Experimenting with the Golden Ratio in Poetry

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Abstract

The number known as the *golden ratio* is one of the few irrational numbers which were instrumental in the development of mathematics, and also captured the imagination of artists and the general public. The focus of this paper is my experimentation with the use of the golden ratio as both content and structure of poems. Along with four of my poems, the paper includes several related poems written by others, as well as brief glimpses of the rich history and the varied mathematical properties of this astonishing number.

Introduction

The number known as the *golden ratio* is one of the few irrational numbers which were instrumental in the development of mathematics, and also captured the imagination of artists and the general public. Denoted by the Greek letter ϕ, it is equal to:

$$\phi = \frac{1+\sqrt{5}}{2} = 1.61803398874989484220 4586\ \ldots$$

The most ancient text in which a definition of the golden ratio has been found is Euclid's (323–285 BCE) *Elements* [8], where it facilitated many geometric constructions, including that of regular pentagons, dodecahedrons and icosahedrons. Its first appearance in a proof in the *Elements* was as the ratio of the sides of a rectangle. Such a rectangle, called a g*olden rectangle*, was used as an aid in finding solutions to quadratic equations by geometric methods. In addition to rectangles, certain triangles, angles, and spirals are also associated with the golden ratio in mathematically significant ways. The golden ratio's surprising multiple appearances at the heart of geometry made Italian mathematician Luca Pacioli (1445–1517) call the ratio the "divine proportion" [25]. Johannes Kepler (1571–1630) considered the golden ratio to be one of the two great treasures of geometry, the other being the Pythagorean Theorem [4, 19]. The golden ratio also makes an appearance in a number of more recent geometric discoveries: Thus, the self-similar logarithmic spiral called the *golden spiral* is a fractal; while Penrose tiles feature the golden ratio prominently in both linear and angular proportions of their various shapes [15, 19]. And geometry is not the only area where the golden ratio works its magic. In addition to exhibiting several unexpected numerical properties, it is closely related to the Fibonacci sequence [19].

Beyond mathematics, the golden ratio appears as a pattern in nature, including the spiral arrangements of certain plants, the flight of peregrine falcons, and the behavior of galaxies [19]. Pacioli's book, *Divina Proportione* (1509) [25], was illustrated by Leonardo da Vinci, thus making the first connection between the golden ratio and artistic expression. But it was not until the 20th century that artists began to use patterns involving the golden ratio in their work. Interesting examples of modern visual art and architectural works engaging the golden rectangle in their design, as well as musical compositions using numerical patterns associated with the golden ratio as structural elements, may be found in [19]. To a lesser extent, the golden ratio also makes an appearance in poetry. There, it is mainly used as a metaphor for great beauty, or for the underlying principle of aesthetic perfection [1, 2, 11, 20]. In the last few years, attempts have been made to use the golden ratio as a structural element in poetry [10, 18, 23, 27].

The focus of this paper is my experimentation with the use of the golden ratio as both content and structure of poems. Along with four poems I composed for this purpose, the paper includes several related poems written by others, as well as brief glimpses of the rich history and the varied mathematical properties of this astonishing number.

The Golden Ratio as a Geometrical Construct

The golden ratio, called in antiquity "extreme and mean ratio," was first defined as a proportion of lengths of line segments. Euclid's *Elements, Book VI, Definition 3,* states:

> *A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less* [8].

In other words, given a line segment AB, the point C cuts AB in extreme and mean ratio if $\frac{x+y}{x} = \frac{x}{y}$, where x denotes the length of CB, and y denotes the length of AC (see the diagram below). This common ratio is the golden ratio, ϕ. Manipulating the equality of ratios yields the equation $\phi^2 = \phi + 1$. Solving for ϕ we obtain the value $\phi = \frac{1+\sqrt{5}}{2}$, whose decimal expansion is given in the introduction.

Ancient Greek mathematics did not have a convenient notation for numbers; its "computations" were carried out by geometric means. And the allowed tools of geometric construction were themselves restricted to the straightedge and compass. The first surprise the golden ratio had in store for mathematicians was the simplicity with which it could be constructed.

Below is my poem "To cut a line AB in extreme and mean ratio" describing the construction of the golden ratio appearing in Euclid's *Elements.* It is a found poem (a literary collage), which uses the description of the construction as paraphrased and illustrated by David Burton [4].

