Physical Representations of Polygon, Wedge, and Arc Squiggles

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Abstract

This document describes the construction and properties of the geometric structures that I call "squiggles", along with some ways in which I use them to create digital and physical art. Brief explanations are given as to how I interpret them in various physical ways including the use of beads, crochet, tatting, and paper folding.

Introduction

Over the years, I have experimented a great deal with how regular polygons fit together, often with the aid of vector graphics programs. A few years ago, I expanded my explorations involving rings of edge-touching polygons to include rings of such rings, since my hardware was now powerful enough to allow such manipulations provided that I was patient enough. Removing portions of the rings, or switching the direction of an "arc of polygons" would sometimes enable me to avoid overlap when putting together these rings. Around the same time, I started experimenting with making rotationally-symmetric Dürer-like tilings with polygons other than pentagons; finding ways of arranging the polygons to fit within a triangular shape. Eventually I noticed that piecing together chains formed by portions of polygon rings in alternating directions would sometimes give rise to figures with interesting properties, such as fitting together nicely to form rotationally symmetric patterns.

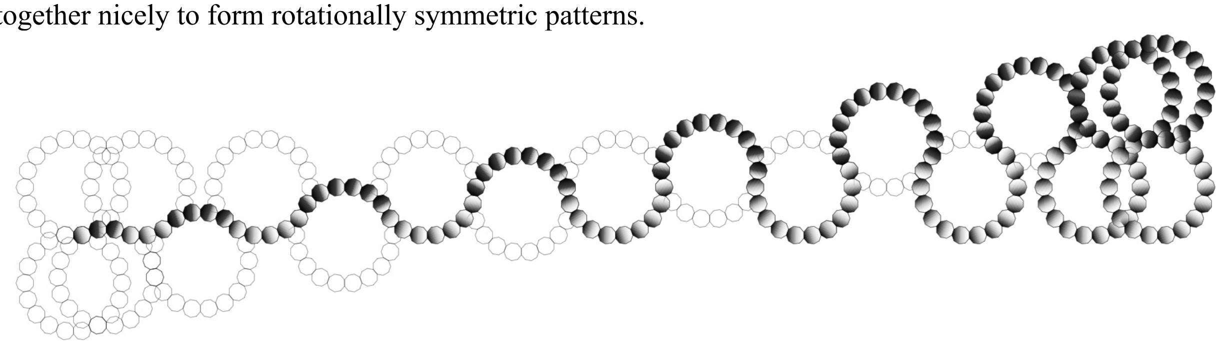


Figure 1: *A squiggle based on rings of edge-touching nonagons.*

Since I was constructing these figures "by hand" in a graphics program by using operations such as copy, paste, and rotate, there was some motivation to search for ways to increase the accuracy in placing shapes, as well as for shortcuts in performing tedious tasks. Finding tricks like grouping a triangle with a polygon to get a handy center-of-rotation to form the polygon ring probably helped me notice that the regular polygons themselves were not strictly necessary. Working with isosceles triangles, circular sectors, or even crescents made of rhombi could also result in figures with similar properties (see Figures 2 and 3). However additional constraints were needed to ensure that a rotational pattern could be formed.

