Rolloids

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Abstract

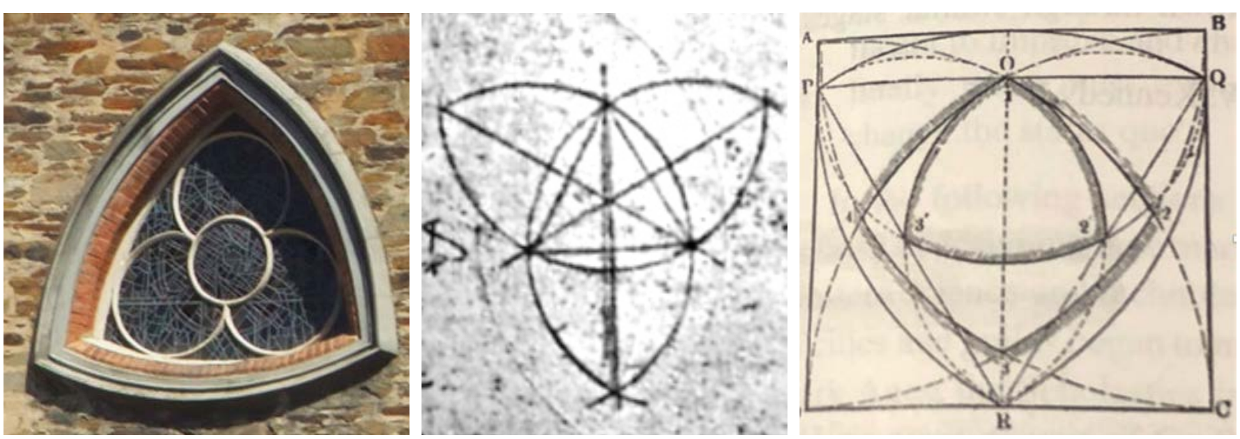
There are many objects that are not spheres but can nonetheless roll like a ball. Here we present an overview of a number of these geometrical shapes and their resulting kinematics and dynamics. Some of these objects were originally designed as artistic sculptures. Others have roots in architecture, others in utilitarian devices, and still others can be appropriately categorized as "play art."

Introduction

There are a wide range of mathematically defined or derived objects that can roll on a flat horizontal surface or down an inclined plane, either in a straight line or with a meandering motion. These include Orbiforms, Oloids, Sphericons, Femispheres, Wobblers, Two-Disk Rollers, the Orbis, Steinmetz solids and others. In this paper we introduce the name "Rolloids" for this broad range of dynamical objects. We should note that while randomly shaped objects, such as garden stones, can roll and may not gather any moss, they are not included here because for objects to be considered rolloids they must be mathematically well-defined objects. Also, though one can roll the dice, we do not include regular polyhedra, only objects with at least one curved surface or edge.

Orbiforms and Solids of Constant Width

Shapes of constant width ("orbiforms") have been known since the Middle Ages [7] [9] [12]. Some two-dimensional versions appear in gothic windows (Figure 1(a)) as early as the 13th or 14th century. Around 1500 AD, Leonardo da Vinci drew what is today called the "Reuleaux Triangle" (Figure 1(b)) and used the curve as the basis of his world map. Reuleaux's own diagram (ca. 1875) of the shape is shown in Figure 1(c). Leonard Euler was the first mathematician to have written about their mathematical properties (ca. 1771). He gave them the name "orbiform" from the Latin term for "circle shaped curves." He was the first to have realized that they are shapes of constant width.



(a) (b) (c)

Figure 1: *(a) Medieval Church window, (b) Da Vinci drawing, (c) Reuleaux's triangle construction [7].*

Today, non-circular constant-width, two-dimensional orbiforms can be found, for example, in various coins, including the Bermuda Reuleaux triangle shaped coin (Figure 2(a)); the 50 pence British heptaorbiform coin (Figure 2(b)); as well as in some Reuleaux triangle shaped manhole covers, for example, such as ones found in San Francisco (Figure 2(c)).

