

Visualizing Virtual Vector Fields

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Abstract

We present our current progress on a virtual reality sandbox experience equipped with a toolset to create interactive vector fields and vector calculus operations. The aim of the project is to empower the student's understanding of vector field and assist in the development of their intuition. The source code for this project is open-source and available at <https://github.com/OthmanAlrawi/Visualizing-Vector-Fields>.

Introduction

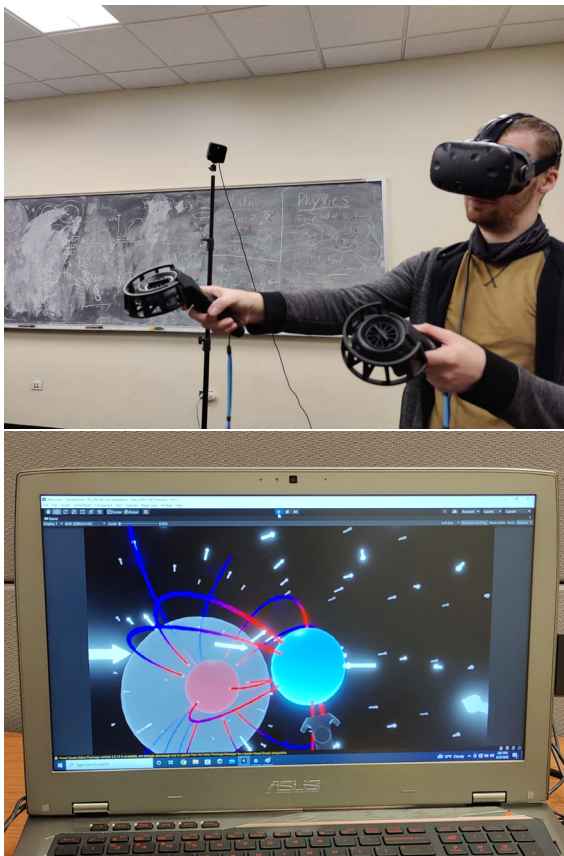


Figure 1: *Demonstration of the electric field module.*

The mathematics taught in classrooms is very often presented under a veil of abstraction. One such concept – the *vector field* – requires a high degree of visualization that cannot be provided by two-dimensional blackboards and static images in a textbook. We used the Unity Game Engine to create a series of real-time visualizations of vector fields and vector calculus in \mathbb{R}^3 implemented in virtual reality (VR). Our objective is to develop a fun and interactive user experience that not only assists students in developing mathematical intuition but also uses modern video game techniques to incentivize learning goals. To construct this “gamified” experience, our simulation employs two key features of game development: real-time and interactivity.

Spatial thinking is a critical skill from vector calculus that is used in many fields, including physics, engineering and chemistry as concepts move from 2D representations to fully 3D systems [7]. Recently, there has been a push in STEM education towards both inquiry based learning models, often using digital “textbooks” with animations and other dynamic graphics. See, eg [8]. However, using interactive media is an additional tool that can empower the students to manipulate concepts graphically to build intuition for mathematical and physical phenomena.

Embodied cognition is one way of building a language for students to both describe and explain spatial concepts [10]. Virtual and augmented reality (VR and AR, respectively) promise to revolutionize learning [11, 9, 6]. Not only do they provide a platform for individualized, embodied learning, but they can help build students agency which in turn can give students a sense of ownership over their

coursework, as demonstrated in Figure 1. This level of interactivity is frequently lost in large or online-based courses. “Visualizing Virtual Vector Fields” is an open-source VR sandbox that can be used in conjunction with formal classes or as a stand-alone learning tool to help students develop intuition about vector fields, scalar fields and vector derivatives.

Vector Fields

A *vector field* assigns a vector to each point in a subset of a space. A *vector element*, specifically a euclidean vector, is a geometrical object that has both a magnitude and direction. Typically, vectors are graphically represented by arrows that connect two points. This representation captures both key aspects of a vector perfectly. For this reason vector fields are usually drawn as a collection of arrows at discretized points in space. While books are limited to two dimensional projections, we are free from those restrictions. In Figure 2, we illustrate the electric field $\mathbf{E}(\mathbf{r})$ associated with a charged particle at position \mathbf{r}' ,

$$\mathbf{E}(\mathbf{r}) = kq \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}, \quad (1)$$

where k is a constant and q is the charge of the particle. Users can place and move charges around the playspace and the field visualization updates in realtime.

Field Lines and Dynamic Interactivity

The depiction of a vector field as a grid of arrows can be misleading to students as it presents them as discrete, rather than continuous objects. We augmented our simulations with a few methods to illustrate the continuity. *Field lines* are one dimensional curves that are tangential to the field along all of its points. For a vector field \mathbf{v} , a field line $\gamma(t)$ parameterized by t must satisfy

$$\frac{\partial \gamma}{\partial t} = \mathbf{v}(\gamma(t)), \quad (2)$$

where t is the arclength parameter. In Figure 3 and Supplementary Video 1 [2], we illustrate the field lines for a configuration of point charges, whose electric field is given by a superposition of the fields of an individual particle.

