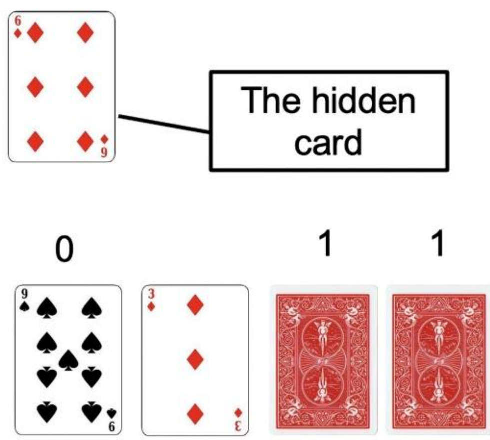

Figure 4: *Set after the assistant organizes the cards*

Faster Than a Calculator

The magician starts by calling a volunteer and then asks the volunteer to blindfold him. Blindfolded, he asks the volunteer to write down two random numbers (one above the other) and under the second number to write down the sum of the two numbers, thus obtaining a third number. He then asks the volunteer to continue writing numbers in the column, adding every last two numbers. The volunteer should repeat the procedure until the column has 10 numbers. This procedure generates the first 10 terms of a generalized Fibonacci sequence whose first 2 terms are chosen at random by the volunteer.

After the volunteer indicates that he already has 10 numbers, the magician asks the audience members to use a calculator to calculate the sum. At this point he removes the blindfold, and quickly, even before the audience has finished calculating, the magician indicates the result.

The magician is able to obtain the sum before the audience because the sum of the first 10 terms is equal to the seventh term multiplied by 11. Fibonacci sequences have many interesting properties that can be used to perform magic tricks.

Curry's Triangle

Geometric paradoxes are apparently contradictory visual puzzles. They have fascinated mathematicians and laypeople alike for centuries. These illusions challenge our fundamental understanding of space, area and perception, and are often presented as simple rearrangements or constructions. They serve as powerful demonstrations of the limits of our visual intuition and the complexities of geometric principles, despite not being true logical contradictions.

An example of a geometric paradox is Curry's triangle. It demonstrates how easily our perception can be fooled by subtle geometric manipulations and is a fascinating optical illusion and dissection fallacy. This puzzle was initially created by American neuropsychiatrist L. Vosburgh Lions, based on a principle discovered in 1953 by New York amateur magician Paul Curry. It involves a set of 6 polygonal pieces that can be arranged in different ways (e.g., a triangle with an area of 60 or a triangle with an area of 58 containing a rectangular hole) like we can observe in Figure 5.

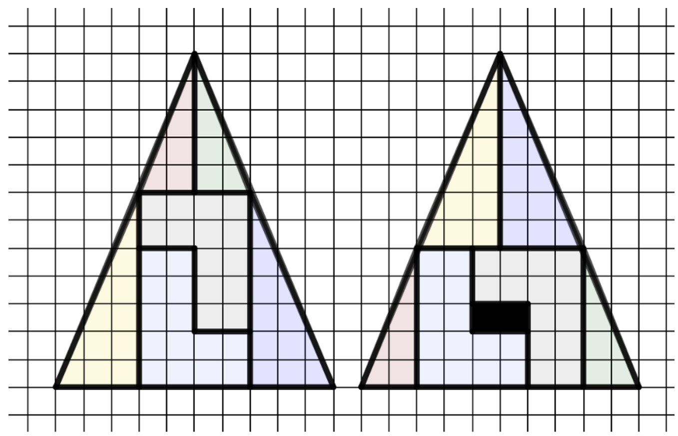

Figure 5: *Curry's Triangle.*

This visual illusion is generated by the strategic placement of diagonal line segments with subtly differing slopes: specifically, 7/3 and 5/2 on the left side of each triangle, and their mirrored counterparts, -7/3 and -5/2 on the right side of each triangle. While the outer polygon in each figure appears to be a triangle, they are in fact pentagons; the left one is concave, while the right one is convex. Superimposing the two figures reveals their non-congruence. Quantitatively, the larger pentagon has an area of 60 square units, whereas the smaller pentagon's area is 58 square units. Consequently, the discrepancy of 2 square units accounts for the perceived 'hole' within the combined image.

Summary and Conclusions

Imagine stepping into a world where math and magic blend together in a fascinating dance. Our workshop invites participants to explore this captivating relationship, where mathematical principles are used to create illusions that will wonder and amaze. Through a series of engaging tricks like the ones described above, participants will uncover the hidden mathematical concepts behind each illusion. These examples show how mathematical magic can make math more enjoyable and accessible, helping students connect with the subject on a deeper level.

Our goal is to empower the participants with the tools to bring these tricks into their teaching or environment, making math more exciting and relevant for everybody. By presenting math in a fun and engaging way, we hope to inspire a genuine appreciation for its beauty and logic. The tricks we share are just a glimpse into the vast array of mathematical magic tricks that can be adapted to fit specific topics in the school curriculum.

As teachers explore these tricks, they'll discover fresh ways to spark curiosity and motivate their students towards math. During the workshop, we expect to surprise participants with a significant number of card effects and other tricks based on number properties, making the experience a memorable one.

Acknowledgements

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