Realizing the Borromean Rings in Wood via Yosegi Zaiku

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Abstract

The Borromean rings, an interlocking knot of three simple curves, has yielded motifs that have been used for centuries around the world. *Yosegi Zaiku* is an old decorative Japanese woodcraft used to realize a variety of geometric patterns. I report on my effort to extend the library of Yosegi patterns to include one based on the Borromean rings.

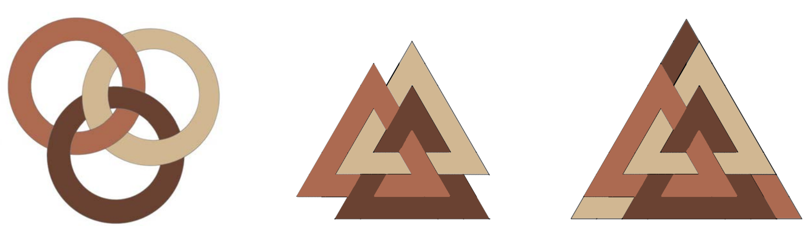
Introduction

The Borromean rings are three intertwined simple curves in three dimensions such that breaking one curve will leave all three curves no longer intertwined. Ancient motifs have been based on them and they are a topic of interest in modern topology. As a motif, they were used by early Germanic peoples and are found on ancient Indian and Japanese temples. As a modern mathematical object, the Borromean rings are relevant to knot theory, algebraic topology, and other fields.

Yosegi Zaiku, or simply *Yosegi*, is a decorative woodcraft technique originating in Hakone, Japan. It was first developed by Nihei Ishikawa in the late Edo period. Different species of wood are carefully cut and intricately combined to yield blocks displaying a chosen geometric pattern. The wood blocks are such that skilled craftsmen can plane off thin (≈ 0.1mm) sheets of veneer showing the entire pattern, similar to the way murrine glass canes are cut to reveal their pattern. The veneer are then used to decorate woodcraft items, the most common being Japanese puzzle boxes.

Designing the Pattern Cell

Viewed mathematically, the Borromean rings are a three dimensional object. Using the usual method of showing over-and-under crossing with hidden segments yields a two dimensional representation (Figure 1(a)). As there are three rings, each is assigned its own color. Such a representation is not well suited for realizing via Yosegi, lacking the straight lines favored by wood working. Also, the presence of empty space prevents tiling without gaps. A smooth transformation provides a representation consisting of only triangles (Figure 1(b)). This is also one of the forms of the *Valknut*, a symbol used by the ancient Germanic people. Examples include the 7^{th}-century *Stora Hammars I* stone in Gotland, Sweden.

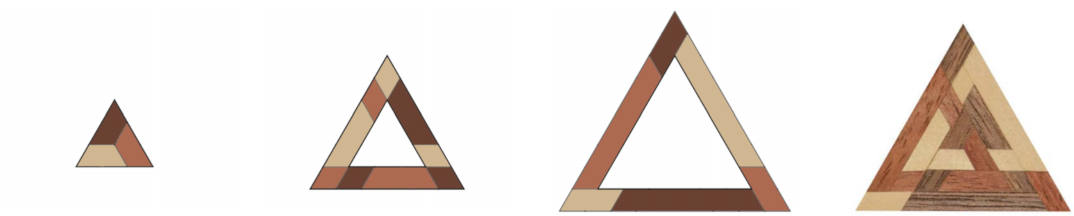


(a) **(b)** **(c)**

Figure 1: *Different two dimensional representations of the Borromean rings: (a) Familiar representation focusing on the intertwined ring structure, (b) Smoothly transformed into a triangular form, (c) With added "filler" to the corners to achieve a space-filling tiling cell.*

Figure 1b is not space-filling when tiled. As each of the "missing" sections are surrounded by only two colors, the representation is made space-filling by adding extra pieces of the third color (Figure 1(c)). Without changing the topology, the ring design has been transformed to occupy an equilateral triangle, one of the three tessellating regular polygons.

Having a design for a space-filling tileable pattern cell, the question is how to adopt it for the Yosegi technique. The goal is a wood pattern block from which thin slices (≈ 0.1mm) of veneer can be planed. This requires the pieces which build up the block fit together with high tolerances. The glue joints between the pieces need to have the wood fibers in contact and not depend of glue spanning gaps for adhesion. A *layer decomposition* of the design will allow assembling the pattern cell in layers. This procedure allows the shape of each layer to be refined and the tolerances maintained as construction progresses.



