Equations in Poetry

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Abstract

In the paper *What is an Equation?* Marcus and Watt provide an informative and entertaining account of the history of the word "equation." They state: "There are few concepts in modern mathematics that are more fundamental or more used than that of the equation." Yet, in spite of its wide and universal usage, the English definition and the French definition of the word are somewhat different. While in English an equation is a mathematical statement that asserts the equality of two expressions, in French the definition requires an additional condition, namely that this equality contains one or more variables. In addition, Marcus and Watt's paper mentions the "huge metaphorical capacity" of the word, which extends its usage from mathematics to language. For someone interested in the connections between mathematics and poetry throughout history, this paper raises several interesting questions: How do equations appear in poetry? Did this appearance evolve over time hand in hand with the development of the role and symbolic appearance of equations in mathematics? The current paper provides some answers to these questions.

Equations in Words

The word "equation" (from Latin *aequationem*) made its first appearance in English in the 14th century, but its mathematical meaning as a statement of equality of two expressions dates from late 16th century [26]. Equations appeared in poetry long before that in connection to solving basic algebraic equations such as linear equations, systems of linear equations, quadratic equations, and equations involving square and cube roots of one or more variables. Since the symbols used in modern algebra took till late 17th or early 18th century to perfect [2, 5, 7], for a long time the appearance of equations in both mathematics and poetry involved only words.

One of the earliest appearances of equations in poetry is in Archimedes' "The Cattle Problem" [22, 25]. Archimedes (287–212 BCE) posed this problem in verse to the mathematicians of Alexandria in a letter he sent to Eratosthenes of Cyrene (284–192 BCE). In twenty-two Greek elegiac distiches, the poem asks for the number of cattle belonging to the Sun god, subject to several restrictions. The restrictions may be divided into three sets. The first two sets of restrictions consist in systems of linear equations. We reproduce here lines 8–16 of Archimedes' poem translated by Hillion and Lenstra [22, 25], which describe the restrictions on x, y, z, and t, the number of white, black, dappled, and brown bulls, respectively. The equations to the right, not part of the original poem, illuminate the description of equations outlined in these lines.

from: ***The Cattle Problem*** by Archimedes

Each herd saw bulls in power unsurpassed,
in ratios these: count half the ebon-hued,
add one third more, then all the brown include;
thus, friend, canst thou the white bulls' number tell.
The ebon did the brown exceed as well,
now by a fourth and fifth part of the stained.
To know the spotted – all bulls that remained –
reckon again the brown bulls, and unite
these with the sixth and seventh of the white.

$$x = \left(\frac{1}{2} + \frac{1}{3}\right) y + t$$
$$y = \left(\frac{1}{4} + \frac{1}{5}\right) z + t$$
$$z = \left(\frac{1}{6} + \frac{1}{7}\right) x + t$$

Similar restrictions are described for the cows of the same respective colours [25]. The Cattle Problem with these two sets of restrictions can be solved nowadays using Linear Algebra. Even in deep antiquity, whoever solved the problem with these restrictions was called by Archimedes merely competent. Lines 33–40 of the poem add the more challenging restrictions, namely the requirement that $x + y$ be a square number and that $z + t$ be a triangular number.

Attempts to solve the problem with the addition of the last set of restrictions gave rise to the *Pell Equation*, $x^2 = dy^2 + 1$, where d is an integer, which is not a square, and the solutions x and y need to be positive integers. The first mathematician to solve The Cattle Problem with all the restrictions posed in the poem was A. Amthor in 1880. The solution generated a number that occupied, in reduced type, twelve journal pages – the number is approximately 7.76×10^{206544}. The Pell Equation continues to pose new solving difficulties to this day, as mathematicians struggle to find efficient computer-based solution methods. Interested readers may find more information and references about The Cattle Problem and the Pell Equation in [25].

