Synthesizing Mathematics, Art, and Synesthesia

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Abstract

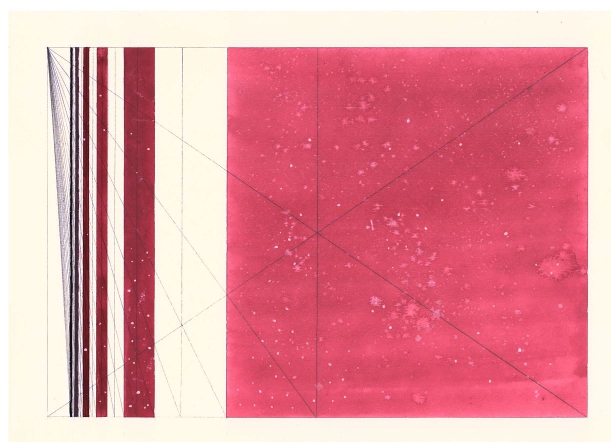
I have been exploring ways to express synesthesia mathematically through art. In this paper, I will describe my attempts at creating art informed by how I experience grapheme-color synesthesia and incorporating my favorite aspects of mathematics that I frequently encounter in my job as a mathematics educator.

My Synesthesia and Inspiration

Synesthesia is a condition where the brain makes connections between different senses. The one I have is grapheme-color synesthesia, where I associate numbers and letters with colors. So far, my experience with this is benign, and I have not found much use for it. Since synesthesia does not seem to be giving me any artistic advantage, I thought I should try to make it happen. The idea of using synesthesia to make art originally came to me when I attended Alice Major's talk "Numbers with Personality" at Bridges 2017 in Waterloo, Canada [1]. I was inspired by Major's expression of synesthesia through poetry so I wanted to try to express my own synesthesia through visual arts.

It took me a few years to start the process of using synesthesia to make art, as I was lacking inspiration on how exactly to do this. I finally had the idea to create a series of works to specifically express how I experience synesthesia. I decided to combine aspects of what I am currently doing with mathematics and art. I combined techniques I learned in perspective drawing with visual representations of convergent series, as I frequently teach calculus. The synesthesia governed the colors I used to express specific numbers and a variety of media was used to represent the qualities of the colors of numbers.

(a)



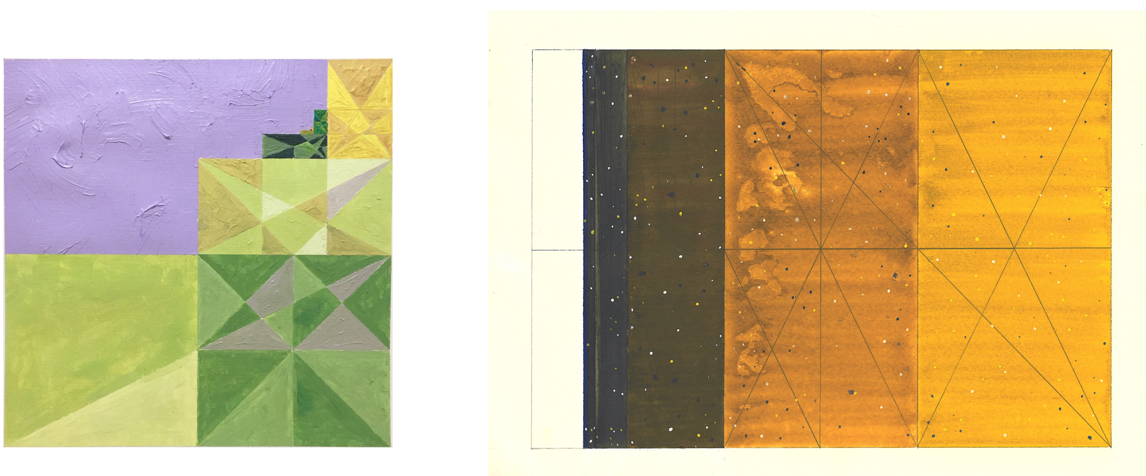
(b)

Figure 1*: Representations of π: (a) Synesthetic Barcode: π, watercolor pencil on paper, 11"×15", 2020, (b) Synesthesia Series: $\frac{\pi}{4}$, gouache on watercolor paper, 11"×15", 2020.*

