Predicting Planned Pooling Patterns

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Abstract

We mathematically analyze the striking visual effect known as planned pooling that arises in knit and crochet patterns when working back and forth with variegated yarn dyed at consistent intervals. Our main result identifies three desirable planned pooling pattern families and provides formulas for choosing row lengths to obtain those patterns.

Introduction and Motivation

Skeins of variegated, multi-color yarn look beautiful in the yarn store but can sometimes produce unsightly "pools" of color that accumulate in particular areas of the finished work. Nobody wants a blotchy sweater, so various techniques exist for hiding this problem, such as double-stranding or alternating skeins every row. Of course, creative knitters and crocheters have also found ways to take *advantage* of color changes in their yarn. One popular example is known as "assigned pooling", in which the knitter creates an intentional feature such as a bauble or special design wherever a certain color appears in the yarn. Yarn dyers create special skeins of assigned pooling yarn with pops of color to specifically facilitate these types of designs. Hand-dyed yarn is not particularly precise, so the areas of color may vary in length and distance from each other.

In this paper we will mathematically analyze another method of leveraging color changes to one's advantage: the crochet method known as *planned pooling*. This technique requires yarn that changes color at consistent intervals so that the crocheter can coordinate their tension and stitch gauge to produce exactly the same number of stitches for each individual color each time it is encountered. For example, a sequence of 10 dark blue stitches, then 5 white stitches, then 4 light blue stitches which repeats over and over again. This simple repeating linear pattern can produce surprisingly regular patterns when worked back and forth, provided that the length of the work is chosen carefully. For example, the yarn in Figure 1 works up as a rather unattractive fabric when crocheted across 26 stitches, but produces a distinctive geometric pattern when worked back and forth in a strict 10, 5, 4 color sequence across 28 stitches.

Figure 1: *Color-changing yarn crocheted back and forth over 26 stitches and then over 28 stitches.*

It is worth taking a minute to highlight what just happened, because it is pretty amazing: the fabric shown in the rightmost image of Figure 1 was created with *one linear strand of multicolored yarn*, crocheted in a fixed numerical sequence that exactly matched the color changes of the yarn, and for a precisely chosen number of stitches per row. Different choices of row length will result in different types of patterns. The reader is encouraged to explore two online apps that preview color pooling outcomes: [3] illustrates pooling patterns for stacked stitches such as single crochet or knitting, and [4], by one of the authors, illustrates pooling patterns for commonplace staggered stitches such as crochet moss stitch or shell/granny stitch. In this paper we will assume that all worked stitches are staggered (each stitch sits between two from the row below) and that work is flat (worked back and forth rather than in the round), because these are the constraints that produce the most interesting pooling patterns.

In this paper we will use mathematics to make these ideas precise, and then to predict which row lengths will lead to what we will call *desirable patterns*. Of course, desirability is in the eye of the beholder, but also "you will know it when you see it." Indeed, much of our preliminary work centered around identifying and categorizing pooling patterns that are discernibly special or desirable in some way. Crocheters employ various techniques to obtain pooling patterns, most often either adding/subtracting one stitch in some way, or instead simply trial and error: crochet a few rows, see if a pattern forms, and if not then start again. In this work we will improve on these techniques significantly by describing three specific desirable pattern families and then determining sequence/row length combinations which guarantee that such patterns will arise.

Notation and Terminology

We begin by establishing some basic notation and terminology for expressing linear color sequences which repeat back and forth across a fixed-length row in a staggered configuration. Given a particular consistently-dyed yarn with repeating *color sequence* $S = \{s_1, s_2, \ldots, s_n\}$, each s_i represents the color of the ith stitch as we work through the yarn, with a pattern that repeats after the *sequence length* $|S| = n$. Each row R_j of work will be the same *row length* L, with odd rows worked from right to left and even rows worked from left to right, in a staggered arrangement, as shown in Figure 2. In this example we have $n = 7$ and $L = 8$, with s_1 and s_2 representing yellow/shaded stitches and $s_3 \ldots s_7$ representing white stitches.

