Kinetic Knots

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Abstract

What happens if you try to tie a (mathematical) knot using not a string or yarn but a stiff, springy cord such as hardened steel wire or basket reed? This is not an easy task, but, if you succeed, you will find that certain knots will settle into very interesting, stable 3D configurations and will spring back to these shapes when deformed. Focusing on two well-known families of knots, we identify the ones that have this property and demonstrate some practical methods for tying them into kinetic decorative objects such as self-tightening bracelets.

Introduction

In this workshop we will create some decorative, 3D objects simply by tying knots. Moreover, these objects will have a kinetic quality that we can use to turn them into interesting pieces of jewelry, see Figure 1. All this is based on the fact that a knot tied with a stiff elastic wire will naturally adopt a stable shape that minimizes the tension created by the bent wire and will spring back to this shape after a deformation.

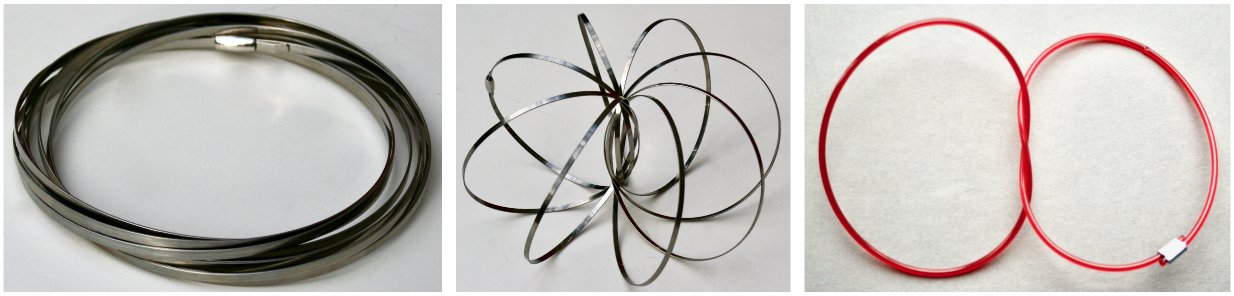
We will focus on *torus* and *Turk's head*, well-known, related knots that depend on two integer parameters. When tied with elastic wire, such a knot will settle into a configuration determined by the relation between its parameters. These configurations were first described in [4] where we viewed the knots as models for a family of polyhedra. We used one of the knot tying methods described here for a very popular activity at a 2022 MoMath summer camp for middle school students.

Elastic wire knots and their kinetic properties are explored in [6], [7] and [8]. Methods of making 2D decorative Turk's head rosettes are presented in [9].

An elastic wire knot can generate a (potentially) infinite number of shapes obtained by twisting the wire ends before joining them together and thus forcing more torsion in the wire. For example, an unknot will settle into shapes with more and more loops depending on how many twists we force on the wire. Based

(a) $(6,7)$ *torus knot* **(b)** $(30,31)$ *torus knot* **(c)** $(4,5)$ *Turk's head knot*

Figure 1: *Kinetic bracelets based on torus and Turk's head knots.*



(a) $(9,8)$, *2D state with 8 coils* **(b)** $(8,9)$, *3D state with 8 lobes* **(c)** $(2,3)$ *or trefoil*

Figure 2: *Torus knots.*

on this idea, N. Hocking created some beautiful knot shapes [5]. We do not explore such shapes here; with one exception, described in the torus knot section, we are tying the wire knots in a straightforward manner, without twisting the wire.

We will use very thick (2.3–2.5 mm) monofilament fishing line which is stiff enough but safe to work with; to complete a knot the wire ends are crimped together. For the other knots shown here and in [4] we used hardened steel wire, memory wire, steel cable, and round reed.

Torus Knot

An (m,n) torus knot (TK), where m and n are relatively prime, is obtained by looping a string through the hole of a torus m times while making n revolutions around the torus before joining its ends [1]. If we loop an elastic wire, we will end up with one of the following shapes after removing the torus:

2D (a coil that wraps around n times) when $m > n$, see Figure 2 (a);

3D (m lobes around a central "column") when $m < n$, see Figure 2 (b), (c).

Since swapping the two parameters of a TK results in a topologically equivalent knot, a TK can have either of these two shapes depending on how the wire is looped around the torus. For example, (a) and (b) in Figure 2 are two different, stable states of the same TK. We can transform one shape to the other by a form of "wire eversion": cut the wire and half-twist both wire ends before rejoining them, see [3] for a demonstration.