Experimenting with Using Synesthesia

As I mentioned, I found synesthesia useless, and I wanted to make this "gift" useful for me. Having studied mathematics for most of my life while also dabbling in art, I am naturally exposed to numbers, symbols, and colors. I started with the numbers π and e, as I am overexposed to these numbers while teaching calculus, and their colors, in my mind, are pretty. One idea was to express digits of numbers as a "barcode" of colors. For example, I did the first twenty digits of π in this way, each digit represented by a bar of the color I perceive that digit to be, and the width of the bar corresponding to the size of the number, as shown in Figure 1 (a). I chose to use watercolor pencils to represent the oscillating quality of the color of these numbers. It is hard for me to pinpoint the exact color of a number, as the color can oscillate between shades of color, so I thought watercolor would be best to represent this as I can draw out a range of colors with dilution.

I thought π would amount to a pretty set of colors in making the barcode, as I perceive π as being rosy, as the 3 is pink, 1 is white, and 4 is red. In my mind, the first few digits usually dominate the color, so I thought it would look nice. When I made the piece in Figure 1 (a), I did not like it, as I thought it had too many colors, as π did not look like that, and my execution with watercolor pencil was not great. I also made a "barcode" for the number e, with disappointing results. As the colors expressed were not true to how I experienced the numbers with synesthesia, I ended my experiment by making a barcode of colors to represent a number.



(a) (b)

Figure 2: *Representations of e, (a) Synesthesia Series: square e, acrylic on paper, 16"×20", 2020, (b) Synesthesia Series: rectangle e, gouache on watercolor paper, 11" ×15", 2020.*

After my failure to correctly express my synesthetic experience of π and e, I moved on. My favorite calculus class to teach is Calculus II, as I am partial to series, and my favorite representation of the number e is as the following series,

$$e = \sum_{i=0}^{\infty} \frac{1}{i!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \cdots.$$

When I teach Calculus II, I enjoy showing students various proofs without words [2] and having them invent their own. My idea was to draw proofs without words for the series represented as units of area and then color the area according to the facets of color that represent e in my mind. I thought this would best represent how I experience e. The letter e to me is a light blue-green, and the decimal form of the number is similar. Since the number e starts as 2.71, its color is a yellow-green-chartreuse. The number 2 is yellowish-green, while 7 is the primary color yellow, and 1 is white. If I include the next three digits, the 828 adds a nice contrast of purple to the color.

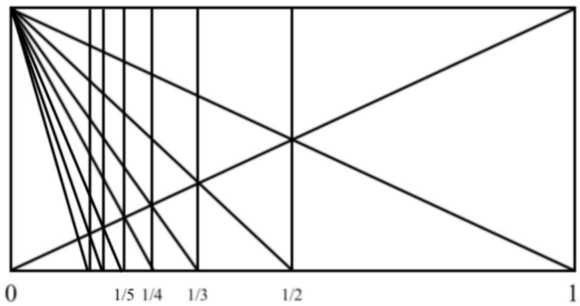


Figure 3: *Division of a rectangle using perspective drawing techniques.*

The result of my first experiment is shown in Figure 2 (a). I used the construction techniques from perspective drawing for duplicating and dividing rectangles using a straightedge, parallel, and perpendicular lines, as shown in Figure 3. The construction starts with a 2 by 2 unit square, with the area of one light green square unit on the lower-left corner representing the $1/0!$ summand in the series expansion of e and a square of the same dimension in the darker shades of green next to it was replicated to represent the $1/1!$ summand. The summation of summands in the series is represented by stacking on a rectangle of that unit of area. For the $1/2!$ summand, the bottom right square was divided into two rectangles of area $1/2$, and a rectangle of that area was replicated on top of it. Then I divided that rectangle into thirds, then replicated one of the thirds on top, then divided that one into fourths, and so on. The process continued until the rectangles cannot be seen anymore, as the factorial process rapidly decreased the area. This demonstrates the series for e summing up to a number between two and three. The lines from the construction of the rectangles were kept in place, and the colors I painted within these lines represent the different hues of e that come from the letter and the first three digits 2.718. I was most satisfied with the piece, as it seemed to represent the number e in the colors I feel it has. I decided to do the background negative space a light blue color, to contrast with the greens, as a way of incorporating the color of the character e. As most of the series rectangles for e were green through yellow, with hints of purple, the result looked like an abstract garden.

