So-Soo-Yoo: A Game of Strategy and Chance on the Number Line

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Abstract

So-Soo-Yoo is a board game based on a number line of integers with each number represented by its prime factors. Optimum movement in the game is analyzed through reachability sets. We discuss how the game can be used to explore multiplication tables, powers, factors and prime numbers. We suggest So-Soo-Yoo could be a useful tool for teachers or students interested in reinforcing these concepts.

So-Soo-Yoo

So-Soo-Yoo, which is Japanese for “prime factor fun,” is a board game created by one of the authors (Jay Dearien) . Figure 1 shows a schematic representation of the first 24 integers using the notation of the game and Figure 2 shows the actual game board with the number line curled around in a spiral [1]. The board consists of the number line from one to 199 with circles of distinct colors and patterns for each prime between 2 and 97 and white squares for each non-prime number. Prime numbers above 100 do not have shading since they do not appear as factors elsewhere on the board. Attached to each non-prime number are the number’s prime factors, drawn as miniature versions (“bubbles”) of the prime circles. The number line extends to 199 since in US schools children learn the multiplication tables up to 144 (12 x 12) and 199 worked well for the dice system in the game. The goal of the game is to be the first to collect a complete set of randomly distributed colored rings, which are replaced each time one is collected.

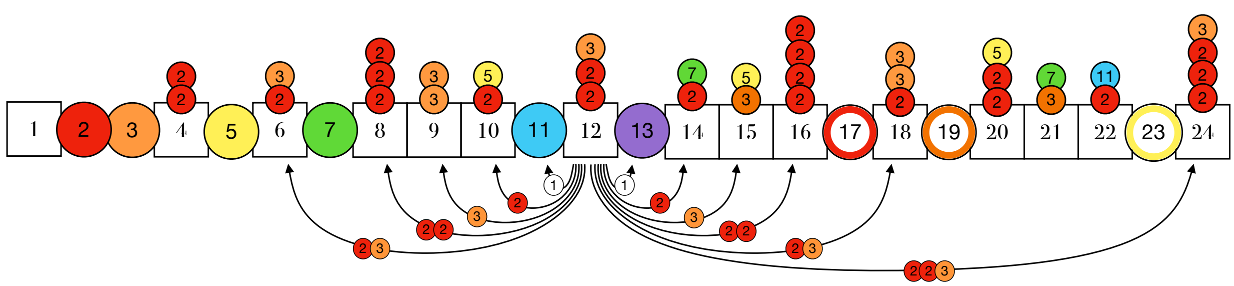

Figure 1: *So-Soo-Yoo number line. Prime numbers are drawn in circles and non-primes in squares. Each composite number has “bubbles” attached that show the number’s prime factors. Beneath the number line is a representation of all the possible moves from the 12 space.*

A turn in So-Soo-Yoo consists of making one to six moves based on a 6-sided die roll. The number of spaces covered by a given move is determined by the factors of the number where the player’s piece sits. For example, if the player is on the number 12, they could move up or down the number line by any of its factors, i.e. 1, 2, 3, 4, 6, or 12 (see Figure 1). Rather than counting, So-Soo-Yoo provides a shortcut. One looks at the prime factors of the given number and then scans along the number line for the next occurrence of that factor. For example, to move up by the prime factor 3, one scans for the next (orange) 3 bubble, which is found on 15. In So-Soo-Yoo parlance, moving by a prime factor is referred to as a jump. One can move by composite (non-prime) factors by scanning for the next *combination* of prime factor bubbles. For example, to move by 6 one looks for the next number that has both a 2 and a 3 prime factor (a red bubble and an orange bubble respectively), which occurs at 18. This move is called a “warp.” This shortcut allows the game to be played visually by young players without math training. One is not permitted to move beyond the ends of the So-Soo-Yoo number line. In other words, one must not drop below 1 or move above 199.

