All You Need is Math… to Write a "Hit" Song!

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Abstract

The authors report on a 2023 collaboration in which they applied their considerable contemporary songwriting expertise towards creating new music in popular song form that stands alone as a "regular song" but also incorporates mathematics in several aspects of its lyrics and music.

Introduction

Everyone is comfortable with the idea that music, from classical to jazz to rock to pop and everything in between, is an art form. And yet, the question is, what tools can accentuate the aesthetics? The emotion? The prosody—the connection between the lyrics and the music? While musicians like to claim that *feeling* is enough, it clearly isn't; if it were, having a good cry or being spitting angry while you perform would be sufficient!

Mathematics is indeed the mechanism (and perhaps *the* mechanism) to add aesthetics and emotion to music. George Martin, The Beatles' brilliant producer, devotes a whole chapter in his autobiography *All You Need Is Ears* [10] to the mathematics and physics of sound and music. Jimmy Webb, who wrote many of the pop hits of the 1960s and 1970s such as "By the Time I Get to Phoenix," "Up, Up and Away," and "MacArthur Park," refers to mathematics in his explanation of melody and harmony in song writing no fewer than 14 times in his tome *Tunesmith* [14]. Max Martin, the most successful music producer over the last 20 years (having worked with artists such as Britney Spears, Justin Timberlake, Katy Perry and Taylor Swift, to name a few), refers to what he calls "Melodic Math" in his approach to writing and producing hit songs. Here are a few math-referencing quotations from other established recording artists:

"If you listen to the great Beatles records, the earliest ones, where the lyrics are incredibly simple. Why are they still beautiful? Well, they're beautifully sung, beautifully played, and the mathematics in them is elegant."

– Bruce Springsteen, in the January 2009 *The Observer*.

"Each time I've learned a little bit more about the inner structures of music, the math of it and the shape of it, my joy in it has increased."

—Peter Mulvey, in the November 2000 *Performing Songwriter*

"Music is *true*. An octave is a mathematical reality. So is a 5th. So is a major 7th chord. And I have the feeling that these have emotional meanings to us, not only because we're *taught* that a major 7th is warm and fuzzy and a diminished is sort of threatening and dark, but also because they actually do have these meanings. It's almost like it's a language that's not a matter of our choosing. It's a *truth*. The laws of physics apply to music, and music follows that. So it really lifts us out of this subjective, opinionated human position and drops us into the cosmic picture just like that."

—James Taylor, in the May 2002 *Performing Songwriter*

There are many songwriting books that take an approach that stresses patterns and ways to repeat and modify them in compelling ways—often what we mathematicians would call *symmetries* and *transformations.* For example, *Great Songwriting Techniques* [11] describes the types of operations that one might preform on a melodic sequence, and a mathematician can look at many of these and see familiar transformations, such as translations, inversions, compressions, extensions/truncations and so on (see, for example, [1] and [13]). Also, see the books referenced at the end of the first paragraph on page 66 of [12].

We have each been songwriters for several decades, releasing albums (Brown's *Songs in the Key of Pi*; Lesser's *Afterglow* and *Sparks*), as well as articles on mathematics and music/song [2, 3, 4, 5, 6, 7, 8, 9, 12]. We assigned ourselves a very specific project—to write a pop song, inspired by and based on mathematics, but with the proviso that the song should stand alone as a pop song that could be appreciated without being conscious of any of the underlying mathematics that went into the process. The result was the song "One More Part of Me" (music and lyrics © 2023 Brown and Lesser); a recording can be found at https://mathstat.dal.ca/~brown/sound/brownandlesser/, as well as lyrics and a lead sheet (that is, melody, chords and lyrics). The structure of the song is shown in Table 1. We describe the process and the mathematically based choices we made (in addition to the choices made for musical reasons).

