Rep-tile Font

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Abstract

The terms *polyomino* and *rep-tile* were introduced by Solomon W. Golomb in the 1950s, and they were popularized in the society of recreational mathematics by Martin Gardner in the 1960s. A polyomino is a simple polygon that can be obtained by gluing unit squares. A polygon is called rep-tile if it can be divided into small congruent polygons which are similar to the original one. They have been well investigated in many societies including recreational mathematics, puzzles, games, tilings, arts, and designs. In this paper, we propose a method to design rep-tile polyominoes. Using this method, we design *rep-tile font*. You can use these patterns to generate tilings based on letters.

Introduction

In some games like Tetris, polygons obtained by joining unit squares edge to edge are used as their pieces. These polygons are called *polyominoes*, and they have been used in popular puzzles since at least 1907. Solomon W. Golomb introduced the name polyomino in 1953, and various sets of polyominoes have been widely investigated in the contexts of puzzle and tiling [1]. It was popularized in the 1960s by the famous column in *Scientific American* written by Martin Gardner [2].

Golomb is also known as the person who first introduced the notion of *rep-tile* and investigated the properties and conditions of the rep-tiles. A polygon P is called rep-tile if it can be dissected into congruent pieces which are smaller copies of P. We can reverse this notion; once we have a rep-tile P, we can form a larger polygon similar to P by tiling some copies of P. Repeating this process recursively, we can tile a plane by copies of P. It is known that some rep-tiles can be used to generate acyclic tiling (i.e., the tiling pattern cannot be identical by shifting and rotation). Both cyclic and acyclic tilings have been well investigated since they have applications to chemistry, especially, crystallography [3]. From the viewpoints of mathematics and art, the notion of rep-tile is popular as we can obtain tiling of the plane with the same shapes of different sizes by replacing a part of the rep-tiles by their copies recursively. In the 1960s Martin Gardner introduced polyomino rep-tiles [3]. Since it is a natural notion, polyomino rep-tiles have a long history mainly in the contexts of puzzles and recreational mathematics. The classic results can be found on a web page by Clarke [4], while Banbara et. al. recently demonstrated more results obtained by computers [5].

In this paper, we give a method for designing polyomino rep-tiles, which allows us to design a variety of polyominoes. Designing fonts is one way of bridging art, mathematics, puzzle, origami, and some others as Erik Demaine and Martin Demaine have designed dozens of fonts in this context [6]. From this viewpoint, we give polyomino rep-tiles font as an application of the method.

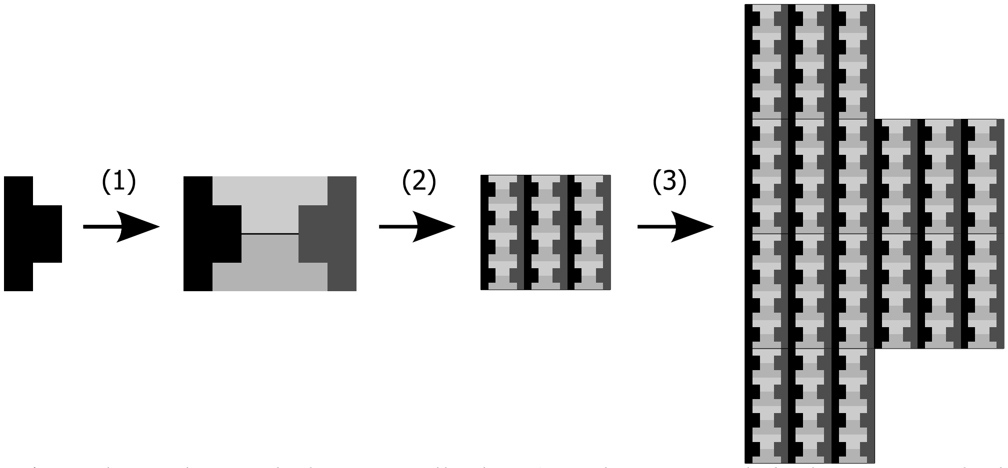

Figure 1 : *Making a large "d" from a small "d": (1) Make a rectangle by four copies of "d", (2) make a square by 12 copies of the rectangle of size* 6×4*, and (3) make a large "d" by 6 copies of the square.*

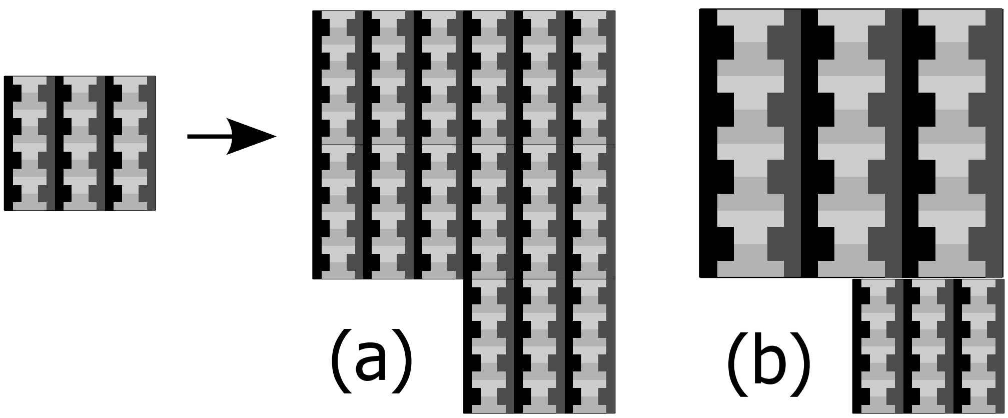

Figure 2 : *Making a large "q" from a small "d" (a) as a rep-tile and (b) a variant.*

Our Method and Designed Fonts

We focus on the polyominoes which can be obtained by splitting a rectangle. In other words, we find our patterns from a set of polyominoes such that we can assemble some copies of a pattern to a rectangle. We assume that c copies of k-omino can be assembled to a rectangle of size $m \times n$. (That is, we have $ck = mn$.) Let L be the least common multiple of m and n. Then, L^2/mn copies of the rectangle (or L^2/k copies of the k-omino) can tile a square of size $L \times L$. This square can be considered as a large unit square, and thus we can represent the original letter by using this pattern. A simple example is shown in Figure 1 in case $c = 4$, $k = 6$, $m = 6$, $n = 4$, and $L = 12$.