Fast forward to the flowering of algebra during the Middle Ages in India with its cultural tradition of recording mathematical results and problems in verse [13]. Some of the most charming mathematical poems involving equations come from this tradition. For example, Bhaskara (1114–1185), the best known of medieval Indian mathematicians, wrote an algebra book intended for the education of his daughter, Lilavati. The book's title is also *Lilavati* (meaning "the beautiful," a reference to both mathematics and his daughter), and it was written entirely in verse. The translation of *Lilavati* into English is in prose [3], but a few of the poems translated in verse appear in other sources [9, 10, 23]. In the introduction to his book, Bhaskara writes (my translation): "I offer this treatise in mathematics, Lilavati, a subject admired for its elegance and clarity, conciseness, precision and literary quality." The topics covered in *Lilavati* include the extraction of square and cube roots, linear and quadratic equations, arithmetical and geometric progressions, and more. Below is my translation of a poem from *Lilavati* that requires a solution to a quadratic equation:

from: ***Lilavati*** by Bhaskara

Ten times the square root of a flock of geese,
seeing the clouds collect, flew towards lake *Mánasa*,
one-eighth took off for *sthalapadminí* forest.
But unconcerned, three couples frolicked in the
water amongst a multitude of lotus flowers.
Please tell sweet girl, how many geese were in the flock.

Let x be the number of geese in the flock. In modern mathematical notation this poem requires the "sweet girl" to solve the equation $10\sqrt{x} + \frac{1}{8}x + 6 = x$ in order to obtain $x = 144$.

The poetry of *Lilavati* inspired educators through the ages to experiment with poetry writing projects in their algebra classes. Interested readers may find more information and references about the modern uses of poetry in college classes in [10, 11].

The word *algebra* has its root in the Arabic word *al-jabru*, meaning *restoration*. In the 9th century, the Persian mathematician al-Khwarizmi (780–850) wrote an algebra book, using the word for the first time, and giving a systematic exposition of the state of knowledge regarding solutions to polynomial equations known in his days. Methods for solving linear and quadratic equations were known since Babylonian times, but the complete solution to cubic equations still awaited discovery. In 1202, the Italian mathematician Leonardo Fibonacci (1170–1240) published his book *Liber Abaci*, which introduced Europe to the Hindu-Arabic numeral system. The development of algebra in Europe went hand in hand with the dissemination of this new representation of numbers. The solutions to both cubic and quartic equations were first published in *Ars Magna* by Girolamo Cardano (1501–1576), although he discovered none of the pertinent formulas. The cubic was solved by Niccolò Fontana (1500–1557), better known as Tartaglia (the "stammerer"), while the quartic was solved by Cardano's pupil, Ludovico Ferrari (1522–1565). Cardano – physician, mathematician, gambler, and eccentric personality par excellence – published Tartaglia's solution in his book in spite of promising Tartaglia under oath to never divulge his solution to a living soul. This act of betrayal resulted in a lifelong feud between the two men [5, 7]. Below is my translation of a fragment from the cryptic verse solution to the cubic given to Cardano by Tartaglia [32], the case of the "depressed" equation $x^3 + ax = b$, where a and b are positive numbers. The equations to the right, not part of the original poem, illuminate the steps outlined in the verse.

from: ***Quando Che'l Cubo*** by Tartaglia

When the cube and the added things
Result in a certain number, find $[x^3 + ax = b]$
Two numbers whose difference is this one. $[u - v = b]$

You should always use these two numbers
Provided their product is equal
Precisely the cube of a third of the thing. $[uv = (a/3)^3]$

Then make it a general rule to subtract the
Cube-roots from each other, in order
To obtain the main thing as your answer. $[x = \sqrt[3]{u} - \sqrt[3]{v}]$

Robert Recorde (1510–1558), a Welsh physician and mathematician, is best known for his invention of the equal sign = which he justified in his algebra book, *The Whetstone of Witte*, with the words "because noe 2 thynges are moare equalle." Although there is no historical evidence that Cardano and Recorde ever met, their lives unfolded in Europe during the same time period, the Renaissance, when algebra – still in its infancy – was called *Cossike Arte* ("The Art of Things") and a card game called *Primero* became all the rage throughout the continent. This is the imaginary setting of my poem "Among Practitioners of Cossike Arte," offered below, a modern poem which refers to equations in words.

