Mirrored Image Montages

Vincent Schumacher

Olympia, Washington, USA; vs3.14159@gmail.com

Abstract

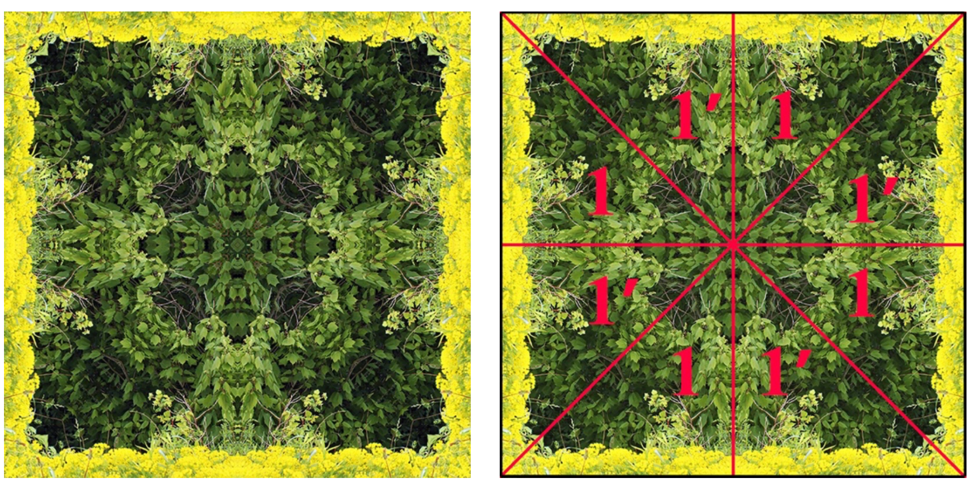
This paper describes a technique for filling a plane with geometric tiles that have no gaps or overlaps; an important feature of these tiles is that each one has an image on it, and the tiles must be connected *seamlessly*. That is, for each pair of adjacent tiles, the edge between them is a line of symmetry for the image formed by those two tiles. To make such an image, multiple copies of a single tile and its reflection can be connected. The artistic purpose of this technique is to ensure that the tiles will form a whole image with no obvious visual breaks between tiles. The mathematical result is that when tiles are connected seamlessly, they may or may not correspond to tessellations, and some tessellations cannot be used as a template for a seamless image.

Introduction

Although I was trained as a math teacher and computer programmer, my interest in the topic of this paper came from photography; I wanted to create a seamless image from a piece of a photograph. In a typical tiled pattern, the edges of the polygons are obvious, but I wanted the edges to be invisible so that the pattern of the image itself would be the focus. From this starting point, I had to assemble the images on the tiles to cover the plane *seamlessly*. This requires mirroring the original tile and connecting it to its mirrored copies. Therefore, the rules for creating such images are different from the rules for creating tessellations.

Mirrored Image Montages

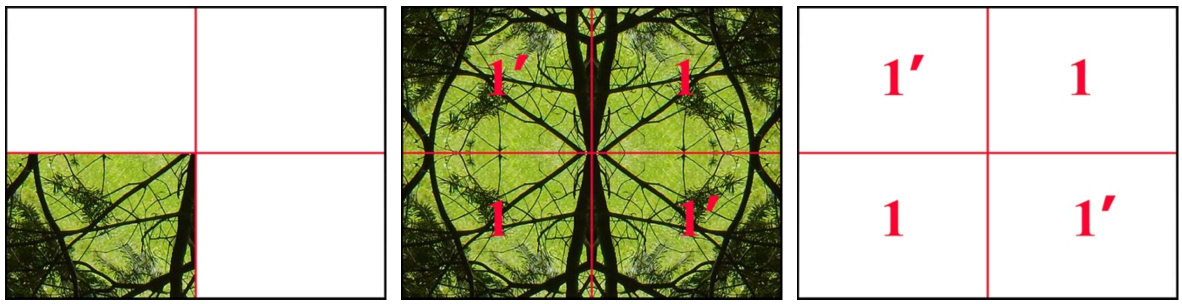
A mirrored image montage (MIM) is a plane surface filled with polygons with no gaps or overlaps, where the face of each polygon contains an image and all images connect seamlessly, as in Figure 1. No assumptions can be made about the image on a tile, including that it has any symmetries. Therefore, to guarantee that the connections are seamless, each polygon—or part (sub-selection) of a polygon—must be connected to a copy of itself that is mirrored across the common edge.