We show our rep-tile font in Table 1. Each pattern is a tiling art, and also a geometric puzzle. These patterns, including S and Z, are all different shapes, which add fun repeatedly.

In our method, we first form a square by some copies of a letter and we then arrange them into a larger polyomino similar to the original letter. Once we obtain a square, we can arrange them into any polyomino. Therefore, we can arrange any letter using the square as shown in Figure 2(a), and moreover, we can mix squares of different sizes in some cases as shown in Figure 2(b). These flexibilities allow us to design not only rep-tiles, but also tiling and some other designs of patterns.

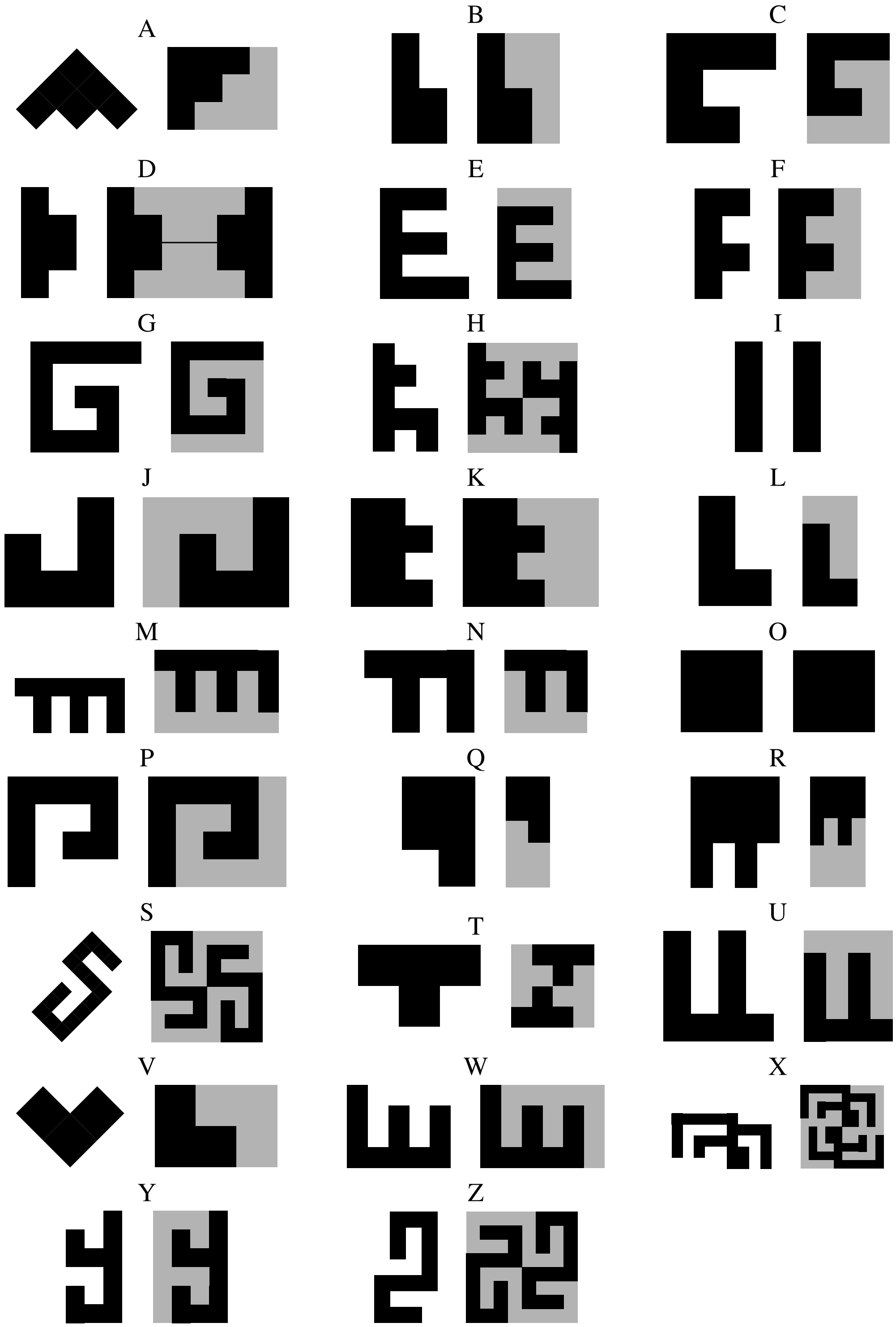

Table 1 : *Rep-tile font. A letter pattern (left) and how to assemble a rectangle (right).*

Figure 3 : *Two disconnected rep-tiles good for "i" and "v".*

Concluding Remarks

In this paper, we give a method for the design of rep-tile polyominoes. We implicitly assume that each polyomino should be a connected polygon. A rep-tile is not necessarily connected. In fact, we have two interesting patterns of "i" and "v" as shown in Figure 3. From the viewpoint of unit figures, the set of polyiamonds on a triangular lattice is another option. Extensions of our method to these options are future work.

References

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