# Two Sculptures Based on Cyclides That Feature Inverted Villarceau Circles

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## Abstract

In previous work at Bridges 2015, I wrote about Dupin Cyclides and their generation by means of inverted Yvon-Villarceau circles, illustrating the idea with a 3D-printed model and a paper slice-form model. This paper goes further by presenting a wire sculpture and a cloth sculpture, each in the form of a cyclide of Dupin, each highlighting inverted Villarceau circles on the surface.


## What is a Dupin Cyclide, and a Yvon-Villarceau Circle ?

A Dupin cyclide is the image of a torus after inversion in a sphere (Figure 1). Inversions preserve circles, so the torus parallel and meridian circles on the torus become parallel and meridian circles on the Dupin cyclide (Figure 3). The Yvon-Villarceau circles (or shortly Villarceau circles) are another family of circles on a torus, obtained by cutting the torus along a bi-tangent plane. These Villarceau circles (the red circles on Figure 2) are transformed into circles on Dupin cyclides (red and blue circles on Figure 3).

**Figure 1:** *A virtual Dupin cyclide*

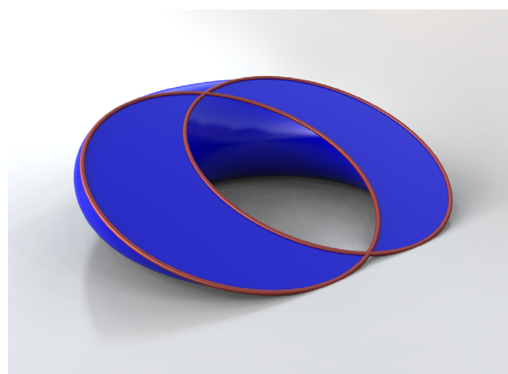

**Figure 2:** *Cutting a torus to create two Yvon-Villarceau circles.*

## Building Tangible Cyclides

After looking at virtual representations of cyclides, like the one in Figure 1, the temptation is strong to realize them physically, by one means or another.

The first possibility that I imagined was to use 3D printing since the equations characterizing the surface are known[2], it is thus easy to code them for printing[1]. I then created several models (Figure 7 and Figure 8 in [1]), by defining a wire frame version instead of a plain surface for several reasons (with the strongest being economic).

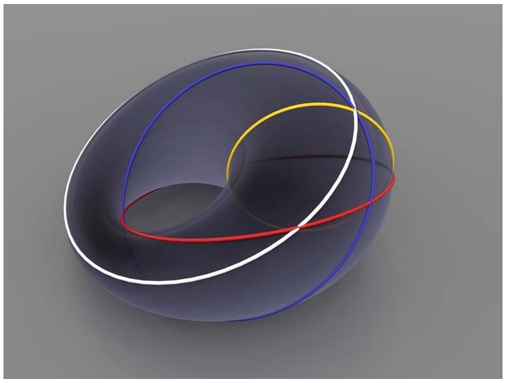

**Figure 3:** *Images of torus parallel, meridian and two Villarceau circles, inverted on a cyclid.*

**Figure 4:** *The set of 20 meridians.*

This approach was not totally satisfactory as the process remains strongly computer-aided and -oriented. I would prefer to be more manually involved in the construction of the object and confine the computer to the computations only.

The first possibility I imagined was a wire frame model made from steel, and the second a model made of woven paper or fabric. I will describe both versions (steel wire frame and woven fabric) in the following sections.

**Figure 5:** *The construction at an advanced step.*

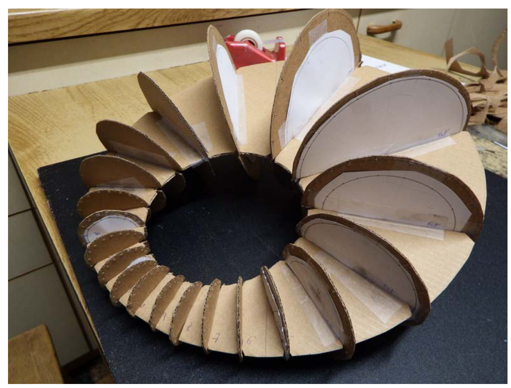

**Figure 6:** *The complete template.*

## Building a Wire Frame Model

Figure 8 in [1] shows what a wire frame model of cyclide could look like. Since this was an early realisation in my study of cyclides, it exhibits two aesthetic problems that could occur when one wants to manually construct one. The first problem is that there are too many circles : I would have to suppress some families. The second one is that the different families of circles seem to intersect at random places : it would be easier for the conception of the objects to have common points of intersections for all the four families of circles.

Figure 7 in [1] looks better, and was the base of my construction. The problem is now a problem for engineers : how to build it ? I decided to use meridians as scaffolding to maintain the cyclid shape during the construction (Figures 5, 6 and 7).

I had to carefully choose the meridians, so that opposite Villarceau circles meet on each meridian and in the middle of the space between two adjacent meridians (Figure 8).