(a) (b)

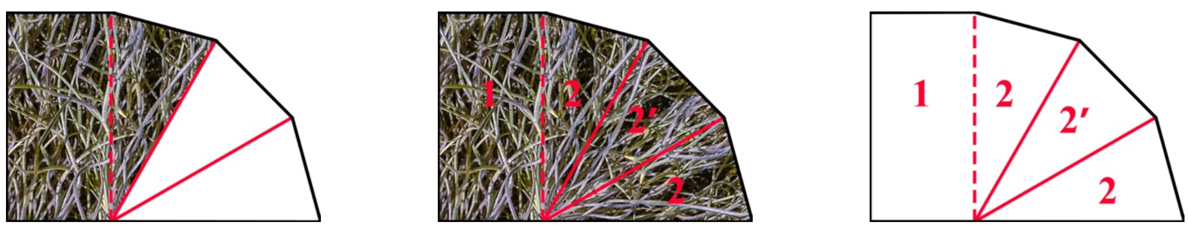
Figure 1: *(a) Sample mirrored image montage (MIM). The tiles are not obvious, and the images on them create the patterns. (b) A template view of the MIM showing how it is constructed. The solid red lines are the edges of reflection. Each image is numbered (1), and a tick mark (1') indicates the reflected image.*

All MIMs start with a single tile. Additional tiles are created by reflecting that tile across one of its edges, as in Figure 2 below, or by cropping it first to create a sub-selection, a tile with a different shape, as shown in Figure 3. The tiles are then connected seamlessly to the appropriate reflected tiles.



(a) (b) (c)

Figure 2: *(a) A starting tile in a skeleton view. (b) The competed MIM labeled with construction details. (c) The template view of the MIM. Applying this template with any image as the starting tile 1 will create a new MIM.*

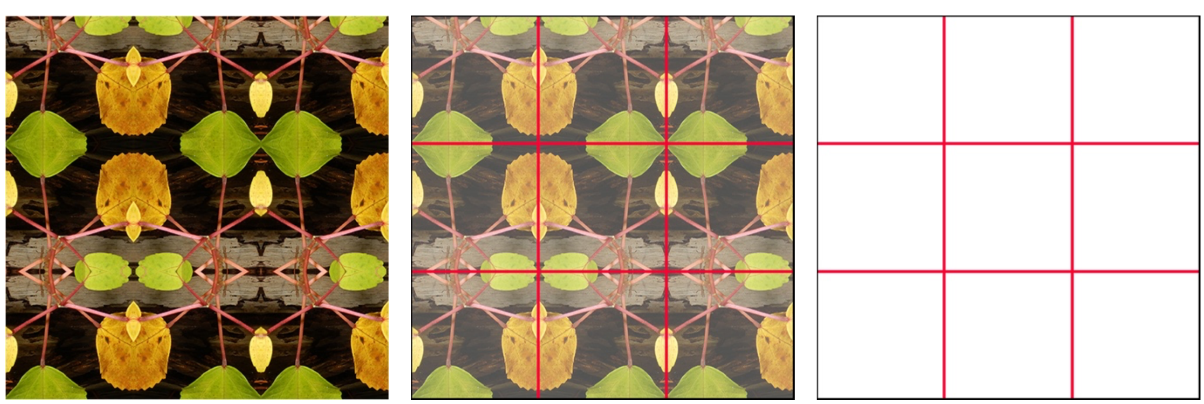


(a) (b) (c)

Figure 3: *(a) The starting tile. The dashed line shows where it will be cropped to create a sub-tile (2) that will be mirrored. (b) The completed MIM that shows how the new tiles (2 and 2') are mirrored. (c) The template view that can be used for future constructions.*

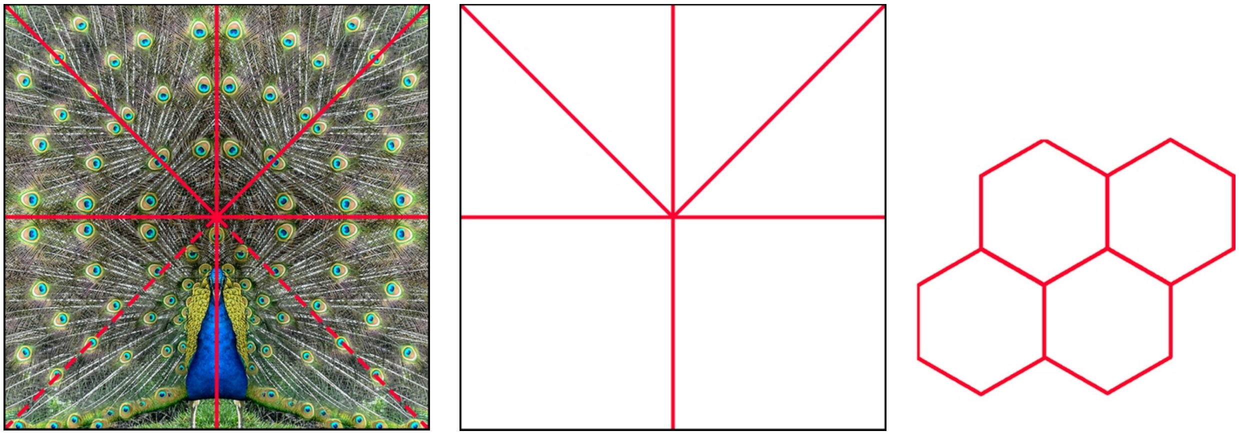
MIMs vs. Tessellations

Some MIM skeletons match up with regular or semiregular tessellations, as in Figure 4. But some MIMs do not form tessellations (Figure 5(a) and (b)), and some tessellations are not MIM skeletons (Figure 5(c)).