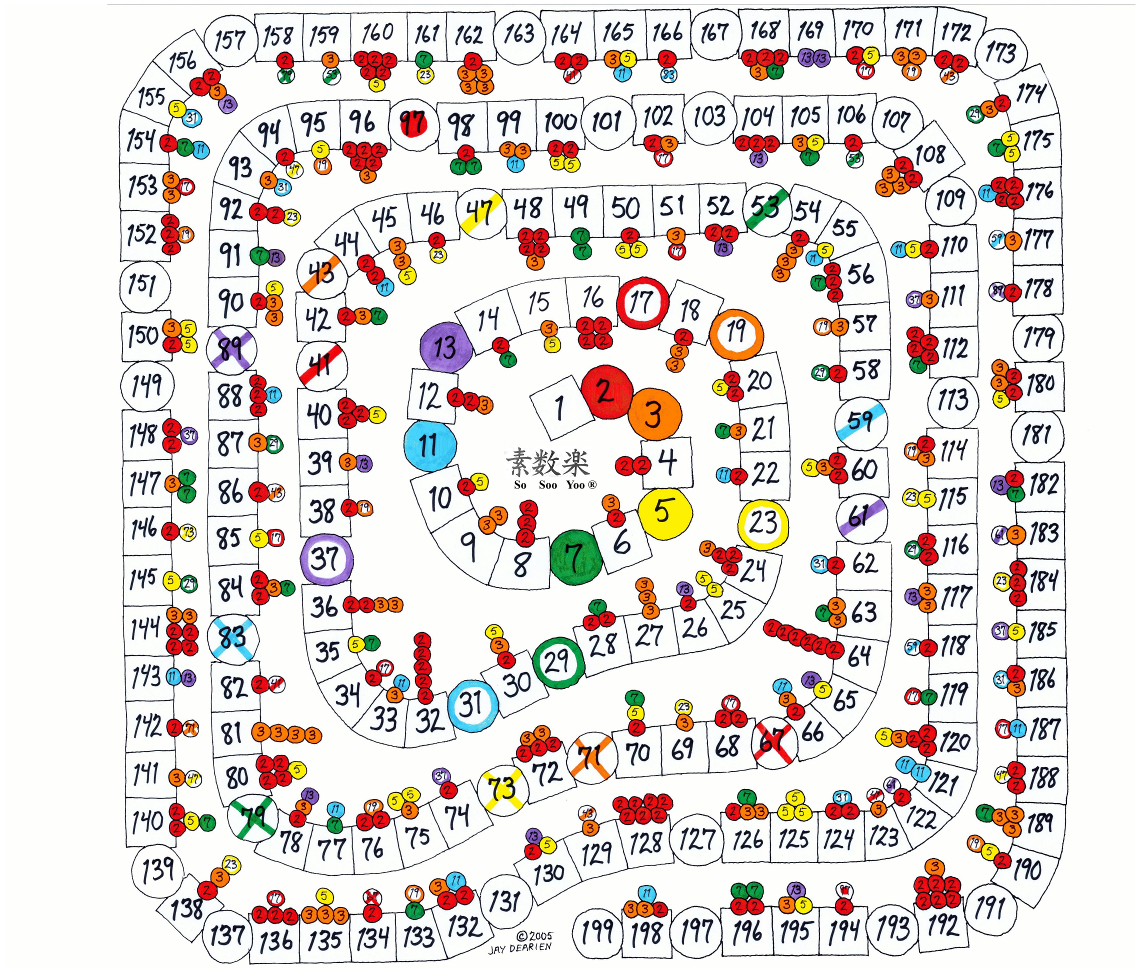

Figure 2: *The So-Soo-Yoo board.*

So-Soo-Yoo Moves and the Multiplication Tables

The So-Soo-Yoo movement rule described above may be thought of as moving up or down the columns of a multiplication table. One may also “switch columns” of such a multiplication table during one’s turn. For example, in Figure 3, we start at five and use the 5 multiplication table to jump to 10, then use the 10 multiplication table to warp to 20, and then the 4 multiplication table to warp to 24. Any legal move in So-Soo-Yoo is a move up or down consecutive elements in some multiplication table column.