(a) **(b)** **(c)** **(d)**

Figure 2: *Decomposing the space-filling tiling cell into layers to facilitate construction via the Yosegi technique: (a) inner layer, (b) middle layer, (c) outer layer, (d) the final pattern cell, as actually assembled.*

Figure 2 shows the decomposition of Figure 1(c) into three layers. Figure 2(d) is an actual constructed Yosegi pattern cell. From this decomposition we extract the *cut-list*: the basic shapes needed to build up the pattern cell.

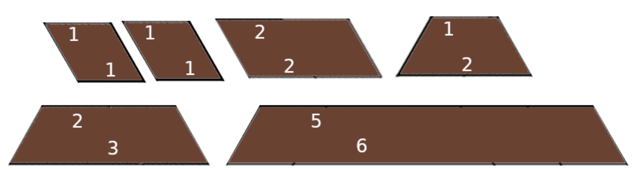


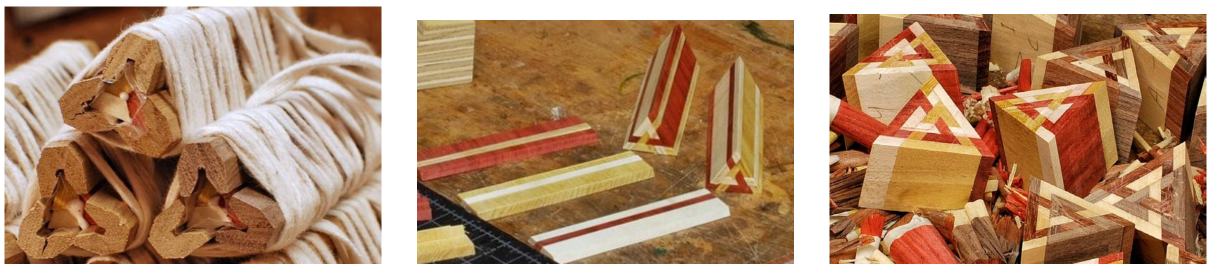
Figure 3: *The cut-list for each species of wood for each pattern cell. All the pieces have the same thickness and the lengths, marked in white, are integer multiples of the thickness.*

Comparing the layer decomposition with the initial pattern cell (Figure 1(c)), it may be tempting to view this as not optimal. It is possible to generate a cut-list with fewer pieces, but such a decomposition would entail assembling the entire cell in one glue-up session. This is impractical as the glue's short *open time*, the time between applying the glue and the glue beginning to cure, doesn't provide sufficient time to arrange the pieces as tightly as needed. It also fails to guarantee all the pattern cells have the same size and shape.

Constructing the Pattern Cell

I realized two different versions of the Borromean rings, the difference being the wood choices for each. As with all Yosegi, wood choice is crucial. On one hand, we want interesting and colorful woods. At the same time, the resulting pattern block needs to allow thin slices of veneer to be cut with a plane. Wood species with fiber structures which allow planing without damaging the resulting shaving, what is commonly referred to as *tearout*, are needed. There is additional challenge of sourcing boards milled with grain oriented as to lend themselves to planing without tearout. My training and experience taught me to look for *rift-sawn* boards, (boards cut from the log with the grain intersecting the faces at an angle between 30^o and 60^o).

With these considerations in mind, I chose Holly, Yellowheart, and Redheart for one block, resulting in a white, yellow, and red pattern. The other pattern was made with Adler, Bubinga, and Walnut, as to have a pattern ranging through typical wood tones.



(a) **(b)** **(c)**

Figure 4: *Building up the layers of the Borromean rings pattern cell. (a) the three pieces of the inner layer, glued and "clamped" via cauls and tightly wound cord, (b) building the middle layer on top of the inner layer, (c) the finished pattern cells, along with shavings generated while refining the shape of the triangles.*

After milling boards of each species to all the same thickness (4mm) and cutting the boards as per the cut-list in Figure 3, I begun assembly by gluing the three pieces of the inner layer together. These are then wrapped in plastic and clamping pressure is applied using corner cauls and tightly wound with cord (Figure 4(a)).