(a) (b) (c)

Figure 4: *(a) A completed MIM. (b) A faded image view of the MIM with its tiled edges shown as red lines. (c) The MIM skeleton, which is also a common tessellation pattern.*



(a) (b) (c)

Figure 5: *(a) A template view of a MIM. (b) The MIM skeleton, which is not a tessellation: even if it were continued indefinitely, not all of its vertices would connect the same polygons in the same order. (c) A tessellation that cannot form a MIM. For details, see the extended example in the next section.*

Extended Example: Hexagons

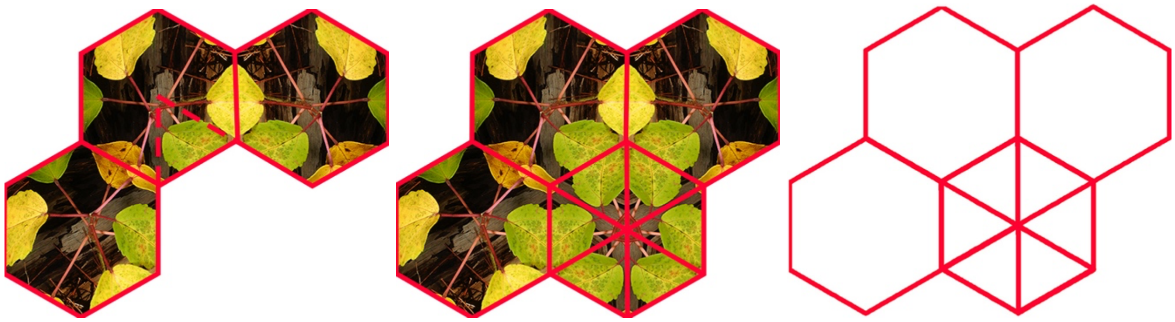
This example is given to clarify why some tessellations cannot be the basis for a MIM. As the MIM is being constructed, it becomes apparent that there is no way to reflect a particular polygon in the tessellation so that its edges align seamlessly with all its neighbors. In the example below, we try to use the hexagonal tiling in Figure 5(c) as the basis of a MIM to see why it is not possible.

(a) (b)

Figure 6: *(a) The middle hexagon can be reflected diagonally down to create a seamlessly connected tile (1') and reflected to its right to continue the pattern along that edge. (b) However, there is no way to reflect, translate, or rotate a fourth hexagon to form seamless connections with all its neighbors.*

In Figure 6(b), the fourth hexagon shares edges with both mirrored and unmirrored hexagons. Each of its edges can match one or the other, but not both. This is because each of its vertices connects an odd number of hexagons. When the tiles in a MIM surround a vertex, the vertex must have an even valency so that the edges that connect at the vertex can remain seamless. Notice that Figure 6 shows the only construction in this paper with a vertex of odd valency.

Sometimes this problem can be fixed by cropping an appropriate polygon out of one of the tiles and mirroring this new polygon to match the edges of the other tiles. This is done in Figure 7 by taking a triangular sub-selection of the starting hexagon. It is first mirrored across the appropriate edge, then is mirrored around itself to form a complete hexagon. This succeeds in making the MIM seamless (and note that all the relevant vertices are of even valency), but it is no longer built of only hexagons.



(a) (b) (c)

Figure 7: *(a) The triangular sub-selection creates a new tile. (b) This new tile is reflected to match the edges of all the surrounding hexagons. This new tile also ensures that each vertex in the montage has an even valency. (c) The resulting MIM skeleton is made of hexagons and triangles, not hexagons alone.*

An Unlimited Variety

Although this paper concentrates on only a few types of MIMs, the possibilities for creating them are limitless. Inspiration for new templates comes from mathematics, imagination, art, and observations of the patterns all around us. Figure 8 shows some MIMs created by using different techniques for filling a plane.

(a) (b) (c)

Figure 8: *Some of the many MIM possibilities. (a) A combination of square and triangular tiles. (b) A tile divided into connecting triangular sub-selections mirrored in opposite directions; the result is flipped horizontally. (c) A series of ever smaller rectangular sub-selections mirrored outward from the center.*

Summary and Conclusions

MIM rules provide a new way of filling a plane with geometric shapes, as well as a way of generating an endless variety of mathematically interesting and visually pleasing pictures.

Acknowledgements

Thanks to Melissa Schumacher, who provided invaluable editing, supportive feedback, and encouragement.