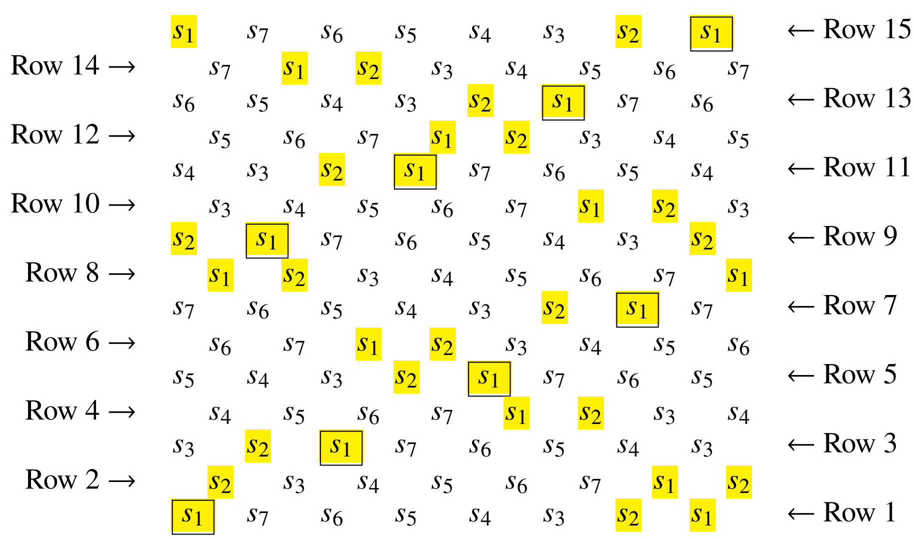

Figure 2: *A color sequence of length $n = 7$ repeated back and forth across a row of length $L = 8$.*

Each of the rows in this arrangement consists of $L = kn + r$ stitches for some number of copies k of the full color sequence S and a remainder $r \equiv L \pmod{n}$ of additional stitches. The first row R_1 starts on the right with copies of S, followed by r additional stitches, and ending on the left with s_r. The second row R_2 begins on the left with stitch color s_{r+1}. This represents a *row shift* of r. This same row shift will happen at every row change, so the next row R_3 will the same as row R_1 but with indices shifted by $2r$. We will call $\delta = 2r$ the *odd row shift*, or the *even row shift*, depending on the parity of the rows we are considering. The proof of our main result will follow directly from an accounting of row shifts.

Desirable Pooling Patterns

Holding sequence length constant and varying row length can result in a surprisingly wide variety of planned pooling patterns, many of which are desirable. Figure 3 shows three such examples, each with sequence length $n = 17$, based on a stitch sequence (2 white stitches, then 3 of each remaining color) that can be created with a size I/5.5mm crochet hook and a mass-market yarn called *Red Heart Berry* that is machine-dyed with highly consistent and contrasting lengths of color. The three examples in Figure 3 highlight the three main results we will prove in Theorem 1. On the left, row length $L = 2n + 1 = 35$ produces a desirable pattern we call *flattened diamonds*; in the center, $L = 2n + \lfloor \frac{n}{2} \rfloor = 42$ produces *square diamonds*; on the right, $L = 2n + \lfloor \frac{n}{4} \rfloor = 38$ produces *elongated diamonds*.