(a) (b) (c)

Figure 2: *(a) Bermuda coin, (b) British 50 Pence coin, (c) San Francisco manhole cover.*

The 19th century German engineer Franz Reuleaux is one of the first to have studied the purely *kinematical* properties of two-dimensional orbiforms [13]. Shapes of constant width can also exist in three dimensions. Ones of varying shapes but with the same width can roll between two straight parallel rods while maintaining contact with both (Figure 3). This is a purely *kinematical* property of the objects.

Figure 3: *Orbiforms of various types placed between two parallel surfaces (from reference [1]).*

There are an infinite number of possible orbiform shapes. In Figure 4 we show four of them: the Reuleaux orbiform, the pentaorbiform, and the heptaorbiform based on the Reuleaux triangle, pentagon, and heptagon, respectively, and one based on the tetrahedron with rounded corners. Different three-dimensional orbiforms of the same width can roll between two flat surfaces and could serve as non-spherical ball bearings (Figure 3). This is a purely kinematic property of the objects. Geometry alone determines their rolling motion. No detailed *dynamical* analysis involving mechanical forces, gravity or friction is needed to describe and determine their motions in this circumstance.

(a) (b) (c) (d)

Figure 4: *(a) Reuleaux, (b) pentaorbiform, (c) heptaorbiform, and (d) tetraorbiform (each ~ 5 cm).*

In Figure 5 we show sculptural variations of the "Meissner" object. The polished bronze sculpture by Canadian artist Gord Smith is what he calls the "Superall," an object based on the tetrahedron with curved edges and surfaces. It was not specifically designed to be a Meissner object, but it is very close to it.

(a) (b) (c) (d)

Figure 5: *(a) 19th Century plaster Meissner Object (~20 cm), (b) J. Valle stainless steel sculpture "Tetrasphero" (~16 cm), (c) G. Smith Bronze "Superall" polished bronze sculpture (~30 cm), (d) Man Ray oil painting "Hamlet" based on the Meissner object he saw at the Institut Henri Poincaré in 1933.*

Detailed searches of the mathematics, physics and engineering literatures have uncovered absolutely no references to the dynamical (including forces) as opposed to the kinematical properties of three-dimensional orbiforms. If the orbiforms are subject to forces, either purely mechanical or electrodynamical, they can move in unusual ways. Then an entirely geometrical (kinematical) analysis of their motion is inadequate, and one must take into account friction, gravity, and other forces–as well as their velocity and acceleration– to determine their subsequent motion. These objects can roll down an inclined plane with a slightly meandering motion. However, playing billiards with them would prove to be quite a challenge!

Oloid

This three-dimensional curved geometric shape was found by the Swiss/German designer/inventor/artist Paul Schatz in 1929 [15]. It can be described as the convex hull of a skeletal frame produced by placing two linked congruent circles in perpendicular planes, so that the center of each circle lies on the edge of the other circle. He found the shape from his study of the eversion of a cube. Schatz himself did not state anywhere the origin of the name "oloid." However, in an introduction to [15] called "Apropos," the writer Georg Unger wrote: "Later on, his main concern was to work out the details of a new science that he named "polysomatology." His original name for the three-dimensional body developed from two opposing rods was "polsomatoloid," but as he thought that no one would be able to pronounce it, he shortened it–half jokingly–to "oloid." Another suggested possible origin of the name "oloid" is the use of the Greek word "holos" or "olos" meaning whole or complete as a prefix combined with the suffix "oid."

(a) (b) (c) (d) (e)

Figure 6*: Oloid sculptures made from (a) bronze (21 cm), (b) stone (~12 cm), (c) brass (~ 11 cm) and (d) wood (22 cm), along with (e) path of the rolling oloid.*

Originally the oloid was considered just a mathematical curiosity. Schatz continued to work on engineering applications of the object for use in mixing dense fluids and powders and, in 1968, he received Swiss patent number 500,000 for its practical application. Oloids made in a variety of materials and sizes are shown in Figure 6 along with the path that they make when rolling [1]. The large bronze and wooden versions are certainly beautiful sculptural art works (Figure 6).