The central column of the 3D shape can be forced to open up, transforming the knot into a flat coil that wraps around n times, but this does not put the knot in the 2D state: the coil will spring back to the 3D shape when released. We can use this property to make self-tightening bracelets as shown in Figure 1 (a), (b).

In the workshop, we will use two methods of tying wire TKs. The first one is essentially the same as the method used for tying Turk's head knots and is described in the next section. The second method can be used to make, without any tools, the 2D shape of a $(m, km+1)$ TK, $k \geq 1$, which we can then transform, as described above, into the 3D shape of the knot. The method consists in making m successive loops and, after each loop, passing the working end of the wire through all the loops k times, see [3]. In the workshop, we will use this method to make 3D $(m, m+1)$ knots which are both attractive and easy to tie.

Turk's Head Knot

Known since antiquity, the Turk's head knot (THK) is used today mostly for its decorative qualities [2]. Typically made by interweaving a string around a cylinder, an (m,n) THK has m *bights* (string bends at the end of the knot) and n *leads* (one more than the number of string crossings between two consecutive bights);

(a) (3,4), *3 bights, 4 leads* **(b)** (5,3) **(c)** (3,13) **(d)** *Forced open*

Figure 3: *Turk's head knots: tied with rope (a), flat (b), and cylindrical (c), (d).*

see Figure 3 (a) and also imagine the diagram in Figure 6 wrapped around a cylinder. An (m,n) THK can be tied only when m and n are relatively prime. When tied with elastic wire, the knot will settle into one of these shapes:

flat when $m > n$, see Figure 3 (b);

spherical (roughly) when $m < n < 2m$, see Figure 4;

cylindrical (tight, rope-like) when $n > 2m$, see Figure 3 (c), (d).

The natural state of the flat knots is a circular coil that wraps around n times; using the friction between coils, the knot in Figure 3 (b) was arranged into an unstable, but more interesting shape. The cylindrical knots are very tight and hard to open up.

An elastic wire THK will spring back to its stable shape when deformed (flattened, forced to open, etc.) For spherical and cylindrical knots, this property can be used to make self-tightening jewelry pieces such as rings and bracelets, see Figure 1 (c); the kinetic effect is similar to the torus knots, but with a different appearance due to its alternating structure.

A figure-eight knot (Figure 4 (a)) can be tied by hand without any tools, but tying a more complex THK requires, in general, the use of a jig such as the one in Figure 5. To tie an (m,n) THK, we will anchor the

(a) (2,3) *or figure-eight* **(b)** (3,4) **(c)** (3,5)

Figure 4: *Spherical Turk's head knots.*

Figure 5: *5-bight jig.*

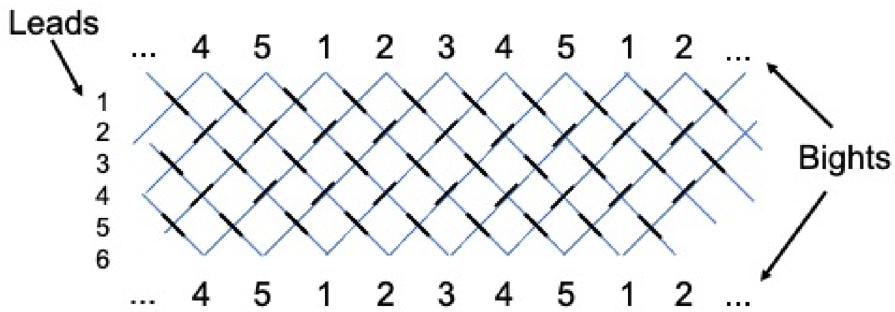


Figure 6: *Diagram for tying a* $(5,6)$ *THK.*

top and bottom *m* bights and run the wire back and forth between them wrapping it around the cylinder in the same direction. A simple diagram, like the one in Figure 6, can be used to keep track which bights to connect and which strand is over and which is under at each crossing.

In general, we need different jigs for different number of bights; the one in Figure 5 has 10 anchor holes at each end and can be used to tie any knots with 5 bights. If the number of leads is even the top and bottom bights are aligned (as shown in the picture), otherwise they are rotated with respect to each other. The jig in Figure 5 can also be used to tie 10-bight knots if the number of leads is even.

The same jigs can also be used to tie torus knots. Wrapping a TK around the jig is equivalent to wrapping it around a torus, with the advantage that, at the end, we can remove the jig leaving the completed knot. The tying method is similar to the one described above, but simpler, since we do not need the crossing information - we will always cross the strands going in one direction over the ones going in the other direction.

Acknowledgment. Figures 1 (a), 2 (a), (b), and 3 (b), (c), (d) appeared previously in [4].

References

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