A Single Stitch

Lisa Marks

Industrial Design, Georgia Institute of Technology; Lisa.Marks@design.gatech.edu

Abstract

Knitting is a ubiquitous craft wherein a single stitch, as a repeated geometric base unit, can be simply modeled mathematically using visual CAD programs. By formalizing the shape of one particular knitting stitch (ignoring forces from surrounding stitches), certain semi-rigid knitting possibilities can be built in an unusual medium such as laser-cut bamboo.

Introduction

Knitting is the process of creating interlocking loops, generally out of a strand of yarn. Because the knit textile is a series of loops as opposed to the straight threads of weaving, the result is a stretchy, flexible fabric used for a broad range of textile applications. There are two basic stitches: a knit stitch, where the loop goes through the previous row from behind, creating a smooth stitch, and a purl stitch, where the loop goes through the previous row from the front, leaving a visible bump.

There has been a lot of work describing the geometry of knitting in many different venues. Much of the work has to do with the tensions arising from surrounding stitches, which is very important when looking at knitting as a textile made of yarn [1, 2]. Some look at a comparison to “scaffolded surfaces” to explore the patterning features [4]. Very little work has been done to look at a single stitch’s actual curvature and features. There is one excellent example of this [5], which looks at the ideal shape. But it is defined in highly technical mathematics, making it inaccessible for designers and hobbyists to reference. It’s understandable why this gap may exist: a single knitting stitch can’t exist in textiles without its supporting neighbors. Yet it is important if we want to explore knitting with threadable media other than textiles, for example, through metal wire or semi-rigid wood veneer [3].

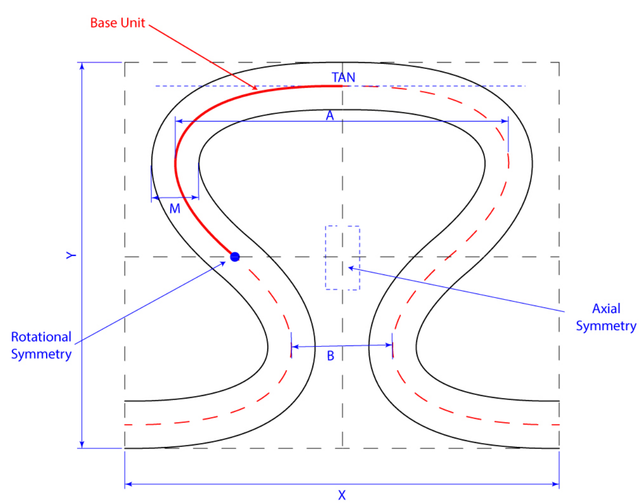


Figure 1: *The basic geometric considerations of a single stitch of knitting.*

Geometry of a Stitch

A single stitch has many simple geometric requirements, as detailed in Figure 1. The first thing that stands out to many is the bilateral symmetry. After looking more closely at inflection points, we can also see the rotational symmetry at certain points. There's some flexibility here. For example, the lower section could be a bit shorter or narrower, but since knit stitches look the same upside down, it's most accurate and efficient to keep the design symmetrical. Because of these two symmetries, we can now identify the base unit. This can be a parabola, a catenary curve, or other similar shapes. In this case, I am using a catenary curve. This curve must be oriented to be tangent so that the overall stitch shape is smooth when reflected, both where it meets for symmetry and where the adjoining stitches may be attached at the ends.

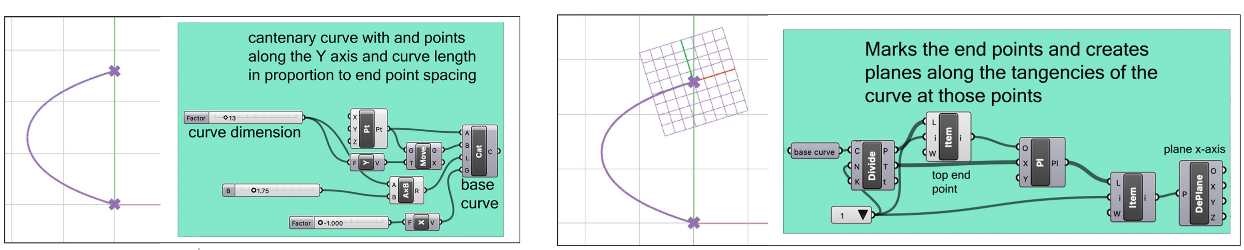
In this model we can define A as the maximal distance of the convex portion of the curve, B as the narrowest distance between the concave portions of the curve, and M as the material representing the material of the stitch. Given this, we can see some other basic requirements. Since A and B are measured to the center line, the larger opening of the stitch, shown at the top of the drawing equals $A - (2 \times 1/2M)$ (half of the material thickness on each side. Also, the outside of the smallest part of the stitch is $B + (2 \times 1/2M)$. The smallest part of the stitch must fit into the upper opening either exactly or with a bit of space. Therefore, $A \geq B + 2M$ and $X \geq A + M$.

