Polystix Sculpture Design Revisited

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Abstract

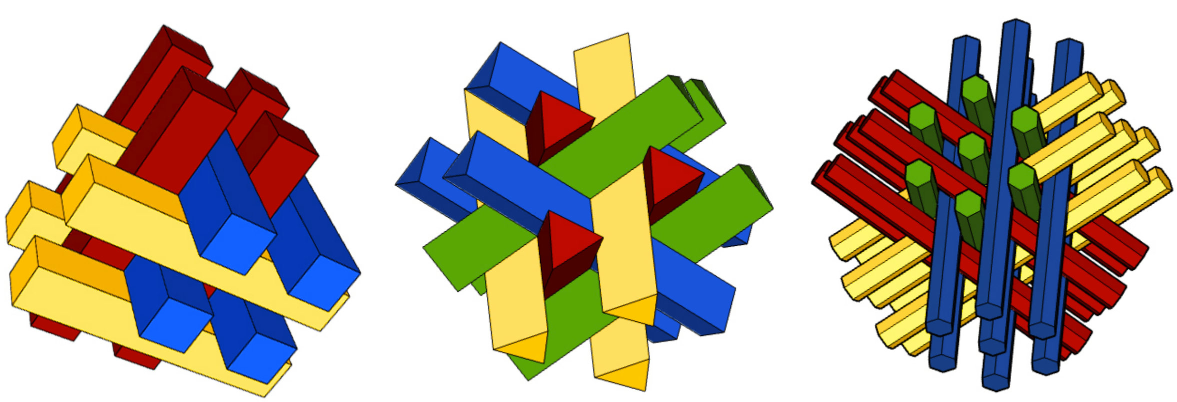
Polystix are arrangements of non-intersecting prisms that exhibit cubic symmetry. In my 2021 paper, "Sculpture Design with Hexastix and Related Non-Intersecting Cylinder Packings," polystix were defined, along with techniques for connecting their components to make complex glass knots. This paper revisits aspects of polystix design, defines conditions of symmetric models, and explores a novel approach to create finite sets with restricted components. I also introduce some related and unresolved polystix problems. Several examples of minimal arrangements are illustrated to highlight the variety of creative and design applications in which polystix can be used.

Introduction

In my 2021 paper "Sculpture Design with Hexastix and Related Non-Intersecting Cylinder Packings" [5], polystix were defined and illustrated with an emphasis on linking elements together to create intricate glass knots. This paper will focus on the specific characteristics of finite polystix configurations and outline several conditions used to design and classify symmetric polystix arrangements. After defining Polystix modeling criteria more precisely, I explain a way of creating sets by limiting the number of sticks used to construct polystix models. Once the finite polystix models are illustrated, I highlight a few unresolved problems and possible design applications.

Polystix Models

Polystix are a family of dense packing arrangements, with cubic symmetry, comprised of an infinite number of non-intersecting prisms [3]. Depictions and models are, by nature, finite things and any representation of an infinite polystix structure necessitates a series of aesthetic considerations, as in Figure 1. To create polystix models, elements can be varied by; the number of elements used, rod shape, color, pattern, and scale. The infinite options available to represent polystix encouraged me to develop a few personal parameters to assist with the classification and design of symmetric polystix sculptures.



(a) (b) (c)

Figure 1: *Polystix models: (a)Tetrastix, (b)Tristix, (c)Hexastix.*

The current conditions I employ to design symmetric polystix models are:

1) The structure is composed of elements that lie on the simple cubic or body-centered cubic lattice.
2) The structure is composed using congruent elements e.g., cylinders or polygonal prisms.
3) The sticks do not intersect.
4) The stick length is at minimum equal to the length of the pattern they are arranged in.
5) The design is stable. The sticks on each axis are in contact and keep each other spaced apart.
6) Each axis appears identical/ octahedral rotational symmetry.

These conditions can also be used to describe other combinations of patterns and stick shapes that are used by interlocking burr puzzles [2]. This list is in no way fixed or a requisite for the creation of polystix, but these particular restrictions have proven suitable for my current purposes of designing polystix sculptures.

Minimal Polystix Arrangements

In addition to the previously listed conditions, limiting the number of sticks is another practical way to discover new design variations. Exploring polystix from a minimalistic approach has inspired several designs used in the creation of my glass sculptures. The geometry of these arrangements can be worked out in two-dimensions [5] because the patterns repeat on each axis and are symmetric through rotation, as in Figure 2.

To begin my minimal polystix investigations, I built tetrastix with only one stick on each axis and found two possible 3 stick configurations, a left and right-handed structure Figure 2 (a). Next, I made tetrastix with six sticks (two sticks on each axis), and found five possible symmetric arrangements, as seen in Figure 2 (b), (c) and (d). The structures in Figure 2 (a), (c) and (d) are enantiomorphic, and their mirror images are not shown. The five six-stick tetrastix were used as the bases for a series of glass links [7], where the ends of the sticks on each axis were joined to create the L6a4, L6n1, and L6a5, links respectively.