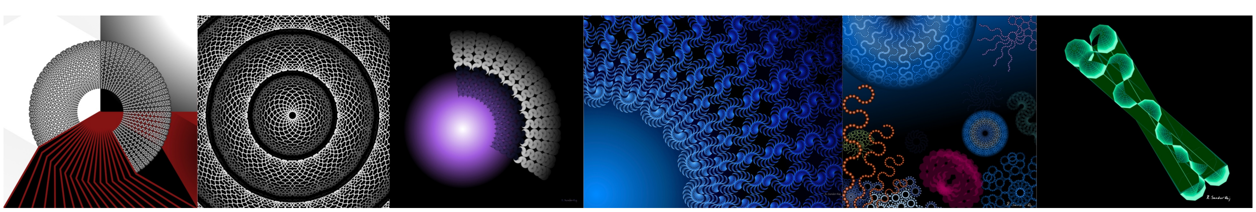


Figure 2: *Some of my digital art involving squiggles.*

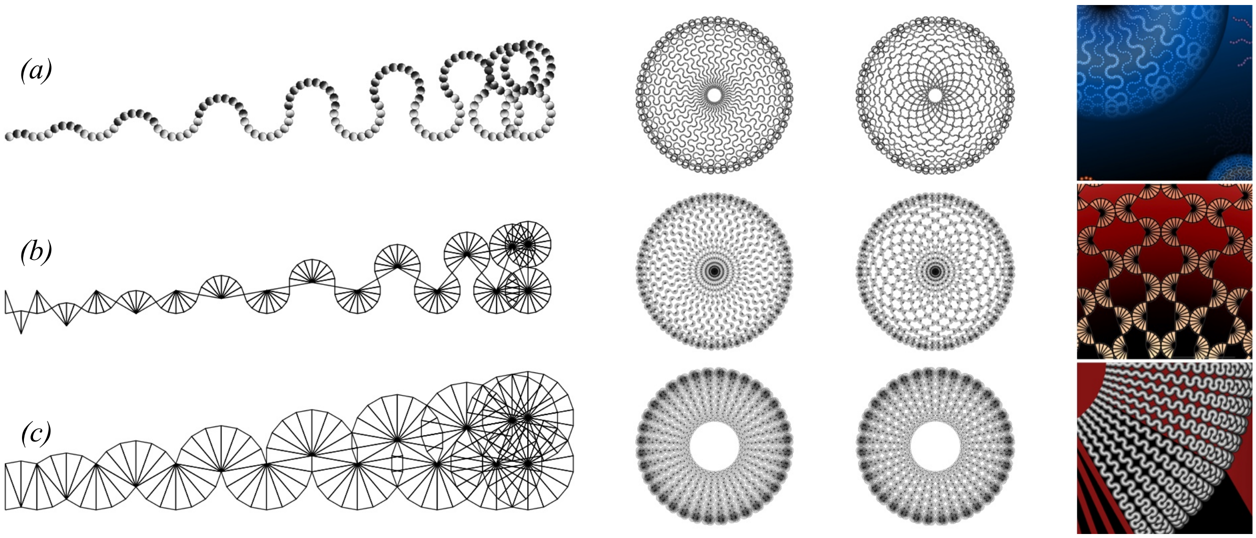


Figure 3: *Some types of squiggles along with how they would look rotated around a point singly, and in mirrored pairs, as well as a sample of my artwork containing that type: (a) a polygon squiggle, (b) a wedge squiggle, (c) an extended wedge squiggle.*

Definitions

We can loosely define a *squiggle* as a figure created by moving along a sequence of tangent circles of constant size, switching direction at each point of tangency. A sequence and a unit (see below) are used to express the relative positions of these circles. I call the single curve formed by portions of such circles an *arc squiggle*, but I am being deliberately vague with the general definition of a squiggle because I would like it to be able to include clusters of shapes, so long as they follow the same form as that curve.

A *polygon squiggle* is a chain of edge-touching regular polygons which is formed in a similar manner, with the tangent circles being represented by rings of edge-touching polygons sharing an adjacent pair of polygons. Traveling from one “circle” to the next involves getting to the first of the shared pair of polygons, then switching to the other “circle” at the second polygon (see Figures 1 and 4(b)).

For a *wedge squiggle*, the circles are represented by regular polygons which touch at vertices. A *wedge* is an isosceles triangle formed by two adjacent vertices and the center of a polygon. Traveling from one “circle” to the next involves rotating a wedge 180° around the touching vertices so that it now has a vertex at the center of the second “circle” (see Figure 4(c)). An *extended wedge squiggle* is just a wedge squiggle whose wedges have been doubled in length causing each wedge to share an edge with each of its neighbors (Figure 4(d)).

Each squiggle is associated with a *sequence* $a_0, a_1, a_2, a_3, \ldots, a_n$, which indicates how far to travel around each “circle” before switching to the next one. This distance is defined in multiples of some *unit*, which I usually take to be a polygon, a wedge, or an arc length. The *squiggle angle* is the angle subtended by this unit at the center of the “circle”. If the sequence is of the form $a, a+k, a+2k, \ldots, a+nk$, then I will refer to both the sequence and the squiggle as *linear*. I sometimes write the terms of the sequence in the center of the “circles” to make it easier to keep track of how many units are being used from each one.

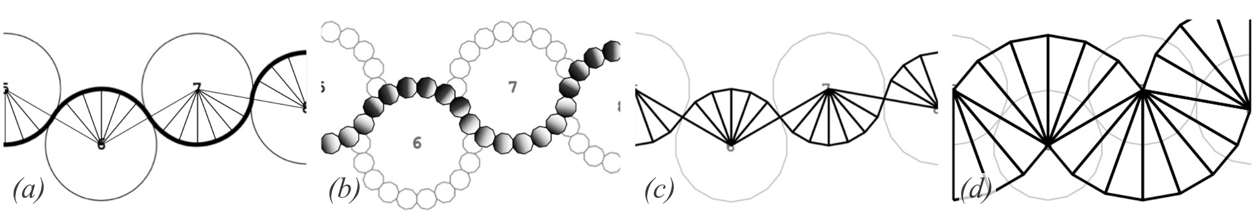


Figure 4: *Close-ups of the transition between “circles” when constructing: (a) an arc squiggle, (b) a polygon squiggle, (c) a wedge squiggle, (d) an extended wedge squiggle.*

Some Properties of a Linear Squiggle

The most notable property of a linear squiggle seems to be the nature of the triangle which bounds it, or in the case of a linear wedge squiggle, bounds everything but a few bits which stick out (see Figure 5). I think that it can be shown that regardless of the starting number in its sequence, the triangle bounding a linear squiggle with squiggle angle β and an increment of k, will contain an angle of $\theta = \beta k/2$. Its other two angles are dependent on the starting number and how many units go around the circle.