Among Practitioners of Cossike Arte by Sarah Glaz

We greet the old mathematicians at the door:
Good evening! Buona sera! Noswaith dda!

I serve a stew made of poetic equations,
while Ludovico – reciting his solution
to the quartic –
fills their goblets with red wine.

"Shall we commence playing *Primero*
after dinner," asks Cardano,
"or shall we play for money…
a lively game of chess?"

Put on the spot, Recorde answers,
"noe 2 thynges are moare equalle" –
although both know that Robert never gambles
and Girolamo would prefer to bet on dice.

I glimpse Tartaglia's scarred face behind a star
casting an evil spell.

This poem appeared in 2020 paired-up with a beautiful collage by Mark Sanders in the online Bridges Math Art Gallery [15] and in print [16].

Other extant poems involving equations written before the Renaissance include the "epigrams" – mathematical puzzles in verse – collected by Metrodorus (5th century, Greece) in the *Greek Anthology* [28], and a 54-line poem composed in Arabic by Ibn Al-Yāsamīn, which he used as a textbook to teach algebra in 12th century Seville, Spain [1].

Equations in Symbols

One can date and place the beginning of modern mathematics to Renaissance Europe, although the symbolic notation for equations took longer to perfect. In the 17^{th} century Calculus was invented by Isaac Newton (1642–1727) and independently by Gottfried Leibniz (1646–1716). This brought with it a new set of concepts and symbols: derivatives, integrals, limits, and infinite summation; as well as a flurry of mathematical activities aimed at exploring the new mathematical concepts in-depth. It was not till the 18^{th} century that the mathematical giant, Leonhard Euler (1707–1783), who made contributions to all areas of mathematics, introduced many of the symbols used today, including the symbol i for $\sqrt{-1}$ and the symbol e for the base of the natural logarithmic function. The development of Calculus brought with it new types of equations, some of which the French would call identities or formulas, involving derivatives, integrals, limits, infinite summations, and various other mathematical concepts [2, 5, 7].

As far as I know, the first poem that included an equation written in symbols was authored by the French poet, Raymond Queneau (1903–1976), the founder of the literary movement Oulipo – Ouvroir de Litterature Potentielle (Workshop of Potential Literature). Oulipo's purpose was to create literary works using constrained writing, and many of the constraints invented by its members were mathematical. Queneau's book, *Exercices de Style* (*Exercises in Style)*, published in 1947, is a collection of 99 prose poems that tell the same story – a man gets into an argument with another passenger on a bus. Each poem is written in a different style. Below is my translation of the story in the style called "mathematical" [29].

Exercices de Style: Mathématique by Raymond Queneau

> In a rectangular parallelepiped moving along a curve representing an integral solution of the second order differential equation:
>
> $$y'' + \mathrm{PPTB}(x)\, y' + \mathrm{S} = 84$$
>
> two humanoids (only one of which, humanoid A, displays a cylindrical part of length $L > N$ whose spherical cap is surrounded by two sinusoids with period ratios $= \pi / 2$) cannot generate a point of contact between their bases that is not a cusp. The oscillations of two humanoids tangentially to the above trajectory causes the infinitesimal displacement of any sphere of infinitesimally small radius tangent to a line of length $l < L$ and perpendicular to the upper part of the median of the plastron of humanoid A.

Amusing piece – a commentary on how the mathematical writing style sounds to a non-mathematician.

Modern poems include equations in symbols in their text or titles for a wide variety of reasons. As a sample, I include below the poem "Give Me an Epsilon and I Will Treat It Well" by Ray Bobo [4, 9].

Give Me an Epsilon and I Will Treat It Well by Ray Bobo

$x = \emptyset$	Found alone and wanting,
	I now languish unbegun;
$\emptyset \rightarrow \{\emptyset\}$	Oh, to have your arms about me,
	Be transformed into the one.
$\delta = \{\boldsymbol{\epsilon}\}$	Closeness for our angels
	Is returned with an embrace;
$x + y \rightarrow \infty$	So that by incorporation,
	We can soar to loving space.
$\partial \int f = f$	In meaning full,
$\int \partial f = f + C$	In asking all, Amen.