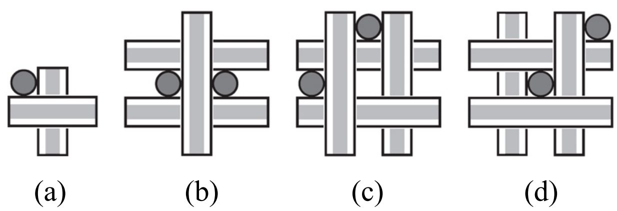


Figure 2: *Minimal tetrastix: (a)3 sticks, (b)Borromean arrangement, (c)6 sticks, (d)6 sticks.*

Tetrastix arrangements, using more than six sticks, were more difficult to find and required a systematic approach. In order to find out how many symmetric nine-point tetrastix arrangements are possible, I began investigating the number of ways n points can be arranged on a n x n grid. My research led me to geoboard geometry and the use of pin boards to approach similar problems. In 1970, J.R. Branfield analyzed all possible triangles on a 3x3 pin geoboard [1], and concluded that there are 84 triangles and eight lines possible. Only eight triangles are possible on a 3x3 geoboard, when considered up though congruency. I graphed and modeled the three-point permutations and found 12 possible symmetric tetrastix of nine sticks, shown in Figure 3. Models of the nine-stick tetrastix were shown at the 2020 Bridges art exhibition [6].

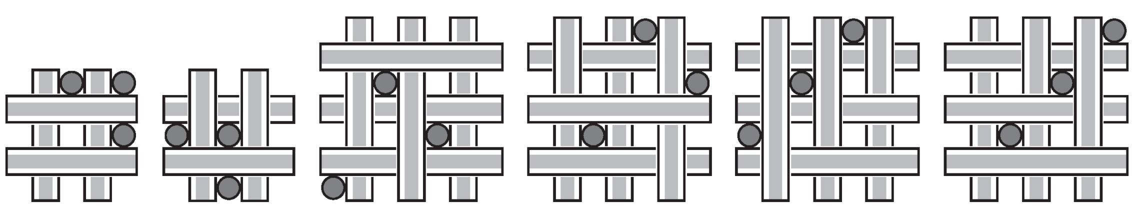


Figure 3: *Six chiral tetrastix arrangements made of nine sticks.*

Making tetrastix with four points on a 4x4 grid, I plotted 188 potential designs and found 40 unique 12-stick tetrastix arrangements, shown in Figure 4. Models of all 40 12-stick tetrastix were built, then used in the design of glass sculptures, as in Figure 5, [7]. The top left arrangement in Figure 4 is shown with perspective in Figure 1 (a), and the arrangement second from left, in the top row, is again shown with perspective in Figure 5 (a).