Maces, Stepped Maces, Sphericons, and Femispheres

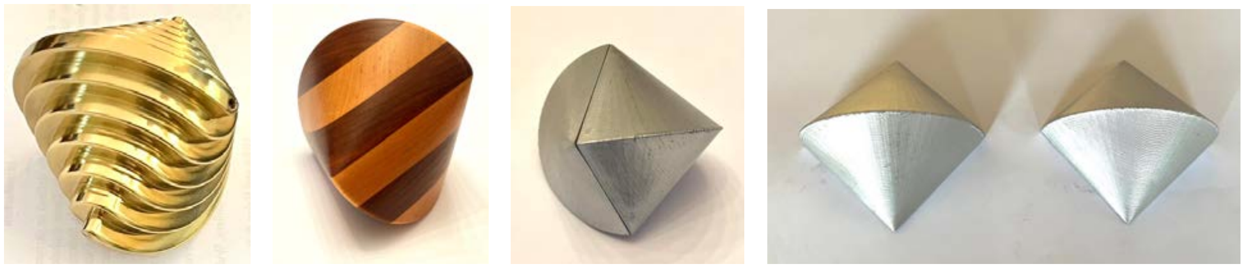
The objects that we discuss here go by many names: maces, sphericons and femispheres (Figure 7). As far as I can tell, the basic shape was first discovered and fabricated by the artist/designer/sculptor Charles O. Perry in about 1964 [14], contrary to the unsubstantiated history of the object reported in the *Scientific American* article by Ian Stewart [20]. Perry's first version of the object was called a "mace," with variants called by such names as "ribbed mace" and "step mace." A version of the "step mace" was independently invented by David Hirsch in 1980 [11]. It was apparently also independently found by the British wood craftsman C. J. Roberts in 1969 who gave it the name sphericon.

It is easiest to begin this section by describing how a "sphericon" can be made. Start with two right angle cones with cone angles 90 degrees. Glue their two flat faces together, then cut the resulting object from one cone tip to the other, rotate the resulting parts by 90 degrees and glue the resulting objects together. Voila, the "sphericon"! This is the shape that Perry called a "step mace" (Figure 8(a)). Perry's topologically related "mace" was first exhibited publicly in large scale sculptures that could not roll, but he also created handheld versions in the 1960s and 1970s that could roll.

(a) (b) (c) (d)

Figure 7: *(a) C. O. Perry bronze "mace" (fabricated by the Centro Duchamp in 1970 (~ 10 cm)), (b) R. Rollings wood "femisphere" (~ 10 cm, (c) M. Salemi wood femisphere (12 cm), (d) H. Schepker stained glass femisphere sculpture (~60 cm).*

The mathematical path by which Charles Perry arrived at his mace is quite different from the one described here for the sphericon (Paul Perry, private communication). In a lecture Charles Perry said "I would take a theme and run it. In this case the theme is very simple: what happens if you turn a sphere inside out on the lines of the stitching of a baseball?" The mace, whether made of bronze or other materials has a marvelous rolling path [2]. (The origin of the name "femisphere" for this object appears to be obscure in the literature. However, the Canadian wood turner and now retired Professor Richard Peter Rand wrote to me that "...they are called sphericons when the cones have straight sides, "femispheres" when the sides of the cones are more … feminine!" In our private correspondence he claims to be the source of the name femisphere. It seems to me remarkable that these objects which can be arrived at by various logical paths had not been discovered until the mid-20th century. Why didn't mathematicians and physicists who had explored double cones in the uphill roller demonstration since the 17th century split a double cone lengthwise to discover the object? Why didn't the Greeks (who were obsessed with the cone and its slices - circle, ellipse, and hyperbola) discover the sphericon?



(a) (b) (c) (d)

Figure 8. *(a) C. O. Perry brass "Step mace," (b) wood sphericon, (c) K. Brecher 3D printed plastic sphericon, pieces held together by neodymium magnets, (d) the 2 components of the same object.*

Generalized Streptohedrons

A generalization of the class of objects just discussed that includes the sphericon, along with many other shapes derived from regular polyhedrons by splitting them in half, rotating the resulting parts by 90 degrees and gluing the parts back together are called "streptohedrons." In this case the name has a rational origin: "strepto" (twisted) combined with "hedrons" (faces). The literature about such objects has been surveyed by the British wood turner David Springett [17] [18]. Beautiful examples of these objects have been made by the Canadian wood turner Robert Rollings (Figure 9(a)), amongst others. An example of one rolling is shown here [3].They form the basis of a very nice exhibit called "Twist-n-Roll" at the Museum of Mathematics in New York shown in Figure 9(b).