Figure 5 shows changes to the bounding triangle of a linear squiggle caused by varying the sequence of the squiggle while keeping its squiggle angle constant. Note that although linear squiggles often form nice rotational patterns as shown in Figure 3, they can also be built so that $\theta = \beta k/2$ does not divide 180° or 360°, for example, $\beta = 10°$and $k = 5$ gives $\theta = 25°$ (see the discussion in the supplement for further details).

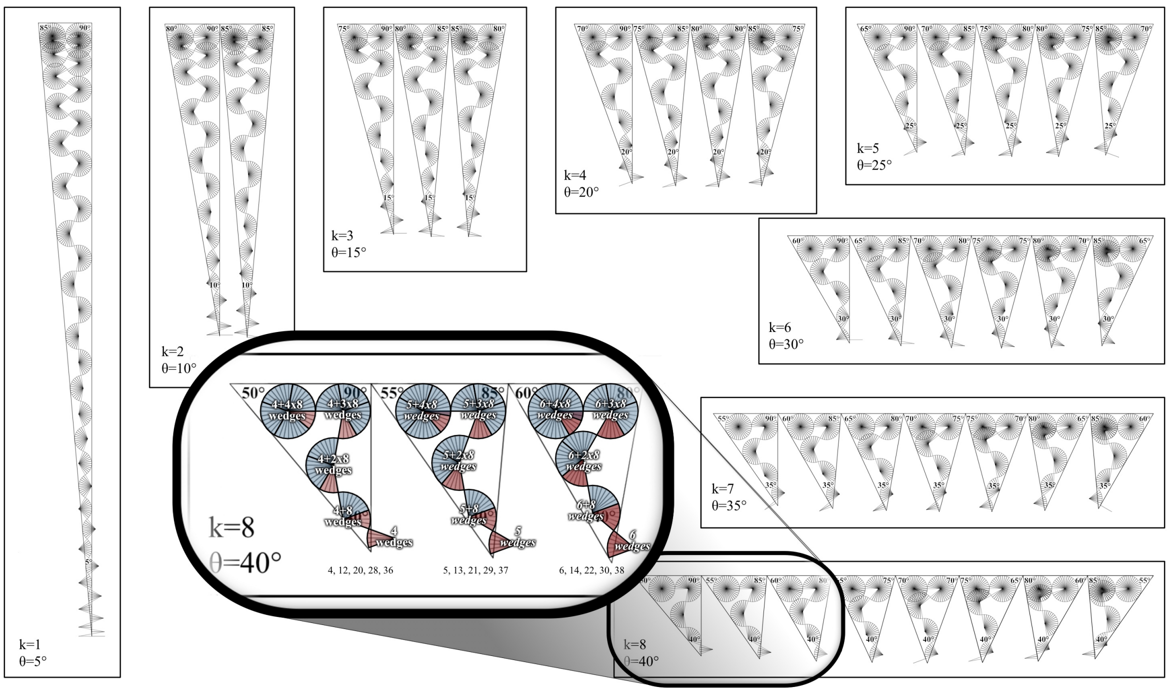
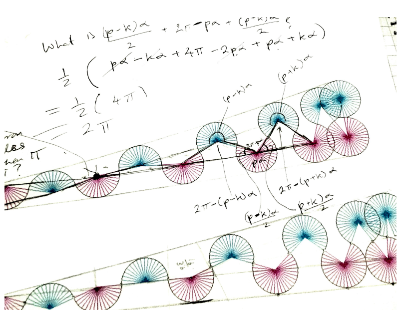


Figure 5: *Linear wedge squiggles constructed using a wedge with squiggle angle $\beta = 10°$, and incrementing by k wedges each time.*

It can also be shown that for every linear squiggle, the centers of the "circles" corresponding to alternating groups of wedges or polygons will be collinear. This can be seen by taking 5 adjacent groups of wedges, constructing triangles with vertices at their centers, and using properties of how the squiggles were constructed (see Figure 6(a) and the supplement to this paper).