Drawing in Grasshopper

While there are many tools we could use to visualize the stitch, I chose to use Grasshopper, a visual scripting plugin for the modeling program Rhinoceros. Once the stitch is drawn, there are excellent capabilities to tile, mapped onto different shapes, and algorithmically play with the outcomes. It also leads to easy outputs for laser cutting, 3D printing, and many other digital fabrication techniques.

Figures 2 through 4 display the Grasshopper command sections highlighted in green, with the output from each section shown to the left. First, I drew a catenary curve starting at (0,0) and moving up the Y-axis, as seen in Figure 2(a). The size depends on how large you want the model. I defined the curve by the endpoints and the length, where the length is proportional to the height of the curve. This proportion is slightly flexible depending on the width of the offset, but to keep our geometric validity, the range is between 1.5 and 2 times the height. I have, therefore, based these drawings on 1.75.

Next, in Figure 2(b), we need to establish the tangent angle at the top end of our curve. Dividing the curve into one segment allows us to mark the endpoints and construct planes where the X-axis coincides with a tangent line to the curve at that endpoint. I then chose the plane at the top of the curve and deconstructed it to get the X-axis of that plane.

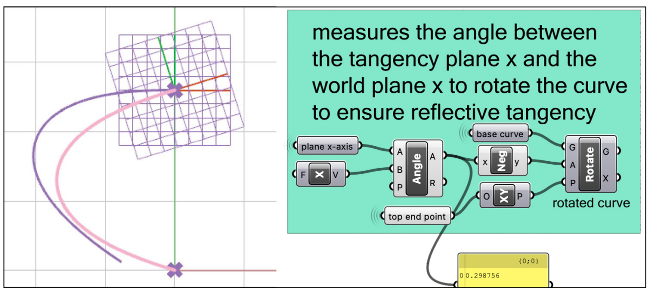


(a) **(b)**

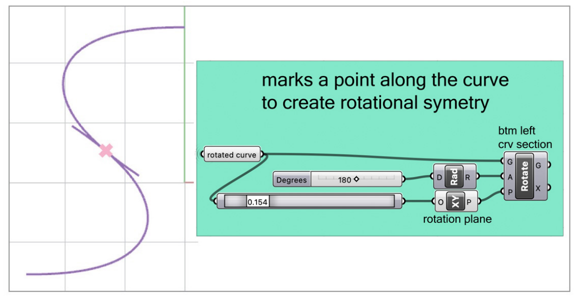
Figure 2: *A catenary curve is drawn (a), and a plane is created tangent to the end of the curve (b).*

By comparing the angle of our constructed plane's X-axis to the world X-axis, we can see the angle at which we must rotate the curve. Since we will rotate this curve clockwise, shown in Figure 3(a), we use that radian as a negative to bring our catenary curve to tangent with a horizontal line so that it will remain tangent upon axial reflection.

From there, we can begin our 180-degree rotational symmetry, as shown in Figure 3(b). If we do this at the end of the curve, the stitch becomes too narrow, so we will trim this end to keep the geometry valid by creating a point along the curve. In this case, the point is defined by a percentage of the curve. This is the second time we have a small range to work with, depending on the width of the offset. This ranges between .1 and .2, so I have drawn this at .15 for this paper.