Reachability Sets

Skillful So-Soo-Yoo play consists of finding the minimum number of moves to navigate the board, either to collect rings or “bump” other players, causing them to lose a ring. One measure of how easy or difficult it

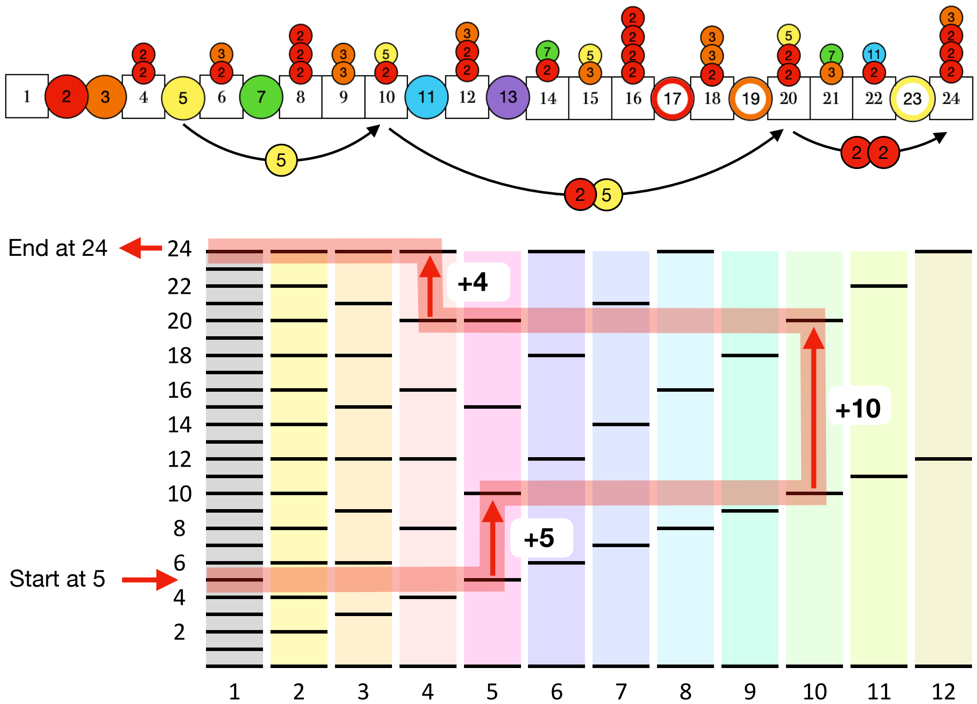

Figure 3: *(a) Example sequence of three moves to go from the 5 space to 24. (b) Representation of the move in part (a) by thinking of the moves as moving between adjacent rungs on a sequence of multiplication "ladders."*

is to choose an optimal series of moves is to compute the reachability set for a given starting number and number of moves. We define the 1st order reachability set $R_1(n)$ for a starting number n as all the numbers that may be reached from it using a single So-Soo-Yoo move. The 2nd order set $R_2(n)$ is all those numbers that may be reached in two moves, and so on. A turn of one to six moves is a path from the first reachability set up through as high as the sixth.

The first-order reachability set ($q = 1$) is

$$R_1(n) = \{s \in \mathbb{Z} : |s - n| \in F(n)\},$$

where $F(n)$ are the factors of n. Example first-order reachability sets are $R_1(1) = \{2\}$, $R_1(2) = \{1, 3, 4\}$ and $R_1(3) = \{2, 4, 6\}$. The first-order reachability set for the numbers on the So-Soo-Yoo board are colored orange in Figure 4 which displays the minimum number of moves required to connect two numbers.

The q-th order reachability set $R_q(n)$ is defined recursively as

$$R_q(n) = \{s \in \mathbb{Z} : |s - R_{q-1}(n))| \in F(R_{q-1}(n))\}.$$

Figure 4 color-codes each pair of starting and ending numbers by the lowest reachability order (i.e. fewest number of steps needed to connect the pair) less than or equal to 6. As the reachability order increases, more pairs of numbers are connected by So-Soo-Yoo moves. While only 4.5% of the number pairs are connected by a single move (reachability order = 1), this percentage increases to 27%, 61%, 83%, 93% and 97% for 2, 3, 4, 5 and 6 moves respectively. Numbers at the lower end of the number line are typically harder to reach due to their reduced number of factors. Numbers that have the most factors can act as "superhubs" that allow the most flexibility. The top three superhubs are 180 (18 factors) and 120 and 168 (each with 16 factors each).