After the glue has fully cured ($\approx$ 12 hours), I refined the shape of the inner layers. Each triangle in turn was placed in a triangular form and a specially prepared hand plane was used to plane each of the faces in turn. The hand plane's blade was shaped to be slightly convex. When the blade was properly adjusted in the plane's body, the only the blade's center extended from the body. Thus the plane only took shavings from the work piece without changing the shape of the form holding the work piece. When each of the inner layer triangles are worked until no more shavings are taken from any of the three sides, all the triangles will have the same dimensions, along with guaranteeing they are equilateral triangles. This process was repeated for each of the next two layers, using appropriately sized forms to hold the work piece. *This is one of the key steps in the Yosegi technique*. It is why the desired pattern was broken into layers. After each layer, the dimensions and shape of all the parts are the same. It is how the tight tolerances required for the final blocks are achieved.

Assembling the Cells Into a Block

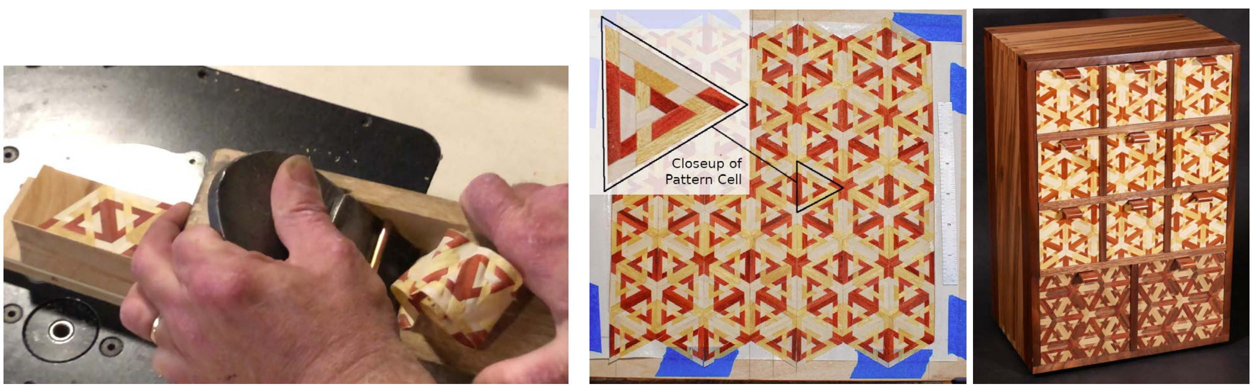
Most traditional Yosegi patterns have cell patterns which readily tile. This isn't the case for this pattern, as the asymmetry of the three colors breaks the 3-fold rotational symmetry of the equilateral triangle (Figure 1(c)). Arrangements which tile aesthetically require additional considerations when assembling the cells into pattern blocks. Reflection transformation are available as either face of the assembled cell can be used.

This reflection was used to match the colors on a single side of pairs of cells (Figure 5(a)). This was done for each color, resulting in three parallelograms. These then were arranged with the colors matching at their boundaries. In practice, this was repeated, yielding a pattern block containing the full pattern twice, allowing longer strips of veneer to be generated in a single pass.

(a) **(b)**

Figure 5: *Assembling the cells into the final pattern block. (a) Arranging the cells into intermediate parallelogram blocks, (b) the two final Borromean rings pattern blocks.*

A Japanese hand plane was used to plane off thin (≈ 0.1mm) shavings (Figure 6(a)) to use as veneer. Taking advantage of either side of the veneer strips being usable, I used the reflection transformation when tiling the veneer. When reversing every other strip, I achieved the final tiled pattern for my interpretation of the Borromean rings (Figure 6(b)).



(a) **(b)** **(c)**

Figure 6: *Using a completed pattern block, (a) a Japanese hand plane,* kanna*, was used to plane off single shaving, (b) multiple shavings assembled to reveal the larger tiling pattern, with a closeup of a single pattern cell included, (c) a decorated piece, "Apothecary Cabinet with Borromean Rings Yosegi", included in the Art Exhibition. Please see the larger version here: https://gallery.bridgesmathart.org/exhibitions/2023-bridges-conference/nicholas-phillips*

Summary and Conclusions

Starting from the classic depiction of the Borromean rings, I have shown how I designed a version useful for the Japanese woodcraft technique of *Yosegi*. I've reported on the technical issues involved with constructing such a pattern. The availability of the reflection transformation facilitated a final piece with additional patterns on different scales, something not typically found with traditional Yosegi.

Acknowledgments

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