To cut a line AB in extreme and mean ratio

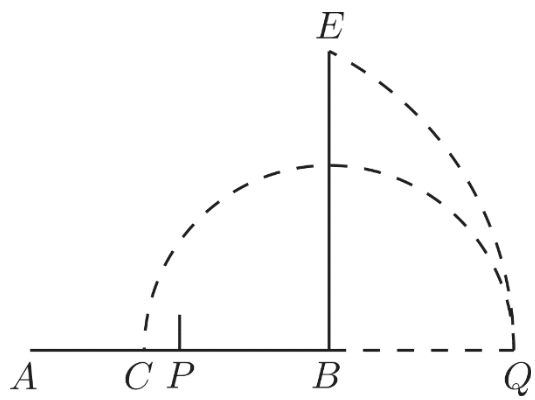
Given a straightedge and a compass,

At the end point B,
Erect a perpendicular BE
Equal in length to AB.

With the mid-point P
Of AB as the center
And radius PE,
Draw an arc cutting
The extension of AB
At the point Q.

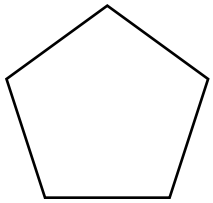
Take B as the center
And radius BQ, and draw
An arc cutting AB at C.

Prove that C cuts AB in extreme and mean ratio.



All the steps of the construction described in the poem are displayed in the associated diagram. This technique, which may be due to Eudoxus of Cnidos (408–355 BCE) [16], has the hallmark of what mathematicians consider a beautiful mathematical result: The construction is simple and elegant, and it has wide ranging implications and applications. These applications start with the ability to use the technique to construct a regular pentagon, which itself led to the ability to both construct and understand more deeply the properties of the five Platonic solids.

I will start the brief discussion on this topic with my poem "I am a pentagon."



I am a pentagon

Deceptively simple,
I show you
only my borders.

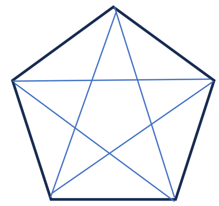
Within me lie
the hidden
roots of everything:

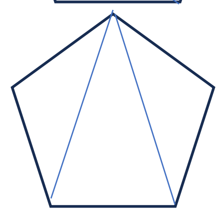
My vertices
inscribe
the five-pointed star.

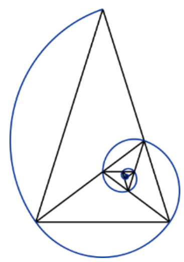
My triangles
are golden.

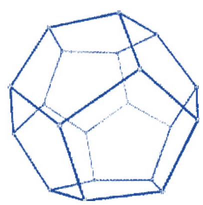
Whirling inside each other,
they form
the curve of life.

When joining edges
with others of its kind,
my face
covers
the universe.









A regular pentagon is a five-sided polygon whose sides have equal lengths. The top image on the right shows the pentagram (five-pointed star) that can be inscribed in the pentagon by connecting its vertices. The pentagram was an important symbol for the Pythagoreans, the symbol by which they recognized each other as members of the brotherhood. The search for a method of construction of a pentagon, in which a pentagram can be inscribed, may have originated with Pythagoras himself [19]. The isosceles triangle in the second image from the top is called a *golden triangle,* because the ratio of its side to its base is equal to ϕ. It is possible to construct a smaller golden triangle inside it, by bisecting the angle on the left side of the base. The process can be continued indefinitely (see third image from the top [15]). Connecting the vertices of the decreasing sequence of golden triangles by arcs results in a logarithmic spiral whose radius grows by the golden ratio per 108° of turn [15]. Spirals, connected to the golden ratio or not, appear often in nature and art. That is why they were called “the curves of life” by Theodore Andrea Cook [7]. The last image at the bottom is a dodecahedron [26]. The dodecahedron’s surface is made up of twelve regular pentagons. Therefore, its construction depends heavily on the ability to construct a pentagon. The golden ratio played a critical role in these constructions. The dodecahedron is one of the five Platonic solids. The other four Platonic solids are the tetrahedron, the cube, the octahedron, and the icosahedron. Plato believed that each of these four solids corresponds to a basic element of matter, while the dodecahedron itself corresponds to the universe [4].

The two poems in this section represent a very small portion of the varied repertoire of geometric shapes, theorems, and constructions closely associated with the golden ratio. Many of them are beautiful and share in the golden ratio's aura of mystery [15, 19]. To date, they still wait for the poet who will immortalize their existence in verse.