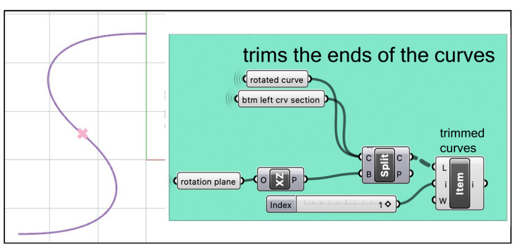


(a)

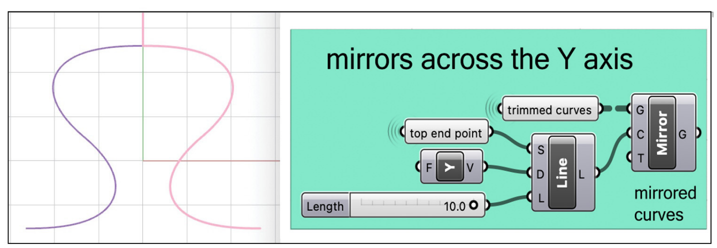


(b)

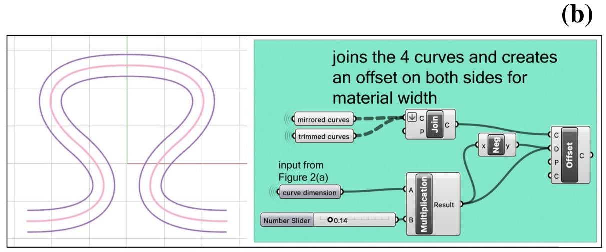
Figure 3: *The catenary curve is rotated to tangent (a), and is rotated 180-degrees from a point along the curve for rotational symmetry (b).*



(a)



(b)



(c)

Figure 4: *The curves are trimmed at the point of rotation (a) and both segments are mirrored across the Y-axis (b). The curves are joined and offset (c).*

Using this same point of rotation, I trim the curves so that they connect cleanly (Figure 4(a)) and mirror both segments across the Y-axis (Figure 4(b)). The four symmetric segments are joined into one smooth curve and offset to create a material width (Figure 4(c)). The material width is once again tied to the original size of the catenary curve so that it adjusts with the size of the model. This is the third and final time when a range could work. In this case, it should be greater than 0 for there to be any material at all and less than 20% of the original height to maintain the geometry.

Summary and Conclusions

With these simple geometric relationships established, we can have a greater understanding of stitch geometry without factors from the surrounding fabric, and also extend this knowledge to other materials. The stitches can be tiled, paneled, and mapped onto different shapes as rows and exported for digital fabrication. Figure 5 shows several possibilities of semi-rigid knitting with bamboo in different-shaped rows and knit together. These were laser cut as 12-15 in. rows. The scale of objects is flexible, and the shape of the pieces changes based on the shape of each row. However, the pieces can be flat or curved, folding or more rigid based on whether the rows are assembled from front to back as a purl stitch or from back to front as a knit stitch.

Figure 5: *Different shapes of geometric knitting stitches that are then knit together*

References

[1] K. F. Choi and T. Y. Lo. "The Shape and Dimensions of Plain Knitted Fabric: A Fabric Mechanical Model." Textile Research Journal, vol. 76, no. 10, 2006, pp. 777–786. https://doi.org/10.1177/0040517507069030.

[2] S. G. Markande and E. Matsumoto. "Knotty Knits are Tangles in Tori." Proceedings of Bridges 2020: Mathematics, Art, Music, Architecture, Education, Culture. C. Yackel, R. Bosch, E. Torrence, and K. Fenyvesi, Eds. Phoenix, Arizona: Tessellations Publishing, 2020. pp. 103–112. http://archive.bridgesmathart.org/2020/bridges2020-103.html.

[3] L. Marks. "Parametric Craft Techniques: Design Methodology for Building on Embodied Cultural Knowledge." Proceedings of Industrial Design Society of America International Conference 2017. 2017. https://www.idsa.org/education-paper/parametric-craft-techniques/.

[4] L. Niu, R. Rimerman, T. Baugh, C. Amanatides, R. Kamien, and G. Dion. "Knitting on Helicoid Scaffolds." Proceedings of Bridges 2024: Mathematics, Art, Music, Architecture, Culture. H. Verrill, K. Kattchee, S. L. Gould, and E. Torrence, Eds. Phoenix, Arizona: Tessellations Publishing, 2024. pp. 409–412. http://archive.bridgesmathart.org/2024/bridges2024-409.html.

[5] S. H. D. Semnani, M. Latifi and A. Jeddi. "A New Aspect of Geometrical and Physical Principles Applicable to the Estimation of Textile Structures: An Ideal Model for the Plain-knitted Loop." The Journal of The Textile Institute, vol. 94, no. 3-4, 2003, pp. 202–211. https://doi.org/10.1080/00405000308630609.