Given the success with e as a proof without words, I thought I would try to do the same with π. The result is Figure 1 (b). The whole frame of the image has area representing 1 and using the division of rectangle method demonstrated in Figure 3, I represented the Leibniz series up to the twelfth non-zero summand, as this was as far as I could go by hand:

$$\frac{\pi}{4} \approx 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \frac{1}{17} - \frac{1}{19} + \frac{1}{21} - \frac{1}{23}.$$

The resulting sum is colored in the pink and red gouache to highlight it, with an ombre effect to represent the colors that the first three digits of π are to me. The 1 being white is represented with the negative space and the white dots and splatter also represent the oscillating aspect of the colors. I was much happier with this piece than my first pieces, as I felt it better represented how pretty the number π looks to me, and went deeper into the number mathematically, conveying more meaning than a superficial coloring of its decimal digits.

With the success I had with $\frac{\pi}{4}$, I did another version of e using gouache, as seen in Figure 2 (b). Gouache, with its opacity and flat matte texture, and its ability to add small dots of color allowed me to more fully express the color of the number. The idea for this piece was to use rectangles as a base, rather than squares as in my first experiment, and the number e comes from the sum of the widths of the colored-in rectangles, using the same series used for e in my initial experiment in Figure 2 (a).

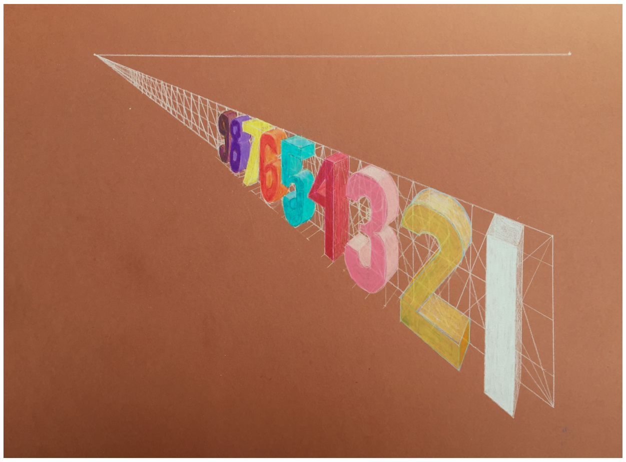


Figure 4: *Synesthetic Integers, colored pencil on paper, 19" ×25", 2020.*

The last idea I had was to represent how I experience the integers in my mind. I imagine the integers with their numerals on a strip, lined up, increasing from right to left. The numbers going into the distance shrink, while the number I am currently thinking about is the closest in view. This strip can move left to right, depending on if I want to zoom in on a number. I teach a general education seminar in mathematics and arts and I also took an art class in the Fall 2019 semester at my local community college. I thought that using perspective would be the perfect way to express how the larger numerals seem to disappear in the distance. I created the piece in Figure 4 using the principles of two-point perspective, and techniques of dividing rectangles to divide the space to draw the numerals. I colored the numerals the best I could to represent their color in colored pencil, to express how I experience these numbers with color in my mind. Since I imagine the numerals as if they are floating in outer space, I chose dark paper for the background so that the colored pencil will highlight the numerals. I decided to show the scaffolding and all of the construction lines in making the drawing because the divisions and replication of rectangles fascinated me the most when I took the perspective drawing class. As this class was an art class, I was taught constructions without explanation so I preoccupied myself with using geometry to prove why these constructions work, and I am still fascinated by this. When I teach my math and art seminar, we spend several classes covering dividing and replicating rectangles as an active learning exercise, so that they can discover and appreciate the mathematics behind this technique as I did.

Summary and Conclusions

Overall, I found this process to be a fruitful exercise for myself and a meditative experience in the slow process of drawing and painting by hand. Attempting to make art informed by my synesthesia allowed me to reflect on how I conceptualize and visualize mathematics. Before attempting my synesthesia artworks, I previously was not aware that I visualize mathematical sums or strings of numbers from right to left, rather than the conventional left to right such as in the number line. I plan on continuing this process to make more pieces and improving on the interpretation and execution.

References

[1] A. Major. "Numbers with Personality." *Bridges Conference Proceedings*, Waterloo, Canada, Jul. 27–31, 2017, pp. 1–8. https://archive.bridgesmathart.org/2017/bridges2017-1.pdf

[2] R.B. Nelson. *Proofs Without Words.* MAA Press, 1993.