The Golden Ratio as a Numerical Value

As a numerical value ϕ has a number of amusing mathematical properties. For example, its square is equal to itself plus one, while its reciprocal is equal to itself minus one (quite easy to check with a calculator). Another strange property of ϕ is the appearance of its continuous fraction expansion (Figure 1): "It's one, one, one,…until distraction," as mentioned along with the other two properties in the humorous poem "Constantly Mean" by Paul Bruckman [5]. Because of this, the continuous fraction corresponding to the golden ratio converges to it very slowly. To quote Mario Livio [19]: "The golden ratio is, in this sense, more difficult to express as a 'fraction' than any other irrational number—it is the 'most irrational' among irrationals." Not surprising, such a statement is irresistible to poets. Below is Kaz Maslanka's visual poem "Golden Fear" [23] inspired by it.

$$\Phi = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}$$

Figure 1. *Continuous fraction expansion of ϕ* [15]

Golden Fear by Kaz Maslanka

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A different property of ϕ that can drive poetic structure is its infinite decimal expansion. The decimal expansions of other irrational numbers, such as π, e, and $\sqrt{2}$, were used to construct poems that follow the sequence as syllable count per line, or other counting patterns associated with the poem. Fine examples of these poems can be found in [12, 13, 14, 17, 21, 22] and other places. Since the decimal expansion of any irrational number is infinite, the poem's counting pattern uses only an approximation of the number itself. Radoslav Rochallyi used a modified decimal expansion of ϕ as word or syllable count per line for his poems. Specifically, he stopped at the 6th decimal place and rounded the last number from 3 to 4 [27].

On the next page is my poem "1.618033988749894842204586 Waterlilies" whose syllables count per line follows the decimal expansion of ϕ to 24 decimal places.

1.6180339887498948482045868 Waterlilies

My
black pond is surrounded
by
mountains. Snippets of memories

rise to the
surface, and
float entangled in dark green water.
I catch them like fish, livid mouth
open, slippery gills glisten,
the sun a pale radiance
not enough for
warmth. At sundown, the waterlilies
fold their petals over hidden
treasures. A giant eye stirs beneath
the oars, eyelid
closed shut. Tear after tear, it fills
the pond sadness.
I like
the pink

waterlilies'
delicate corals,
and the yellow ones, a golden
glow borrowed from the moon.

The last property of the golden ratio I will consider in this paper is its relation to the Fibonacci sequence. The Fibonacci sequence is the sequence made up of the following numbers, called Fibonacci numbers:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

Starting from the number 2 in the third position, each number in this sequence is constructed by adding the previous two Fibonacci numbers.

The Fibonacci sequence appeared in ancient Indian mathematics in connection with Sanskrit poetry. The first appearance of the sequence in western mathematics was in 1202, in the book, *Liber Abaci* [9], written by the Italian mathematician Leonardo Fibonacci (1170–1250). In *Liber Abaci*, the Fibonacci sequence was produced by the pairing of rabbits and represented the ideal growth of a rabbit population over time. The Fibonacci sequence is intriguing, not only because it exhibits an attractive numerical pattern, but also because it makes frequent appearances in natural settings and in the arts. The sequence has been as extensively studied as the golden ratio. More information about this fascinating sequence, and the Fibonacci poems whose counting patterns follow it, can be found in [3, 6, 13, 15, 19].

The relation between the Fibonacci sequence and the golden ratio of interest in this paper is the following: Denote the n^{th} Fibonacci number by F_n; then the ratios $\frac{F_{n+1}}{F_n}$ converge to ϕ, as n goes to ∞. In other words, as n increases, the ratios of consecutive Fibonacci numbers become a better and better approximation of ϕ. We can use this connection between the Fibonacci sequence and the golden ratio to structure poems in which approximations of the golden ratio appear in more than one way. I will start the brief discussion about this poetic form with an example, my poem "Golden Days."

Golden Days

It is summer! Summer!
The woodpecker sounds the alarm
in the backyard. With a hammer
hammers on the hard tree bark.
Come to see the sun
shining through the leaves!
Come to smell the perfumed air,
and to gulp delight!
It is truly summer! Yammer cockatoos
wearing Queen Anne's lace.
It is really summer! Rings the phone
bird's call as it clings in place,
thrilling up the trunk.

It is cheaper here!
Cheaper here by much!
Cheers the chirper in the cherry tree:
Chokecherries for free!
We can later come to your barbeque,
eat and drink on cue
trilling as we do:
Beery beery beer! Beery beery beer!

Both the line count and the syllable count of the two stanzas of "Golden Days" are consecutive Fibonacci numbers. The top stanza has 13 lines and 89 syllables, while the bottom stanza has 8 lines and 55 syllables. The line count ratio is 13/8 = 1.625, and the syllable count ratio is 89/55 = 1.61818, both approximations of the golden ratio. If we let the blank line of the stanza break stand for the line denoting division, the poem's structure uses approximations of the golden ratio twice, once as ratio of lines and again as ratio of syllables. This two-stanza format can use other consecutive Fibonacci numbers as line count or syllable count. The larger the numbers the better the approximation to the golden ratio.