(a) (b)

Figure 9: *(a) Hexagonal Tunbridge Stickware Streptohedron by R. Rollings, (b) rolling paths of various streptohedrons (from "Twist-n-Roll" exhibit designed by G. Hart & T. Nissen, courtesy MoMath).*

Two-Disk Rollers and Wobblers

It is not clear when these objects were first discovered/devised [10] [19] [21]. Reference [10] says that the object was first devised by Frederick Flowerday. The earliest paper about them that I am aware of [19] was written by A. T. Stewart, published in the *American Journal of Physics* in 1966. He pointed out that if two round disks are joined at right angles, they will roll on a flat surface or on a slightly inclined plane [4]. He noted that if the centers of the disks are separated by a distance d equal to their radii $r/\sqrt{2}$, the resulting object–the two-disk roller or "wobbler"–will roll with its center of mass remaining a constant distance $r/\sqrt{2}$ above the rolling surface. Varying the separation between the centers only marginally varies the wobble up and down motion. A wide variety of such objects have been constructed on many size scales and made from a variety of materials. Some are intended as plays objects, others as works of art.

Frederick Flowerday produced and marketed a 3-dimensional plastic closed surface version of the wobbler (Figure 10(d)). David Hirsh has designed wooden versions of what he calls “ellipsoloids” made by starting with ellipses instead of from circular disks (Figure 10(c)). I have made many versions of wobblers using 3D rapid prototype printing and varied the parameters to assess their dynamical properties. I have also made two-disk wobblers by cutting grooves in coins and joining them together (Figure 10(a)). Large scale sculptural versions of the wobbler have also been constructed and exhibited.

(a) (b) (c) (d)

Figure 10: *(a) two disk roller made from quarters by KB (~4 cm), (b) brass two disk roller by P. Kim (~6 cm), (c) wood Ellipsoloid by D. Hirsch (~9 cm) (d) plastic wobbler by F. Flowerday (~5 cm).*

The Orbis, Sin, and the Holey Roller

Hoops have been a staple of play for millennia. They have appeared in art works since at least 500 BC. They have appeared in painted images on vases as well as in Roman mosaics (cf. Figure 11(a)). Probably the most famous medieval image of play with hoops appears in Pieter Bruegel the Elder’s painting “Children’s Games” painted in 1560. As far as I can tell, hoops have always been rolled as separate, single hoops, whether for rolling with a stick, or as “hula hoops” for osculating around a person’s waist in modern times. It is remarkable that only in the 1960s did anyone consider combining two hoops together to form a novel kind of rolling object. Peer Clahsen’s two ring device was called the “Orbis,” one of several “Objeux” (play objects) that he designed [8] and that were manufactured by the Swiss Kurt Naef toy company. A related object that Clahsen designed also appeared by the name “Sin,” short for sinus or sinusoidal.

With my artistic collaborator, the expert wood craftsman Randy Rhine, we decided to produce a large-scale sculptural artistic version of the Clahsen “Orbis.” Our resulting rolloid that we named the “Holey Roller” (Figure 11(d)) consists of two tori made of multiple wood segments conjoined at an angle of about 45 degrees. It can be seen merrily rolling along here [5]. Placing the resulting Holey Roller on its side on the surface of a rotating turntable produces a version of the “barber pole” illusion which was possibly first demonstrated at the Exploratorium in 1973.

(a) (b) (c) (d)

Figure 11: *(a) Roman mosaic of play with two hoops, (b) play with hoops from P. Bruegel’s “Children’s Games” (c) P. Clahsen “Orbis” (~5 cm), (d) K. Brecher & R. Rhine, “Holey Roller” (~ 20 cm).*

Assorted Other Objects: Steinmetz Solid, Parabolicon, Anti-oloid, Gömböc

Here we consider the dynamical properties of some other mathematical objects (Figure 12) that are worth mentioning in a discussion of what I am calling "rolloids."