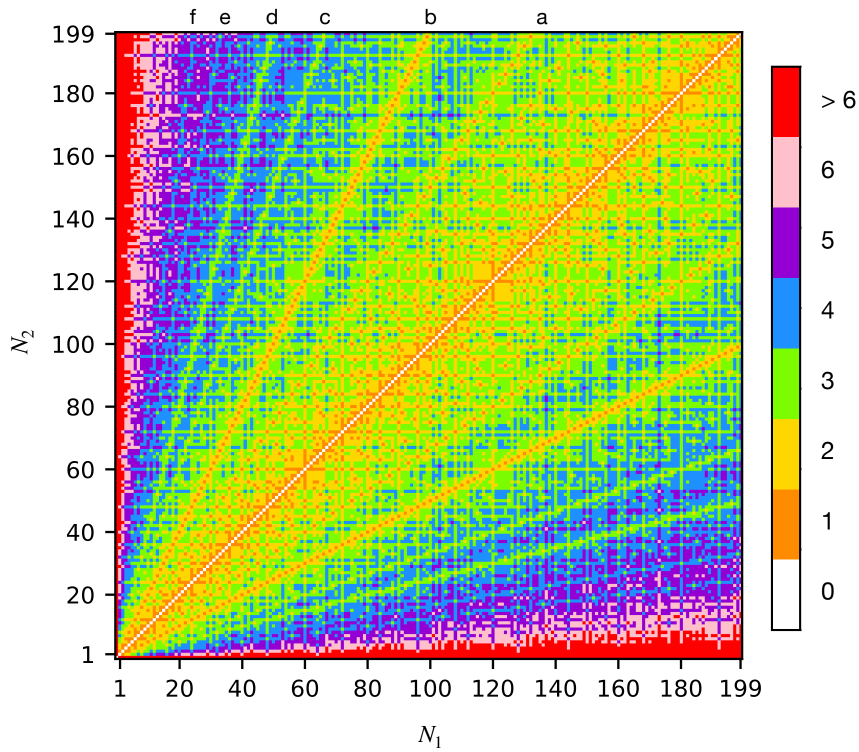

Figure 4: *The minimum number of moves needed to connect two numbers N_1 and N_2. The "rays" seen in the figure labeled a-e are described in the text.*

The prominent "rays" in Figure 4 show pairs of distant numbers that are connected by relatively few moves. For example, the number pairs in the ray labeled b satisfy $N_2 = 2N_1$. This is the upper limit of the N_2 values reachable from a given N_1 value with a single move. The next largest number reachable from N_1 is $N_2 = N_1 + N_1/2$ assuming N_1 has 2 as a prime factor. More such rays with smaller $(N_2 - N_1)/N_1$ values blur together as one approaches the diagonal where $N_2 = N_1$. The rays marked c and d show the two largest values of N_2 reachable in two moves and e and f show the largest values of N_2 reached in three moves. The figure is symmetric around the diagonal so similar arguments apply for moving down the number line.

Summary and Conclusions

Playing So-Soo-Yoo is rehearsing the multiplication tables (and other math concepts, such as primes, factors, powers), sustained by a system of visual rules and driven by other players. We have found that both children and adults can quickly and easily become skilled at So-Soo-Yoo, which is to say make optimal series of moves to collect the maximum number of rings (or "bump" other players), even looking out as far as six moves at a time. This is equivalent to choosing from a large number of paths through higher and higher order reachability sets, which leads us to wonder if there exist optimized algorithms that are more efficient than brute-force searches. Another avenue of further research is how well So-Soo-Yoo actually teaches the multiplication tables. Do players perform better at math tasks once they have stepped away from the board?

References

[1] *So-Soo-Yoo* website. http://so-soo-yoo.com/ (as of May 1, 2024).