The equations to the left enrich both the form and the content of the poem, contributing to the poem's visual appeal, as well as adding to the message conveyed in words both clarification and an element of mystery.

Another example of a poem that includes equations in its text makes use of a classical proof, which contains a logical fallacy resulting in the conclusion that 2 = 1. Each step of the proof is an equation of the kind the French would call an identity. In the hands of the poet, H K Norla, this flawed proof becomes a complex love poem. The poem appeared in a now defunct blog in 2005. All attempts to trace the poet led nowhere. The full image of the blog entry in which the poem originally appeared, a visual-art rendition by Kaz Maslanka, and a commentary can be found in [27].

Of Xs and Ys by H K Norla

Who am I to say who I am?
$x = y$
You are me.
$x^2 = xy$
You are part of each of us.
$x^2 - y^2 = xy - y^2$
And we are each less without each other.
$(x - y)(x + y) = y(x - y)$
But are we not each separate?
$x + y = y$
Look beyond, and we are the same.
$2y = y$
You will then see who I am.
$2 = 1$
Who are you to say who you are?

In contemporary poetry, a different use of equations written in symbols employs them as shape makers, which also add nuances of meaning to the content. To compose such poems the poet replaces some or all of the symbols appearing in an equation with words. Early examples appear in the first anthology consisting entirely of mathematical poems *Against Infinity* [30], which was published in 1979. Below is a poem from this anthology [19, 30], whose lines are equations in which words replaced symbols.

Non-additive Postulations by Scott Helmes

random order + preposterous outcry = negative time

negative time^2 = relationships + 3

relationships = $\frac{\text{rudders}}{\text{udders}} + \sqrt{\frac{\text{alphawakes}}{\text{oscillations}}}$

ϕ + π = blueberryohio to the tenth power

Ohio = $\sum_0^\infty \frac{+\text{antioc}h}{\text{trying}} \sqrt{\text{power} + \phi}$

equality + three equality + 5 = race^2

without (recognition) + negative sex = tomorrow

Jefferson + $\frac{\text{airplane}}{\text{6+3pee}} = \frac{\text{pee} + \infty}{\text{green ddt}}$

$\frac{\text{negative}}{\text{sex}}$ + i.u.d = $\sqrt{\frac{\text{time}}{\text{communicate}}}$ + 1^2 + c

$\frac{\text{time}}{\text{telepathy}}$ = 2' + c = $\frac{\text{noosphere}}{\text{RBF}}$ =

terminate

It is fun to navigate through this poem's "postulations" discovering new meanings in the deceptively random word choices and mathematical expressions.

Kaz Maslanka is a major contemporary practitioner of poetry in which some or all the symbols appearing in an equation are replaced by words. Below is Kaz Maskanka's poem "Sacrifice and Bliss" [9, 27].

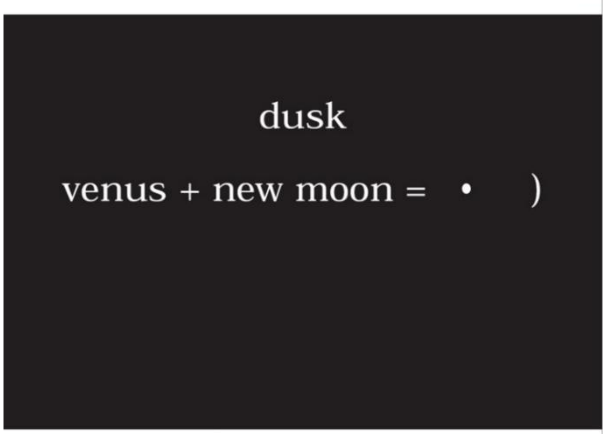
Sacrifice and Bliss by Kaz Maslanka

$$\text{love} = \lim_{\text{ego} \to 0} \frac{1}{\text{ego}}$$

The poem requires the reader to actually "do the math" on the words in order to understand the poem. Specifically, as the denominator of the fraction, the ego, shrinks to 0, the fraction grows larger and larger, till – at the limit – it becomes infinite. In other words, love's strength is inversely proportional to the size of the ego. Maslanka's poetry involves a wide variety of equations. He also embeds many of his poems in computer enhanced visual environments that intensify their visual impact. In addition to his own work Maslanka's blog [27] showcases poems involving equations, written by others.