Table 1: The Structure of the Song "One More Part of Me" (Brown/Lesser)**.**

Song building block	Lyrics (line numbers)	Bars
Intro	--	1–8
Verse 1	1–4	9–24
Prechorus	5	25–26
Chorus	6–9	27–44
Verse 2	10–13	9–24
Prechorus	14	25–26
Chorus	15–18	27–42
Bridge	19–21	45–58
Verse 3	22–25	59–74
Prechorus	26	75–76
Ending choruses	27–36	77–112

Lyrics

We began with a discussion of what mathematical concepts might form a basis for the lyrical content of the song. After some discussion, we centered on fractals. The construction of the Cantor ternary set on [0,1] involves repeatedly removing the middle thirds of the remaining intervals, until only dust is left. We thought that this could be an analogy for a relationship in which the singer bemoans repeatedly losing part of his identity until he feels there is almost nothing left! The analogy seemed strong enough (and distinctive) as the storyline. The first verse sets the stage.

Verse 1: They say it takes compromise in matters of the heart
There must be give and take, so I played my part
At every fork in the road, I let you decide
You're always at the wheel, I'm just along for the ride

While not overtly mathematical, we did utilize words and phrases like "part", "fork in the road" as part of the construct. Still, we ensured that the storyline played the pivotal role in the decision making.

The prechorus is the small section that connects a verse to the chorus.

Prechorus: What have I given up?

This is again a reference to the loss in the next stage of the construction of the Cantor ternary set, but seemed apropos in terms of the singer's distress at giving up a part of himself.

Chorus: Losing one more part of me
How long can I go on before there's nothing left?
Losing one more part of me
How long can I go on before there's nothing left, nothing left

The chorus states the main theme of the song and is the lament of the singer (as well of the Cantor ternary set!). We decided early that the storyline of the song would have an arc, with movement in the emotional content. Verse 2 tells us more about what the protagonist has given up for love:

Verse 2: I've given up my friends, my hobbies and my faith
How much can I give up, before I start to break?
I've worshipped at your altar, offered up myself
But the spirit's left, now I'm someone else

Again, there are passing references to the Cantor ternary set, especially in line 2, as both the intervals and narrator are even more broken up. After another chorus, we proceed onto the bridge, which is a section that typically has new music and something new to say.

Bridge: Rain pours down as I stuff my bags
With all I can take from this place,
'Til a flash of lightning reveals your face

The fractal lightning is the mode by which a stark change occurs in the mindset of the singer. He realizes that perhaps he has been too self-centered, too certain that his view point is the only truth. An epiphany ensues, as described in the last verse, prechorus, and ending choruses:

Verse 3: Now I reflect when I see your tears.
I unpack my bags and unpack my fears
Standing up for myself can still leave room for you.
At a bigger scale, there's a higher truth.
Prechorus: Do we have to give up?

Ending Choruses: Losing one more part of me
I went on far too long, there was almost nothing left
Leaving, we can both agree,
Is just an easy answer, never puts us to the test

Losing one more part of me
Is not the only lonely road that I can't take
Leaving can't be our destiny,
There has to be a way that isn't a mistake
Where we don't have to fake,
With everything at stake …

This third verse, in addition to a re-evaluation of the narrator's assumptions, introduces some mathematical terms "reflect," "bigger scale," and "truth" that do connect to fractals. The prechorus has undergone a transformation as well—instead of "What have I given up?" it has now morphed into "Do we have to give up?" The last choruses are variations of the original, but with more nuance— just like real life!

Music

Mathematics affords the opportunity to build and break expectations (with the ensuing emotional effects) and inject consistency or variability into the patterns. As most rock/pop songs. The song has 4/4 time signature, so each measure (or bar) of music has four beats. The song starts out on a variation of a standard chord progression in the key of A minor: Ami, C, G, F, each a bar long (see Figure 1), that underlies the verses as well (there is a G/B chord as well, but it is rather incidental as a passing chord). We chose a minor key for the verses to emphasize the sad nature of the lyrics. (For those of you wishing to play along, the notes in the chords are as follows: Ami: A,C,E; C: C,E,G; G: G,B,D; Fadd9: F,A,C,G; G#˚7: G#,B,D,F; Emi7: E,G,B,D; D: D,F#,A; E: E,G#,B.)

