Sidewalks of Lisbon and Azores

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Abstract

Part of the beauty of Lisbon and Azores lies under the feet of anyone who walks in the streets. Given their highly symmetrical nature, Portuguese sidewalk drawings are ideal to be analyzed geometrically. Furthermore, these drawings provide opportunities for promoting mathematical concepts through exhibitions, card decks, and other forms of dissemination.

Introduction

Designs that repeat in one or two directions can be classified based on their symmetries, resulting in seven frieze patterns and seventeen wallpaper patterns [4][6]. Another type of pattern is the rosette, which repeats around a single point within a limited area of the plane. Rosettes can be classified as dihedral or cyclic depending on the presence of mirror symmetries. Many beautiful Portuguese pavements feature these patterns, including all seven friezes, cyclic rosettes, dihedral rosettes, and twelve out of seventeen types of wallpapers in Lisbon. In the Azores islands, all seven friezes, cyclic rosettes, dihedral rosettes, and some of the seventeen types of wallpapers were also found. These findings have been shared widely and have inspired the creation of decks of cards and museum exhibitions. This work highlights the importance of promoting the beauty, cultural and mathematical significance of these designs.

Portuguese Pavements

Portuguese pavements are unique due to their designs, stone production, and the skill of the pavers. There are several motifs, including geometric, figurative, and specific designs [5]. These pavements are a heritage of Roman culture, and many examples of Roman-style pavements can still be found in Portugal.

During the reign of King John II (1455–1495), the growing commercial profitability of Lisbon and Oporto prompted the construction of "New Streets" (*Ruas Novas*) in areas where the main stores were concentrated. The earthquakes of 1531 and 1551 led to a natural need for the creation of new streets, but the most significant moment came after the Lisbon earthquake of 1755. A massive project was initiated, not only for the reconstruction of the streets but also for the creation of new ones. One of the most important figures in this process was Sebastião de Carvalho e Melo (1699–1782), the Marquis of Pombal.

In 1848, Lisbon became the first city to use decorative paving in urban spaces with the *Mar Largo* project. This wave-shaped composition was constructed in the D. Pedro IV Square (now known as *Rossio*). However, six years prior to this project, Lieutenant-General Eusébio Pinheiro Furtado paved the narrow streets leading to the São Jorge Castle with white (limestone) and dark (basalt) stones. This illustrates the rich history and tradition behind Portuguese pavements. Limestone displays a smooth and glossy surface, and the most typical Portuguese pavements are made from a combination of limestone and basalt. The process of laying paving units is carried out by skilled technicians known as *calceteiros*. They use small hammers to place the stones over a layer of fine material. Some of the designs feature remarkable symmetry, such as the wave motif known as *Mar Largo* (Figure 1), which has been replicated around the world.


Figure 1: *Wave motif ("Mar largo", designed by Eusébio Pinheiro Furtado): (a) Rossio, Lisbon, Portugal, and (b) Praceta Roberto Mesquita, Santa Cruz, Flores, Portugal.*

Regarding the Azores, it is worth mentioning that part of the islands' beauty is often found right beneath one's feet while walking the streets, as Teixeira pointed out [7]. The traditional Portuguese pavement serves as a decorative feature in public spaces, adding a unique aesthetic to the area.

Symmetry

Understanding the symmetry of a figure involves identifying the transformations that leave the figure unchanged. It is not the motif itself that is of utmost importance, but rather how the invariant transformations manifest. There are three objects that frequently serve as artistic motifs: (1) *Friezes*: patterns that repeat infinitely in one direction; (2) *Wallpapers*: patterns that repeat infinitely in two directions; (3) *Rosettes*: designs with rotational symmetry that radiate out from a single point and are confined within a limited region.

Many of the beautiful pavements found in Portugal are composed of friezes, wallpapers, or rosettes. To better understand the examples, although not exhaustively, it is crucial to highlight some fundamental concepts that enable the mathematical classification of these three objects. In the case of two-dimensional figures, four types of symmetry exist: (1) *Reflection symmetry*, or *mirror symmetry*, which is defined through a line known as the axis of symmetry; (2) *Rotational symmetry*, which relates to a point called the rotation center, and an oriented angle; (3) *Translational symmetry*, which is linked to a vector; (4) *Glide reflection symmetry*, which results from a composition of reflection and translation in the direction of a vector that runs parallel to the axis that defines the reflection.

Friezes, wallpapers and rosettes can be classified using these four types of symmetry [4][6]. Mathematical analysis has shown that there are precisely seven distinct types of friezes and seventeen distinct types of wallpapers. In [2], the authors provide flowcharts to help in the classification of these objects. It has also been demonstrated that only two possibilities exist for rosettes: cyclic rosettes (without mirror symmetry) or dihedral rosettes (with mirror symmetry).

Lisbon

The sidewalks of Lisbon are artistic works beneath our feet. In 2010, the Calouste Gulbenkian Foundation supported the *Simetria Passo a Passo* project, which aimed to detect patterns on the city's sidewalks [1]. By June 23, 2010, the project had identified all seven friezes, cyclic rosettes, dihedral rosettes, and eleven out of the seventeen types of wallpapers on the sidewalks of Lisbon. Only six wallpaper groups are missing, the ones denoted by 632, 333, 333, 22X, 42, and O in the Conway/Thurston orbifold notation [4].

In [2], another wallpaper was found. That wallpaper is in Rossio dos Olivais, an interesting place in Parque das Nações in Lisbon. The architect Fernando Conduto executed the artistic concept of the pavement [3]. Figure 2 displays a panoramic view of the place obtained from Google Earth. The sidewalk was constructed with Portuguese pavement, and the design resembles a water channel. At the bend, a one-dimensional frieze "transforms" into a two-dimensional wallpaper, giving the impression that the water is flowing towards the river. The frieze is classified as ∞∞, and the wallpaper is classified as O. Hence, there are only five remaining wallpapers yet to be discovered.