A one-equation poem made out of words and typographical marks is Karl Kempton's visual poem "Dusk" [24, 27]:

Dusk by Karl Kempton



The equation in Kempton's poem enhances the visual effect, giving the poem its minimalist elegance, as well as conjuring an atmospheric image: darkening sky, crescent of waxing new moon, Venus glowing with intense luminosity….

We conclude this section by paying homage to the equation that inspired this paper, Euler's Identity:

$$e^{i\pi} + 1 = 0$$

Euler's Identity is a special case of a more general equation, involving trigonometric functions, discovered by Leonhard Euler. Euler's Identity is considered to be one of the most important equations in mathematics, as well as its most beautiful [8]. It is linked to several areas of mathematics, and it connects the five fundamental numbers 0, 1, i, π and e with precisely three basic arithmetic operations: addition, multiplication and exponentiation, each occurring exactly once. Reading in a book that this identity was called in English "an equation" prompted Marcus and Watt to investigate the word "equation" and write their paper [26], and therefore, indirectly motivated the writing of my paper. Thus, although the difference between the French and the English definitions of equations/identities had no particular impact on their use in either mathematics or poetry, it played an important role in motivating investigations into the links between mathematics and language. Euler's equation is celebrated in many poems, some of which appear with interesting commentaries in JoAnne Growney's blog [17]. These include my own poem "The Enigmatic Number e" [11, 14, 17], and Neil Hennessy's concrete poem "The Transcendence of Euler's Formula," [17, 20, 21] reproduced below.

from: ***The Enigmatic Number e*** by Sarah Glaz

> *e*'s abstract beauty casts a glow on Euler's Identity:
>
> $$e^{i\pi} + 1 = 0$$
>
> the elegant, mysterious equation,
> where waltzing arm in arm with *i* and π,
> *e* flirts with complex numbers and roots of unity.

"The Enigmatic Number e" traces the history of the number e and the mathematics associated with it, presenting each mathematical idea using both the symbolic language of mathematics and its reflection in the natural language of poetry. More information about this poem can be found in [11].

The Transcendence of Euler's Formula by Neil Hennessy

```
epitome
epitome
epitome
epi+ome
ep1tome
———————
epit0me
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Hennessy's poem is an example of a novel way of interweaving words and symbols to include an equation in a poem. It is also a clever and appealing way to pay tribute to a perfect equation. More information about this poem can be found in [20, 21].

More poems involving equations appear in the already cited references as well as in [6, 12, 18, 31].

Concluding Remarks

The brief historical overview presented in this paper reveals an interesting connection between the change in representation of equations in mathematics, from words to symbols, and the purpose of their appearance in poetry. When algebra was rhetoric and equations were described in words, poetry involving equations was written with the intention of presenting mathematical problems or ideas. Poems were used to present a challenging mathematical problem, the solution of an equation, an appealing way to teach mathematics to youngsters, or entertain elders with mathematical puzzles. Poetry served mathematics. As mathematics matured and developed its own language, the presentation of its concepts became more and more symbolic. The purpose for including equations in poetry underwent a change as well. The symbolic language of mathematics offered poets new ways to extend the capacity of natural language to convey their poetic message. Equations began being used in poetry to enrich both form and content of the poems themselves. Mathematics serves poetry. I hope that this paper succeeds to give a taste of the rich and multi-dimensional links between equations and poetry both past and present.

Space limitations do not allow us to consider another aspect of the connection between poetry and equations. Many poems that are far removed from mathematics, use mathematical language in purely metaphorical ways. It would be interesting to explore how the word "equation," whose meaning acquired layers of complexity from its association with mathematics, is used as a metaphor in poetry. But this is a subject for another paper. Mostly, it is my hope that this paper inspires poets to experiment with new and creative ways to use equations in their poems.

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