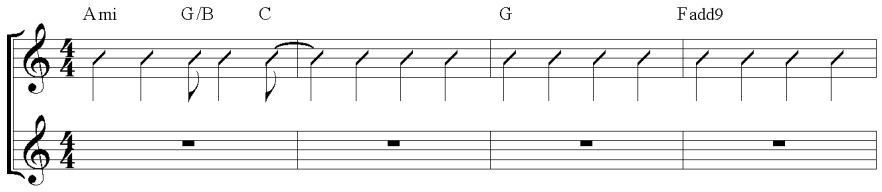


Figure 1: *Intro.*

The tempo of chord changes decreases in the prechorus (see Figure 2), where there is just one chord over two bars:

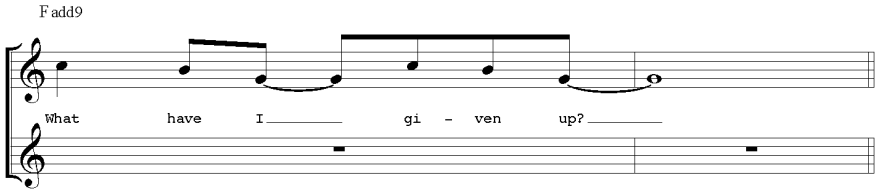


Figure 2: *Prechorus.*

This slowdown of the harmonic pace continues throughout the chorus (see Figure 3), where we chose to use the same set of chords, but in an alternate permutation: C Ami, G, F, effecting a key change into the more "hopeful" C major from the A minor of the verses. The choruses each have the standard length of 16

bars, except for the first which is extended to 18 bars long—the unusual length of this first chorus has the effect of propelling the song into the second verse.