Mathematically speaking, there are 7 friezes, 17 wallpapers, and 2 types of rosettes, for a total of 26 cards, which is equivalent to 2 suits. This appears to have been intentionally designed for a deck of cards!

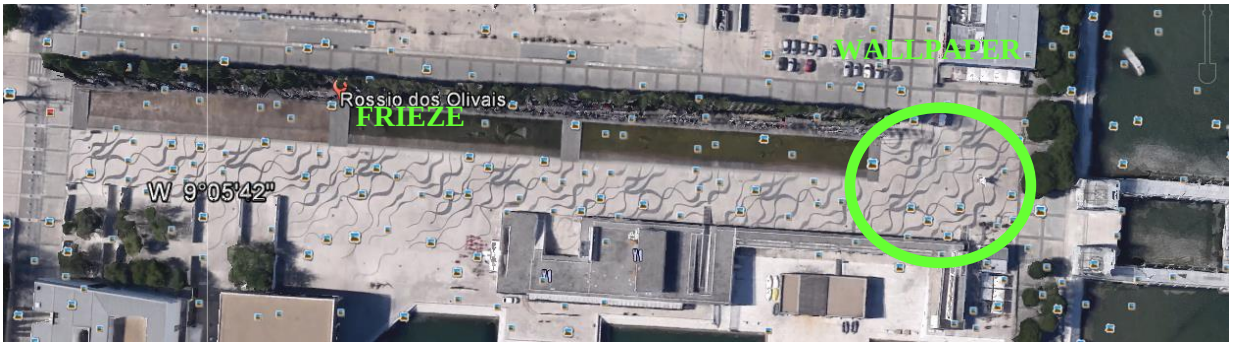

Figure 2: *Rossio dos Olivais, Lisbon.*

Under a joint project between the Ludus Association and the University of Lisbon, the authors of this article created a card deck to explore the subject of symmetries. The deck, entitled *Baralho de Simetrias - Calçadas de Lisboa* [4], features problem-cards with motifs that can be found in Lisbon. People with mathematical knowledge can also attempt to classify the symmetries. The solution-cards show locations in Lisbon where the motifs can be found, along with the classifications. The cards are organized by color, with, for example, the solution to the 10 of spades being the 10 of clubs, and the solution to the ace of hearts being the ace of diamonds. Since there are five missing wallpapers in Lisbon and there are five Platonic solids, we decided to name these imaginary locations using the names of these solids (Figure 3). The jack of diamonds shows Rossio dos Olivais. The deck also includes two mirror cards, which are useful for identifying mirror reflections and dihedral effects. Finally, the deck box exhibits an ambigram.

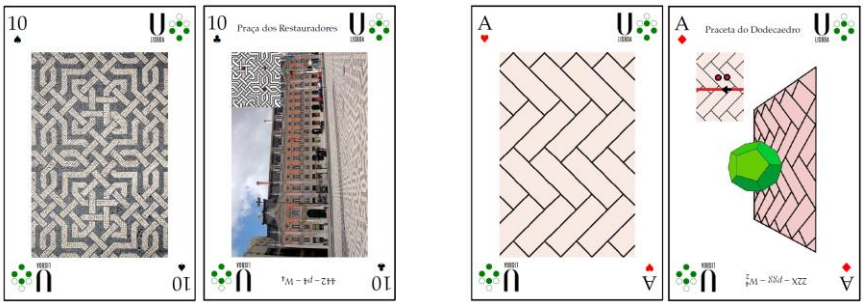

Figure 3: *Baralho de Simetrias - Calçadas de Lisboa*: *Examples of pairs of cards (problem/solution).*

Azores

The sidewalks in the Azores are also works of art beneath our feet. However, the drawings often have inverted colors compared to those in Lisbon. What is light in Lisbon is black in the Azores and vice versa. This is due to the fact that the Azores are volcanic islands and the limestone used for the sidewalks has different nature than that found in Lisbon.

After launching the deck of cards on Lisbon's sidewalks, the authors of this article have also released the *Baralho de Simetrias - Calçadas dos Açores* (Playing Symmetries: Azorean Sidewalks), a deck with a similar concept (Figure 4). This deck encourages visiting the 9 islands of the Azores as well as appreciating the symmetries of its pavements. Since there are not as many squares in the Azores as there are in Lisbon, fewer wallpapers were identified. Therefore, the deck focuses on friezes and rosettes, with the additional goal of representing the nine islands. The cards have four wallpapers (**, *2222, 22*, *442), fourteen friezes (two examples of each), and eight rosettes (four of each). Analogously to the first, this second deck has mirror cards and the box has an ambigram.

Figure 4: *Baralho de Simetrias - Calçadas dos Açores*: *A pair of cards (problem/solution).*

After creating a deck of cards, University of Azores, Ludus Association, and Expolab collaborated to launch the exhibition *Por entre calçadas dos Açores: à descoberta de simetrias* (*Amidst the Sidewalks of the Azores: Discovering Symmetries*, Figure 5). This is a traveling exhibition that, after visiting several islands, was inaugurated at Expolab on the island of S.Miguel on November 24, 2022. Visitors are given an initial explanation about the four types of symmetry, using sidewalk designs as examples. After the explanation, visitors are presented with a sidewalk motif and challenged to discover its location in the Azores and classify its symmetries. To complete the challenge, participants navigate through some stations, carefully observing the details, manipulating the motif with mirrors or acetates, eventually turning it upside down. The visit culminates with a detailed explanation of the symmetries present in the motif, as well as its location in the Azores.

Figure 5: *Amidst the Sidewalks of the Azores: Discovering Symmetries*.

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