(a) 

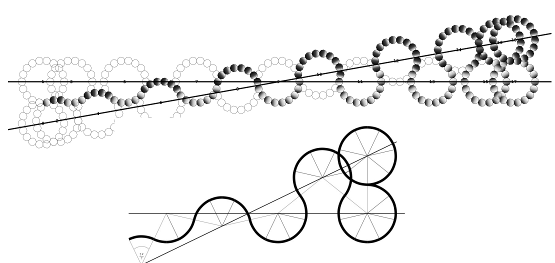
(b) 

Figure 6: *Collinearity of the centers of alternating parts of squiggles:*
(a) an excerpt from my notes, (b) linear squiggles showing the lines through the centers.

Figure 7: *Early attempts at physical squiggles.*

Constructing a Physical Squiggle

Probably my simplest attempt at making a physical squiggle involved drawing a line down the center of strip of paper, fan-folding it, then taping together groups of folds on alternating sides. This resulted in something like an extended wedge squiggle, with the center line giving the appearance of an arc squiggle (Figure 7(a)). Figures 7(b) and (c) are similar, but pairs of beads at the edges were used to curve the line of single beads down the center. Figure 7(d) involves a folded ribbon, but in this case, the curves are forced by only stitching beads between folds on the side where the squiggle should curve outward.

I have recently learned to tat, which involves making lacy patterns by tying lark's head knots around a core thread. I have found that I can force circular arcs on the core thread by choosing which side the thicker part of the knot is on. So to switch directions in a curve, I can either switch which of my threads is the core thread (for example by flipping stitches) as in Figure 8(a), or simply leave a bit of loose thread and flip the entire piece around to work on the reverse side (Figure 8(b) and (c)). In Figure 8(c), I have also attached spokes made of eye-pins by threading them onto the core thread.

When crocheting, I have found it convenient to use a central line of some sort, be it a pre-made length of sequins, beads, or even just a piece of thread, and nudge it to the left or right by means of making stitches around it that are unequal in size. I can approximate circular curves in the central line if I am careful about choosing my stitches; this can involve making many test samples to get the inner and outer curves matched well enough for the piece to lie flat. If I alternate groups of inner-curve and outer-curve stitches to match a squiggle sequence, the result is the part of an extended wedge squiggle surrounding the main curvy bit, but possibly missing the outer edge and the centers of the circular arcs (see Figures 8(d), (f), and (h)). In Figure 8(e) I have digitally copied and rotated Figure 8(d) to see whether it would be worth crocheting more copies. I have simply used a needle, thread, beads, and sequins for the squiggle in Figure 8(g), and Figure 8(i) shows an interpretation one of my polygon squiggle pictures using a form of bead crochet [1].

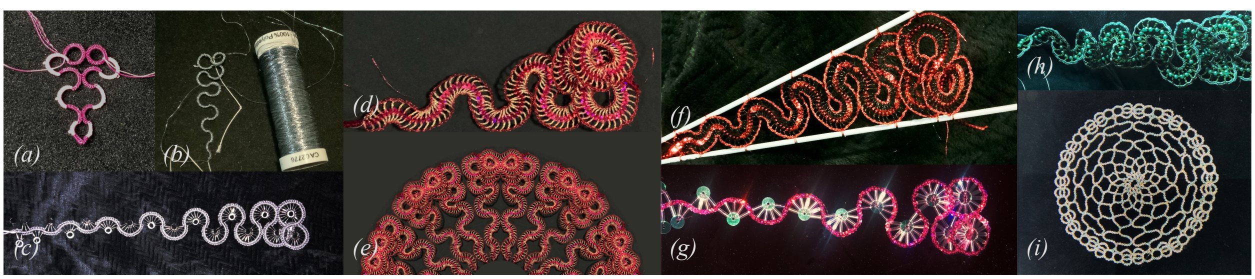


Figure 8: *Physical squiggles using tatting, crochet, and beading.*

Concluding Remarks

There are more ideas and properties that I do not have the space to discuss here such as a link with rhombus rosettes, construction with a compass and straightedge, the use of angles which do not divide 360°, complementary squiggles, recursive squiggles, and non-linear squiggles. I have documented some of these explorations on Twitter as I was doing them [2]. The supplement to this paper contains proofs, more detailed examples, and discussion of some of these topics.

References

[1] R. Sunder-Raj. "Approximating Edge-Touching Regular Polygon Patterns Using Crocheted Bead Lace." *Bridges Conference Proceedings*, Online, Aug. 2-3, 2021, pp. 281-284. http://archive.bridgesmathart.org/2021/bridges2021-281.html

[2] R. Sunder-Raj. *Polygon, Wedge and Arc Squiggles*. 2018. https://twitter.com/i/moments/1050938075222855680