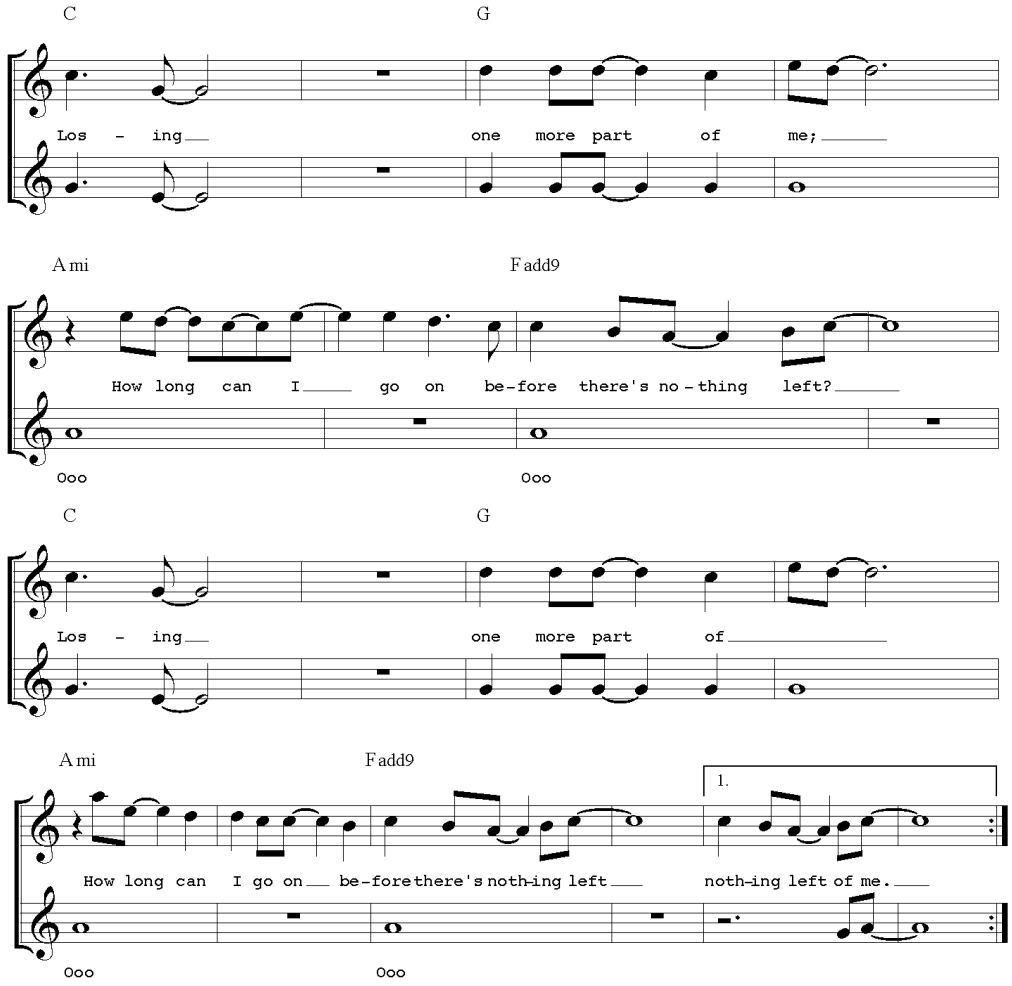


Figure 3: *Chorus.*

The choruses also introduce a vocal harmony that runs in some places parallel (moving in the same direction, essentially a translation of the lead vocal) and in other parts obliquely (on a constant pitch that differs from the main vocal).

The bridge is of unusual, unstable length—13 bars (see Figure 4). The final line of the bridge crescendos up to the song's climax as the chords follow a rising bass line. There is a large melodic jump of a major sixth on "lightning," and the revelation of the "face" leads to a protracted silence of two bars, leading into the last verse (see Figure 5).

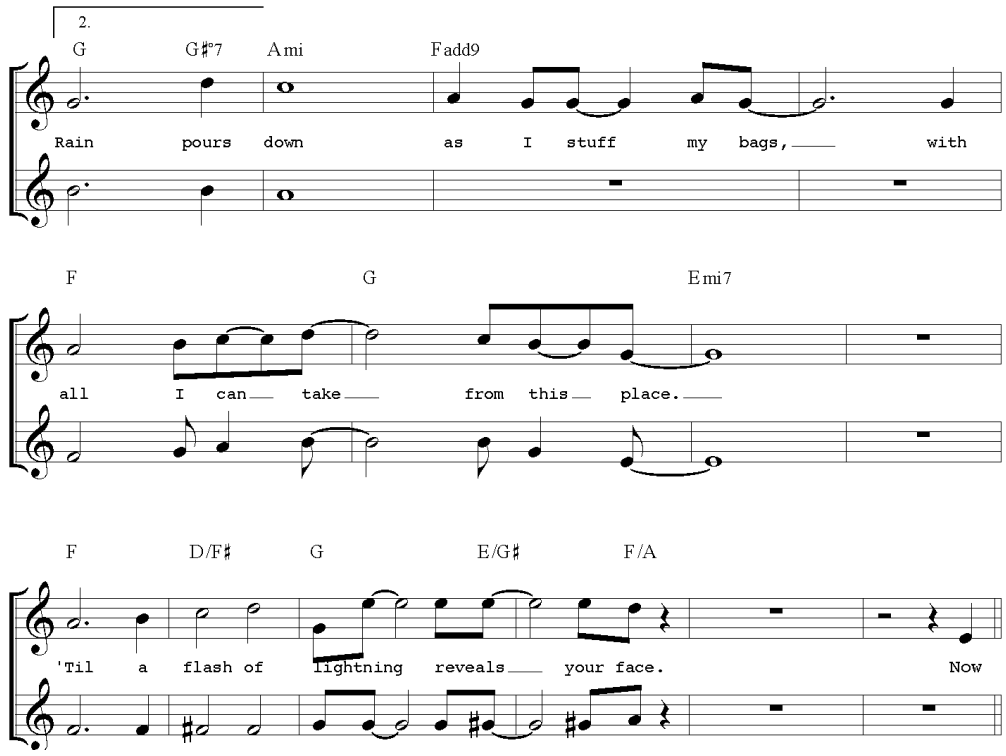
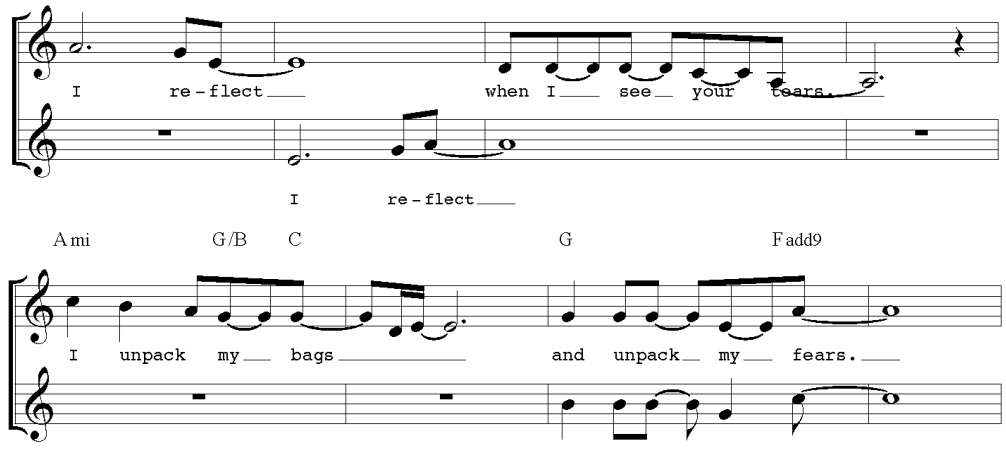


