Origami Birthday Gifts: A Preliminary Report

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Abstract

We describe ways to use origami to produce birthday gifts that are interesting mathematically. While these gifts are artistic and can be appreciated without mathematical understanding, (i.e., they are "stand alone" beautiful), we strive to engage the curiosity of the recipient, resulting in new mathematical understanding. The paper is a preliminary report of our project to develop one or more ways to construct an origami birthday gift for each age from 1 through 101.

How It All Started

More than a few years ago I (Charlene) began to create origami birthday gifts that represented the age of the friend celebrating her/his birthday. The first gifts mainly consisted of modular origami models that were made of a number of pieces of paper that matched the friend's age. For instance, an origami polyhedron made of 30 units as a gift for a friend turning 30 or an origami paper quilt made of 60 pieces of paper for a friend turning 60. There are many models that can be created in this way to represent virtually any age. While this method created visually attractive models, it eventually became less challenging and the search was on for more complex and less obvious ways of representing integers with origami. The main goal remains the creation of something beautiful that can be appreciated without an understanding of the mathematics used in creating it. But another goal is to provide mathematical interest for those who wish to dig in, and to engage the interest of those who think they are "not good at mathematics." Through the creation of birthday quilts we have had quite a few interesting conversations of this nature. In the following sections we provide details about several types of birthday quilts.

The Challenge of Prime Number Birthdays

One of the interesting problems we faced was to find origami and mathematical techniques to deal with birthday years that are prime numbers. Although prime numbers are hard to represent visually as they have no proper factors, origami, along with mathematics, gives us a way to represent them beautifully. Consider the prime 41, which is the sum of two consecutive perfect squares, 4^2 and 5^2. An origami way to represent this sum is a 5×5 array of 25 squares held together by a 4×4 array of 16 connectors. We frequently focus on the quilt squares and often do not count the connectors since they are usually mostly hidden and made to disappear into the background. A quilt having 25 squares could easily be seen as a gift for a 25-year-old, but counting the 16 connectors in addition makes it a gift for a 41-year-old as well. Figure 1 shows a quilt with 9 squares and 4 connectors, made for a 13-year-old. Note that all figures can be found at the end of this paper.

It was fascinating to realize that an interesting fact has been hidden from our view because of the filters through which we were viewing it. Once we realized that quilts of this form are composed of an $n \times n$ array of squares joined by an $(n-1)\times(n-1)$ array of connector units, we got the prime numbers 5, 13, and 61. Although not all sums of consecutive squares are prime, the method gives us a way of representing other prime numbers. Those consecutive sums that are not prime can also give interesting options for other birthdays, such as 25, represented with a quilt having 16 squares and 9 connectors.

Two Other Approaches to Making Origami Birthday Gifts

The first method is another $n \times n$ array of quilt squares held together by connectors that fit them together in a completely different way than described above. A 2×2 version of this quilt is shown in Figure 2. The connectors are fit into triangular spaces on the edges of the squares and each connector holds a pair of squares together. The connectors show as purple squares on point in this example. The small, light-color triangles on the outer edges of the quilt are ready to accept connectors if more squares were added for a bigger quilt. The 2×2 quilt requires 4 connectors. Figure 3 is a sketch of a 3×3 quilt of this type. It has 9 quilt squares and 12 connectors for a total of 21 pieces of paper. We invite you to stop and ponder for a moment how many connectors are required for bigger square quilt arrays. As it turns out, for any $n \times n$ quilt of this type, the total number of pieces of paper, T, required to make the quilt (squares + connectors) can be expressed as $T = 3n^2 - 2n$, where n is the number of quilt squares along an edge of the quilt.

The second method, one of the most productive strategies for producing interesting integer results, is not focused on total number of pieces of paper, but on color. This technique does not work for all integers, but only for those integers that are the solution to an (n choose m) = x equation where n = a specific number of colors, m = how many different colors are chosen at a time, and x = the total number of unique sets of m colors that can be formed from these n colors. We call these numbers "combinatorial numbers." For example, a gift for someone turning 15 could be made by starting with 6 colors of paper, choosing 4 at time (thus $n = 6$ and $m = 4$). The number of unique sets of 4 colors is 15 (i.e., $x = 15$). We then form 15 different origami squares using each of the sets of 4 colors and interlocking the squares together (no separate connectors needed). Figure 4 shows the quilt made using this method. Gifts for quite a few other birthdays (e.g., 21, 35, 56, and 70) can be made using this technique.

Figure 1: *Jellyfish Quilt for a 13th birthday gift.*

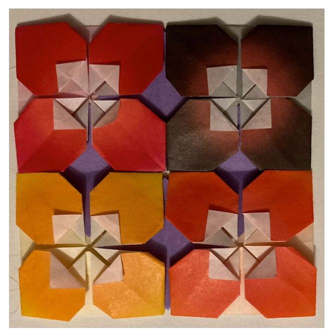

Figure 2: *Flower Quilt.*

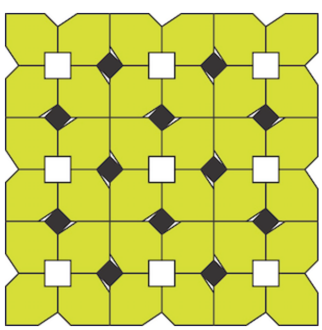

Figure 3: *Sketch of a 3×3 Flower Quilt.*

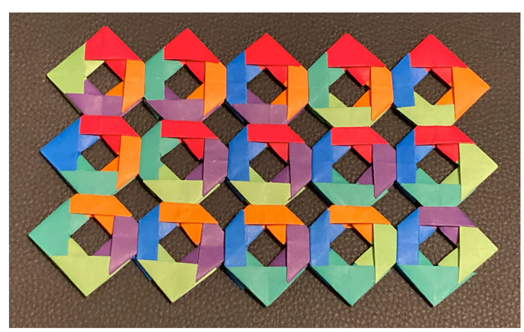

Figure 4: *Pascal's Quilt.*

Conclusion

Thus ends our whirlwind tour of origami quilts as birthday gifts. I hope to write in more detail about this topic in the future, and expanding to other origami objects that make interesting birthday gifts.