A third way in which the golden ratio is part of this poem's structure is geometrical, rather than numerical. Start by drawing a vertical line from the top of the first line of the poem to the bottom of its last line, and use Euclid's method of dividing this line in extreme and mean ratio. To accomplish this, follow the previous section's straightedge and compass method outlined in the poem "To cut a line AB in extreme and mean ratio" and its associated diagram. The point C, which cuts the line in the desired proportion, falls in the blank line of the stanza break. Nowadays we do not need to use a straightedge and compass to verify this result; we can do it with a calculator. Here are the measurements I obtained for a version of the poem typed in Times New Roman 12 pt, single spaced: The vertical line from the top to the bottom of the poem is 10.5 cm long. To obtain the longer segment of the division, we need to divide this length by ϕ. Thus $10.5/\phi = 6.489$ cm is the length of the longer segment. Then, the shorter segment is $10.5 - 6.489 = 4.011$ cm long. We now verify that these line segments indeed give the proper ratios: $\phi = 10.5/6.489 \approx 6.489/4.011 = 1.6178$. Note that the 6.489 cm mark from the top of the vertical line falls in the blank line of the stanza break.

On the same principle, Lawrence Mark Lesser's poem "Convergence" [18] is composed of 6 couplets, each of which consists of words whose letter count are consecutive Fibonacci numbers. The words converge down to the one letter word "I," while the couplets' ratios "converge up" to ϕ. Here is the poem:

Convergence by Lawrence Mark Lesser

incomprehensibilities
rationalizing

architectures
generate

nautilus
ratio

shell
may

fit
in

as
I

We conclude our deliberations on the relation between the golden ratio and the Fibonacci sequence with another aspect of the discussed convergence that inspired poetry. Figure 2 shows a possible tiling with squares whose side lengths are consecutive Fibonacci numbers and the Fibonacci spiral resulting from drawing connecting arcs between opposite corners of the squares. If we start with a golden rectangle and remove from it a square whose sides are equal to the short side of the rectangle, we are left with a new, smaller, golden rectangle. This process may be continued for as long as we wish. When the squares are marked rather than removed, the process yields a tiling of the large golden rectangle by smaller golden rectangles. It is then possible to fit a logarithmic spiral to the same square corners visited by the Fibonacci spiral. This particular logarithmic spiral is called the *golden spiral* (its radius grows by the golden ratio per 90° of turn). In each rectangle appearing in Figure 2, the ratio of the long side to the short side is equal to $\frac{F_{n+1}}{F_n}$ for some *n*. Since $\frac{F_{n+1}}{F_n}$ approximates Φ, such a rectangle is an approximation of a golden rectangle, and the resulting Fibonacci spiral is an approximation of the golden spiral. A tiling of this nature inspired Emily Galvin's concrete poem, *Spiral* [10, 13].

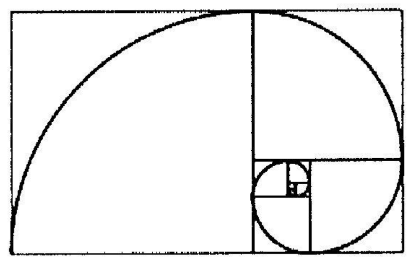

Figure 2. *Logarithmic spiral* [15]

I will end with a few words about the two poems I wrote for this section. In an attempt to experiment with golden ratio constraints for a wide range of poems, I picked poems whose content is not mathematical. Nevertheless, I tried in the spirit of OULIPO [24], to choose poems whose content speaks of the constraint used to form them, even if only metaphorically. Each of the two poems has something "golden" about it, the moon glow of the waterlilies or the euphoria induced by a perfect summer day. Both poems were written a few years ago, in free verse. But they both fit into the golden ratio mold I had chosen for them naturally, without forcing, and with very little editing. The process of shaping an existing poem by syllable and line count of the golden ratio, like any other form of editing, is a collaborative work between the poem and the editing poet. I picked poems whose syllable count was slightly over the one I needed, on the principle that it is easier to subtract than to add. For example, "1.618033988749894848204586 Waterlilies" needed 127 syllables and the original poem had one superfluous syllable (thus "surface" became "oars" in line 15). This poem required a further adjustment, the first sentence of the poem was originally its third sentence. It took me a while to realize that a switch was necessary. In the end, regardless of how hard or easy it is to shape the poem, the final version has to work well as a poem. I have not identified any patterns that guarantee success. But, when it works, it is a joy! For poets who would like to give these constraints a try, I recommend googling for an online syllable counter. It will save you hours of tedious syllable counting.

Concluding Remarks

This paper was inspired by reading Mario Livio's entertaining and informative book, *The Golden Ratio*, [19]. But, as I progressed through the project, it took a life of its own. The techniques and poems (more than I included in this paper) that came out of my experiments surprised me in many ways. But then, it is no wonder. Where the golden ratio is concerned, expect the unexpected! My hope is that this paper will, in turn, inspire other poets to experiment with the inclusion of the golden ratio in their own poems.

Acknowledgements

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