Figure 4: *Bridge.*

Here we explicitly utilized some mathematical transformations in the backup vocals. The first of these is an echoing of parts of the first and third melodies with a vocal reflection of the tones in reverse order (this adds to the prosody, as the reverse order indicates that the singer is reflecting on and re-evaluating the past). In the final line, "At a bigger scale, there's a higher truth," the harmonizing voice sings a line that runs at a slower pace (which highlights more deliberation).



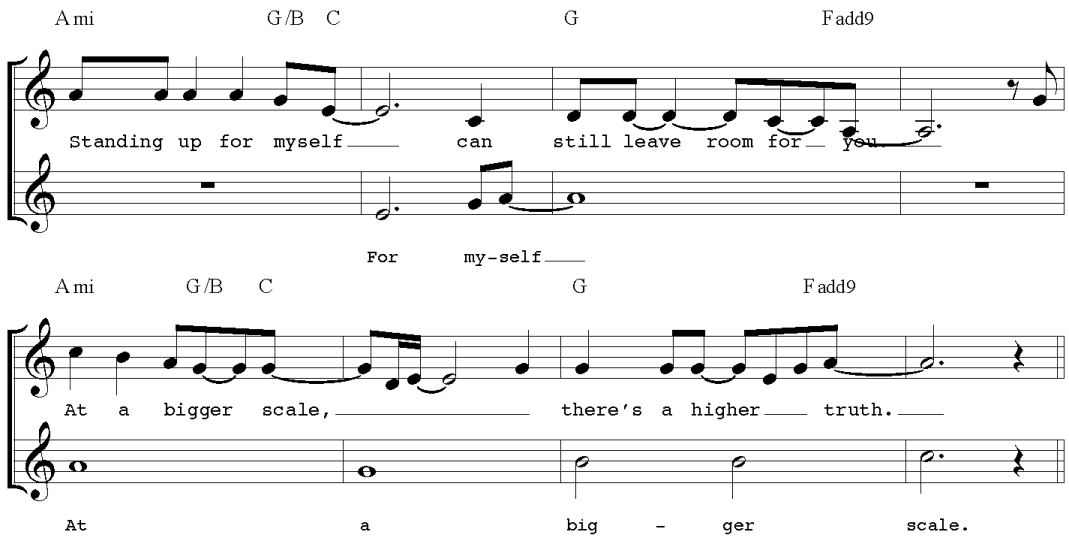


Figure 5: *Verse 3.*

The choruses (see Figure 6) end with elements of a canon, where one voice follows the other, offset in time (examples include the classic "Row, Row, Row Your Boat" and the ending of The Beatles' "Good Day Sunshine"). This technique adds to the prosody of the singer and his significant other chasing after one another in the frantic hope of saving the relationship:

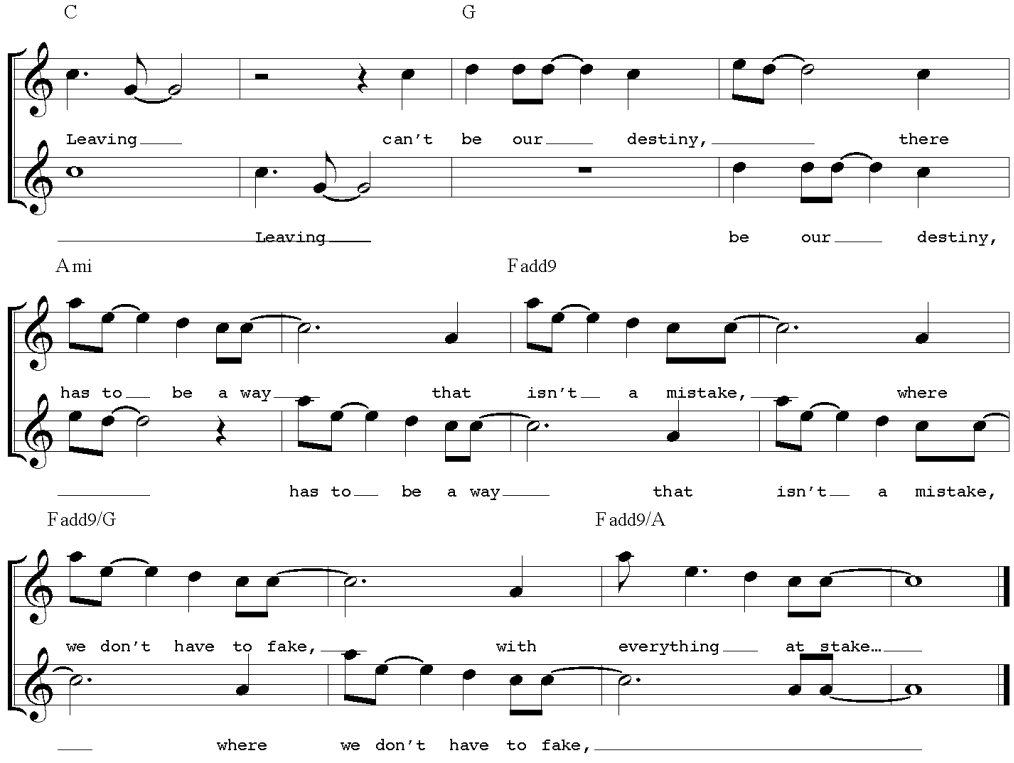


Figure 6: *Ending.*

Summary and Conclusions

Overall, we were happy with the results. Playing the song for various songwriting groups, we received appreciation for both the music and the lyrics (and the storyline). Moreover, the musical but non-mathematical audience were unaware that there was any mathematics in the background, and surprised to hear how mathematics entered the process. Patterns and their transformations in music are perceived implicitly by all listeners. As the famous German mathematician Gottfried Wilhelm von Leibniz (1646-1716) wrote, "Music is the pleasure the human soul experiences from counting without being aware that it is counting." Finding inspiration for songwriting can be hard, but we found that mathematical concepts (like fractals and the Cantor ternary set) could serve as useful metaphors, even for a relationship story. And the explicit consideration of mathematical ideas like transformations could add to the interest and the prosody. We look forward to writing more pop songs with a similar sensibility or approach. Yeah, yeah, yeah!

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