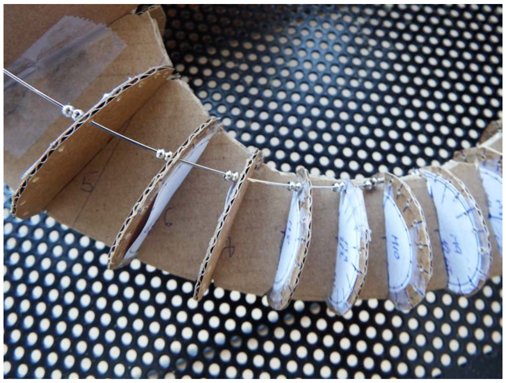

**Figure 7:** *A single Villarceau circle passing through its dedicated holes.*

**Figure 8:** *Intersections of Villarceau circles.*

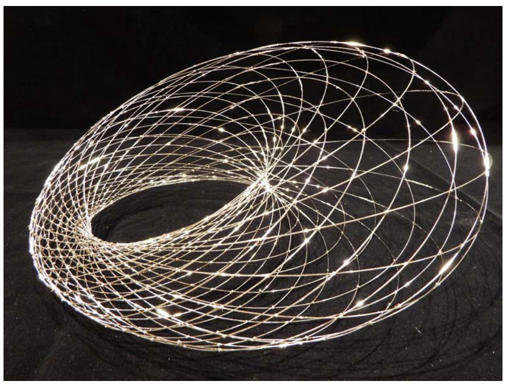

**Figure 9:** *The finished model (32x32x14cm)*

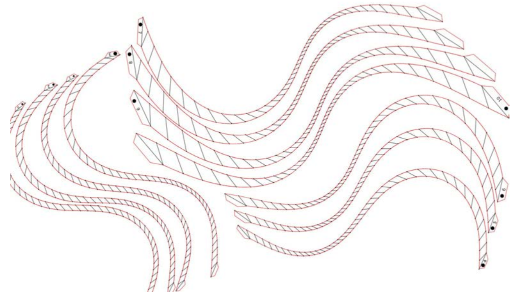

**Figure 10:** *A set of strips in one direction.*

I made a template in cardboard (Figure 6), figuring the main cyclide parallel and twenty meridians (Figure 4), with holes that allow the Villarceau circles to pass through. Beads were used to join crossing Villarceau circles, and tin solder their wires.

Figures 7, 8 and 5 show successive steps of construction, while figure 9 is the finished model, after the templates have been destroyed. When I show the finished model to mathematician François Apéry, who works with the *Maison des Mathématiques* in Institut Henri Poincaré, in Paris, he proposed to integrate it into the collections of the museum. This model is now part of the permanent exhibition (together with models that inspired Man Ray !).

## Building a Fabric Model

If we consider two adjacent Villarceau circles going in the same direction, we can imagine a surface connecting them. Surfaces going in different directions can be weaved. It would be nice if this surface would be developable. I am just a poor programmer, and not fluent enough in maths to compute the exact shape of those strips (which might not exist). However I can approximate it by splitting this surface in very small triangles, computing the shape of the next triangle, based on the previous one, and the distance (on the cyclide) between the new and previous vertices. Figure 10 shows a set of such strips, where the black lines indicate were to sew. Figures 11 and 14 show some steps of construction, and Figure 13 shows the finished model, filled with silk cotton. Figure 14 summarizes several monthes of trials and errors.

**Figure 11:** *Beginning to weave.*

**Figure 12:** *An advanced step of realization.*

**Figure 13:** *The finished model (28x28x13cm)*


**Figure 14:** *The way to success is not straightforward.*

Weaving the cyclide was quite a long work, but was made possible by the Covid lockdown. The fabric can be replaced with paper, but I found it very difficult to make a clean job at the tiny part of the cyclide. Fabric is also more flexible, and can correct the approximations occuring either from the computations of the strips, or my lack of sewing skills.

## Conclusion

Computing beautiful mathematical shapes is a rewarding activity. Making 3D models of those shapes is also very satisfying, since you can handle them, and see them from all points of view, maybe finding new properties or new ideas.

But constructing models without the help of machines, or at least limiting their role to their conception, make you feel like an engineer confronted with new problems, and having to find your own way to solve them. The feeling of being the first to try to solve them is also a very thrilling experience.

## References

[1] F. De Comité. "Yvon-Villarceau Circle Equivalents on Dupin Cyclides." *Bridges Conference Proceedings*. Baltimore, July 29-August 1st 2015. pp. 253–258. http://archive.bridgesmathart.org/2015/bridges2015-253.html.

[2] L. Garnier, H. Barki, S. Foufou, and L. Puech. "Computation of Yvon-Villarceau Circles on Dupin Cyclides and Construction of Circular Edge Right Triangles on Tori and Dupin Cyclides." *Computers & Mathematics with Applications*, vol. 68, no. 12, 2014, pp. 1689–1709. http://dx.doi.org/10.1016/j.camwa.2014.10.020.