(a) (b) (c) (d)

Figure 12: *(a) Copper Steinmetz Solid (~5 cm), (b) D. Hirsch aluminum Parabolicon (~8 cm), (c) wood anti-oloid (~9 cm), (d) aluminum Gömböc (~10 cm).*

A Steinmetz solid is the three-dimensional solid that results from the intersection of two or three cylinders of equal radius at right angles. The intersection of two cylinders, shown in Figure 12(a), is called a bicylinder. It is named for the physicist Charles Proteus Steinmetz (best known for his defense of Einstein's theory of relativity in the early 20th century) who determined its volume. However, the shape and its volume had already been known by Archimedes. When placed on an inclined plane and given a nudge, it will roll in a straight line [6].

David Hirsch has generalized the sphericon to what he calls "platonicons" [16]. The example shown in Figure 12(b) is what he calls a "parabolicon." It has 6 parabolic edges and 8 vertices. It is a new member to the line segment bodies (solids whose surfaces are created by self-intersecting ruled surfaces that create 3D space). The whole surface is made of a single smooth developable surface (with zero Gaussian curvature). While rolling, its center of mass maintains a constant altitude, so its motion is smooth.

The "anti-oloid" has a shape that is a continuous surface created by the trajectory of a line connecting two points that travel along two congruent circles in perpendicular planes so that the center of each circle lies on the edge of the other. Like the oloid, it can roll in a wobbly path down an inclined plane. It appears to be a new mathematical variation on the oloid. I know of no mathematical literature about the object.

Finally, the "Gömböc," [22] is quite a different kettle of fish–or turtle. It is the result of an investigation of a problem posed by the Russian mathematician Vladimir Arnold in 1995 concerning the stability of objects. Arnold had conjectured that there might be three dimensional homogenous objects that are mono-monostatic which, when resting on a flat surface have just one stable and one unstable point of equilibrium. The resulting object was found by the authors of reference [22] in 2006. The "Gömböc," as they called it, has a novel shape, and can roll back and forth on a surface, eventually settling in a stable equilibrium position. The authors partly discovered the shape by studying how turtles can right themselves when placed on their backs. While the Gömböc cannot roll along a flat surface in a straight line or with a meandering path, I include it here for its novelty and because it indeed rolls, even if only back and forth about an equilibrium position. It is a beautiful and novel mathematical object. A 4.5-meter tall sculpture of it sits in the Corvin Quarter of Budapest.

I will end this section by posing some questions to the art and mathematics members of the Bridges community: Just as the Mace can be found by everting a sphere and the oloid can be found by everting a cube, what does everting any other platonic solid produce? Or the Meissner object? Or the Steinmetz solid?

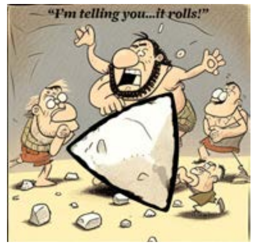


Figure 13: *Discovery of Rolloids. (initial Midjourney AI concept image adjusted by Kaz Brecher.)*

Summary

In this paper I have discussed a broad range of mathematically defined kinetic art objects whose chief dynamical feature is that they can roll. I call these objects “rolloids.” Though the *geometrical* properties of each member of the class have been discussed individually over the years, this is the first discussion that I am aware of that has considered them together and compares their dynamical properties. My main motivation for presenting all of these objects together is to try to motivate more investigations of their mathematical, physical, and artistic possibilities. I would also like to encourage people to try to discover new and unusual geometrical objects that have interesting artistic, physical, and dynamical properties.

Acknowledgements

I would like to thank Peer Clahsen, Gord Smith, Hans Schepker, David Hirsch, David Singmaster, Bob Rollings, Christian Ucke, Gabor Domokos, Charles O. Perry, and Paul Perry for fruitful discussions about various aspects of these objects. I thank my collaborator on many projects Randy Rhine. And I would especially like to thank my daughter Kaz Brecher for help with many aspects of this project.

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