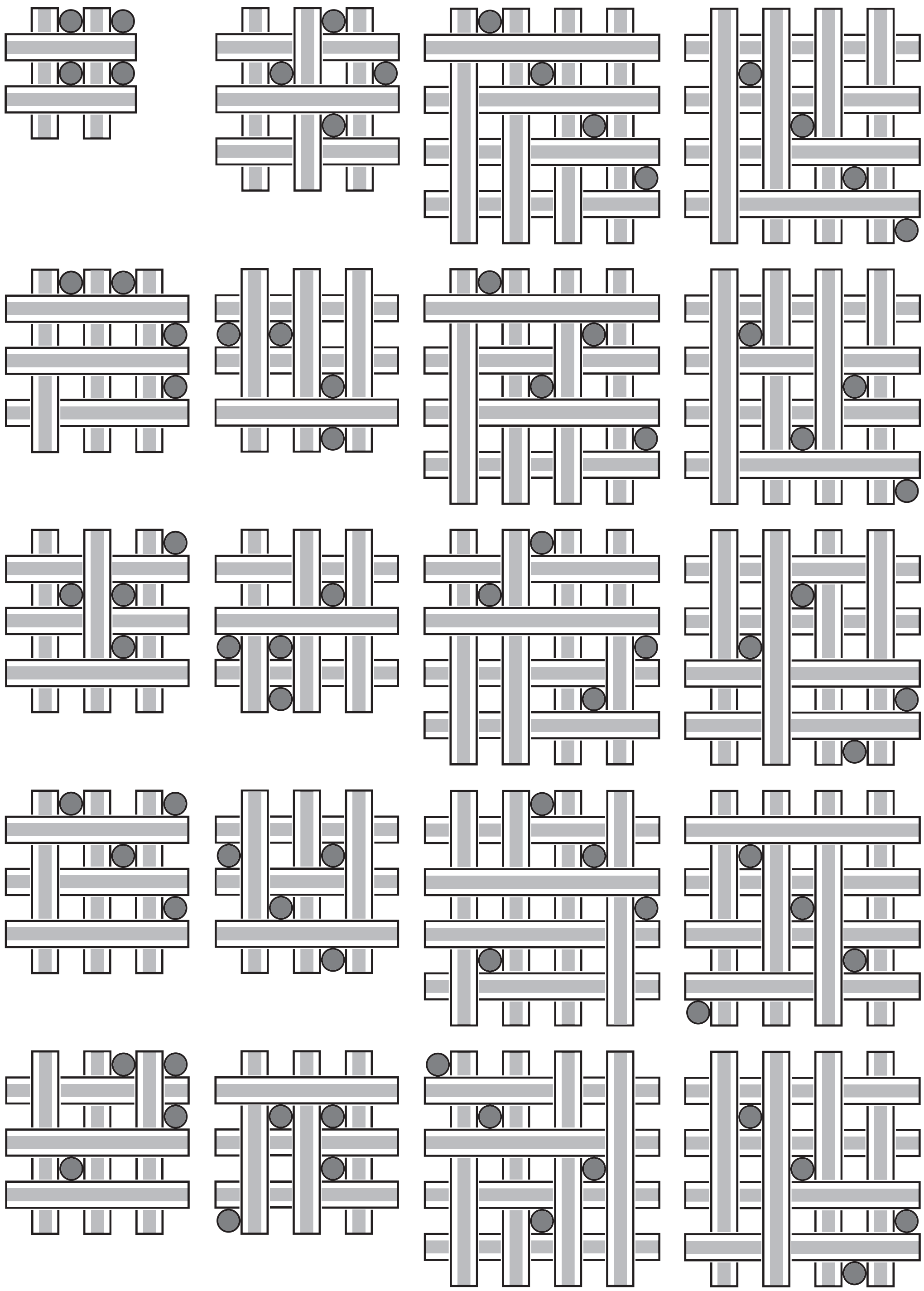
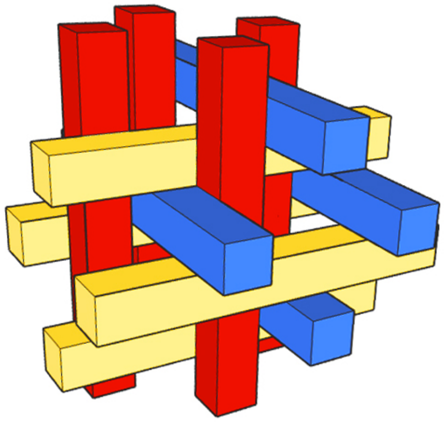
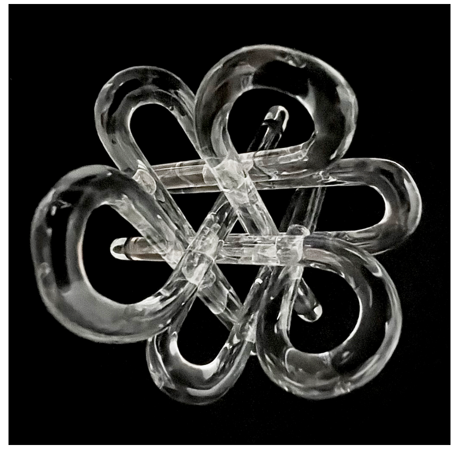


Figure 4: *20 chiral tetrastix made of 12 sticks.*





(a) (b)

Figure 5: *12 stick tetrastix; (a) The second from left top row configuration in Figure 4 in perspective, (b) Tetrastix glass knot made from 12 sticks.*

Similarly, using a limited number of sticks works to create minimal arrangements of both tristix and hexastix. The 4-axes tristix and hexastix arrangements that were made by using only 12 sticks revealed an additional 12 symmetric arrangements. To date, only sets of polystix using up to 12 rods have been resolved. The polystix sculptures that I have made using more than 12 rods [7] were designed by improvising with subjective aesthetic considerations and the previously listed conditions. Cubic packing arrangements that have more axes have been described as burr puzzles, by Coffin [2]. I have yet to find a complete analysis of how many 6, 12, and 24 directional homogeneous arrangements are possible [4]. As a puzzle, directionality and color can also be introduced into polystix as a fun way to increase complexity. Developing polystix with a limited number of sticks has not only produced stimulating geometric problems, but is also a rewarding technique for sculpture design, as in Figure 5.

Conclusion

The open-ended nature of polystix modeling can provide many opportunities for both creative and systematic explorations. My hope is that these polystix investigations encourage others to use mathematics to make beautiful art, and to use art to engage with mathematics.

References

[1] J. R. Branfield. "Geoboard Geometery." *The Mathematical Gazette*, vol. 54, no. 390, 1970, pp. 359–361.

[2] S. Coffin. *Geometric Puzzle Design*, A K Peters Ltd, Massachusetts, 2007.

[3] J. Conway, H. Burguel, C. Goodman-Strauss. *The Symmetries of Things*, CRC Press, Boca Raton, FL, 2008.

[4] M. O'Keeffe, J. Plevert, and T. Ogama. "Homogeneous Cubic Cylinder Packing Revisited." *Acta Cryst.* a.58, 2002, pp.125–132.

[5] A. Widmark. "Sculpture Design with Hexastix and Related Non-Intersecting Cylinder Packings." *Bridges Conference Proceedings*, 2021, pp. 293–296. https://archive.bridgesmathart.org/2021/bridges2021-293.html.

[6] A. Widmark. " 12-9 Match-Match." *Bridges 2020 Art Exhibition Catalog.* http://gallery.bridgesmathart.org/exhibitions/2020-bridges-conference/anduriel-widmark.

[7] A. Widmark. 2022. www.andurielstudios.com.