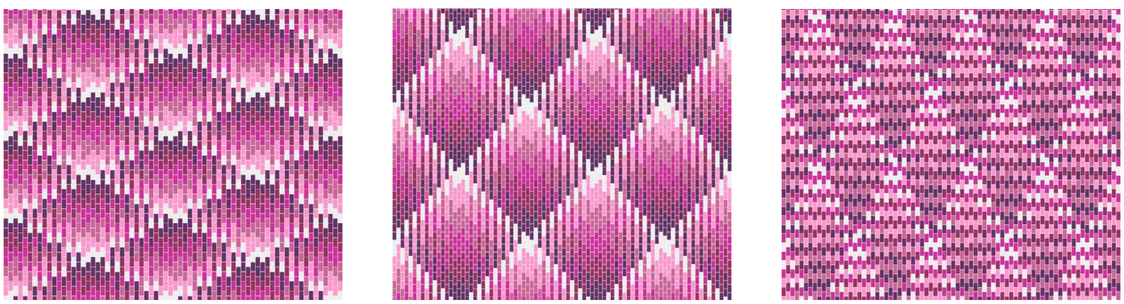

Figure 3: *Desirable patterns with sequence length* $n = 17$ *and row lengths* $L = 35, 42,$ *and* 38.

We have determined that these three desirable pattern families can be constructed with any consistently dyed yarn and any odd sequence length, provided that the row length is chosen appropriately. Figure 4 illustrates four different hypothetical yarns worked up at various row lengths, generated by the app in [4], with sequence lengths recorded in the leftmost column and row lengths described across the top. Note that at multiples of the sequence length we get vertical stripes, because in these cases the row shift is 0. Our three desirable pooling patterns appear in this figure in predictable locations, which we describe in Theorem 1.

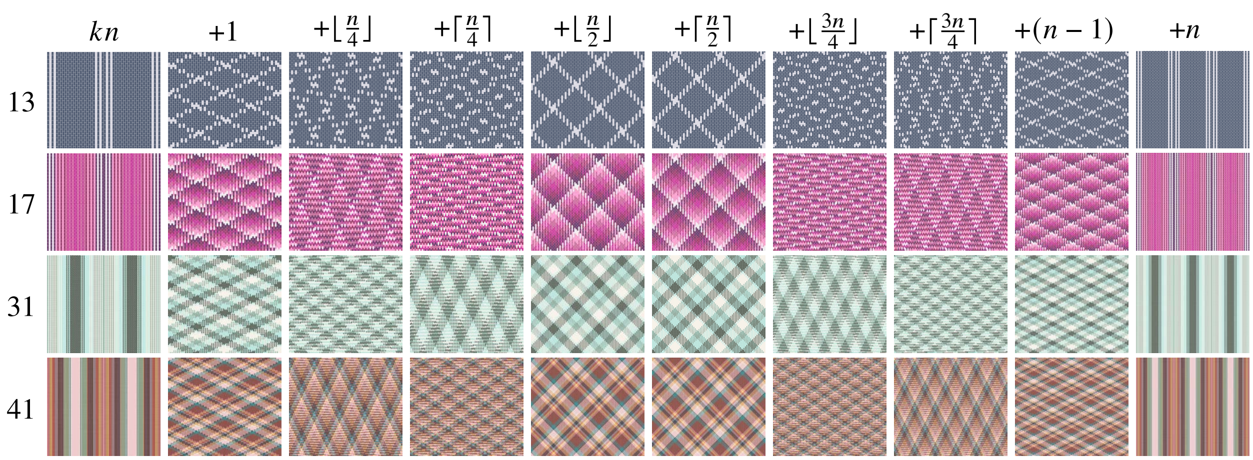

Figure 4: *Row lengths which produce desirable patterns for sequence lengths* $n = 13,$ $17,$ $31,$ *and* 41.

Theorem 1. *Suppose that* $S = \{s_1, \ldots, s_n\}$ *is a repeating color sequence that is crocheted in moss stitch back and forth in rows of* L *stitches. Desirable pooling patterns will occur in each of the following cases:*

a) If $L \equiv \pm 1 \pmod{n}$*, then the resulting pattern will consist of flattened diamonds.*

b) If n *is odd and* $L \equiv \lfloor \frac{n}{2} \rfloor$ *or* $\lceil \frac{n}{2} \rceil \pmod{n}$*, then the resulting pattern will consist of square diamonds.*

c) If $n \equiv 1 \pmod{4}$ *and* $L \equiv \lfloor \frac{n}{4} \rfloor \pmod{n}$*, or if* $n \equiv 3 \pmod{4}$ *and* $L \equiv \lceil \frac{n}{4} \rceil \pmod{n}$*, then the resulting pattern will consist of elongated diamonds.*

Proof. For part (a) we will consider the case when $L \equiv 1 \pmod{n}$. In this case the odd row shift is $\delta = 2$, which means that each color s_i in an odd row moves two steps to the right in the next odd row. This produces a diagonal repeat with slope $\frac{1}{2}$ (see for example the boxed entries in Figure 2). At the same time, we have diagonal repeats of slope $-\frac{1}{2}$ in the even rows, resulting in an overall flattened diamonds pattern.

For part (b), consider the case where $L \equiv \lfloor \frac{n}{2} \rfloor \pmod{n}$. Since n is odd we have $n = 2m + 1$ for some integer m, and therefore the odd row shift is $\delta = 2\lfloor \frac{n}{2} \rfloor = 2\lfloor \frac{2m+1}{2} \rfloor = 2m = n - 1 = -1 \pmod{n}$. Each odd row color s_i moves right by one stitch in the next odd row, creating a diagonal pattern with slope 1. At the same time, the colors in even rows create a diagonal pattern with slope -1, resulting in square diamonds.

For part (c), consider the case where $n \equiv 1 \pmod{4}$ and $L \equiv \lfloor \frac{n}{4} \rfloor \pmod{n}$. Since $n = 4m + 1$ for some integer m we have odd row shift $\delta = 2\lfloor \frac{4m+1}{4} \rfloor = 2m = \frac{n-1}{2}$. Each odd row color s_i shifts nearly halfway through the length of the color sequence in the next odd row, and over two odd rows, this color shifts by $n - 1 \equiv -1 \pmod{n}$. Thus every odd-row color moves two odd rows up and one position to the left, resulting in a stretched diagonal with slope -2. Together with the even rows, this creates elongated diamonds. □

Conclusions and Future Work

In this work we have developed a framework for discussing planned pooling patterns, and established mathematical formulae for reliably producing three very basic types of desirable patterns. This only scratches the surface of possible pooling patterns and what we can say about them. Future work is suggested by the images in Figure 5. The first two patterns have the same n and L values and the same stitch color counts, but in different orders and positions. The third and fourth patterns illustrate that the diamonds are not in fact all identical. The fifth pattern shows the complexity that can occur for large values of n and L, and the sixth is a starting point for thinking of planned pooling as an extension of *sequence knitting*, as described by Cecilia Campochiaro in [1] and analyzed extensively in the context of group actions by Sara Jensen in [2].

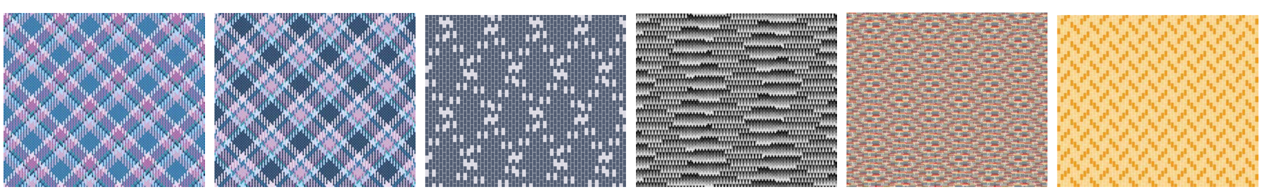

Figure 5: *Additional pattern previews generated by [4], suggesting possible directions for future work.*

References

[1] C. Campochiaro. *Sequence Knitting: Simple Methods for Creating Complex Reversible Fabrics*, Chroma Opaci Publishing, 2015.

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[3] *Planned Pooling*. https://plannedpooling.com/

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