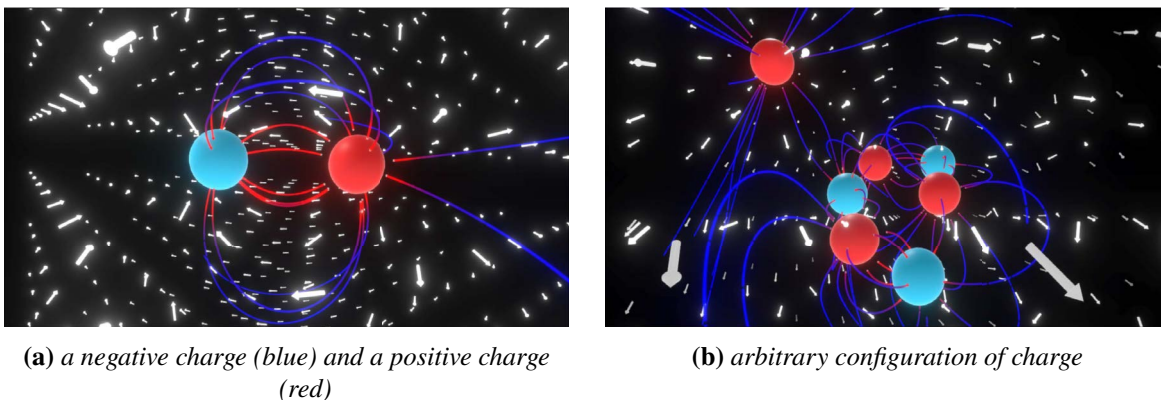
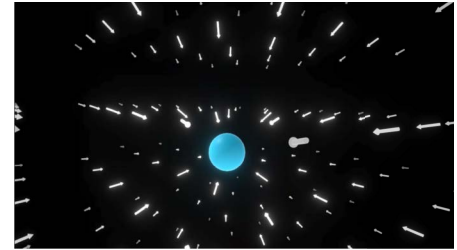
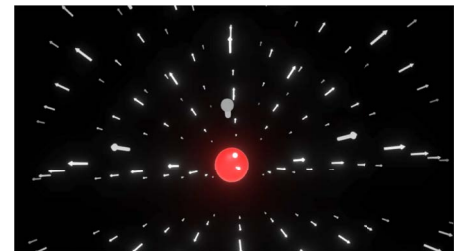


Figure 3: Fields lines of vector fields coming from two different charge distributions.



(a) negative charge



(b) positive charge

Figure 2: The electric field generated by a point charge.

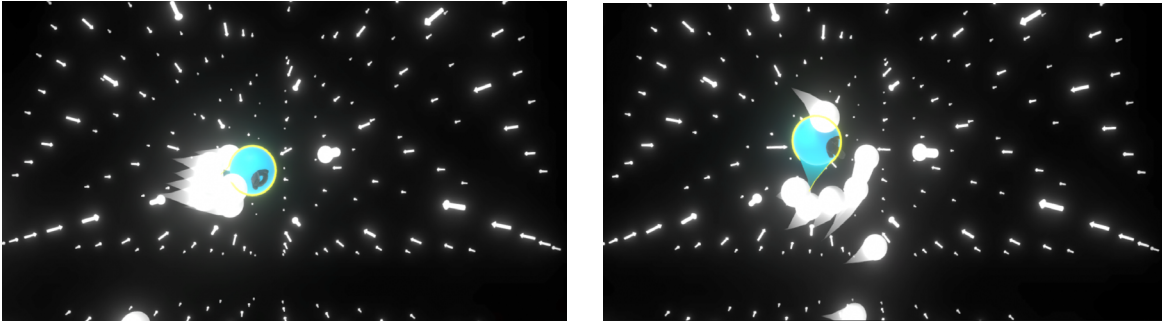


Figure 4: *Interactive test particles.*

Drawing the electric field lines frees us from the restrictions imposed by a 3D arrow mesh that portrays the field at discretized points, giving us the ability to express fields in a more continuous manner. Along these lines, the vector field is no longer represented by a set of local vectors with a direction and a magnitude. At each point along the curve, the tangent to the curve gives the direction of the vector field. We use color to quantify the magnitude of the field. The color gradient is created by linearly interpolating between two colors based on the maximum and minimum values of the field. Ultimately, we find a combination of both illustrations provides more intuition.

To make the visualization more interactive we introduce “test particles” that flow along the vector field. These are interactive game objects that undergo forces according to the direction of the fields in real-time, as seen in Figure 4 and Supplementary Video 2 [3]. These test particles are simulated using Unity’s 3D physics that implements Nvidia’s PhysX Engine, which uses a semi-implicit Euler integrator [1]. Since we are not trying to find exact numerical solutions but rather are mainly concerned with the general qualitative behavior, we believe the accuracy lost by using a fast integrator designed for video games doesn’t come at the cost of user experience.

Vector Derivatives

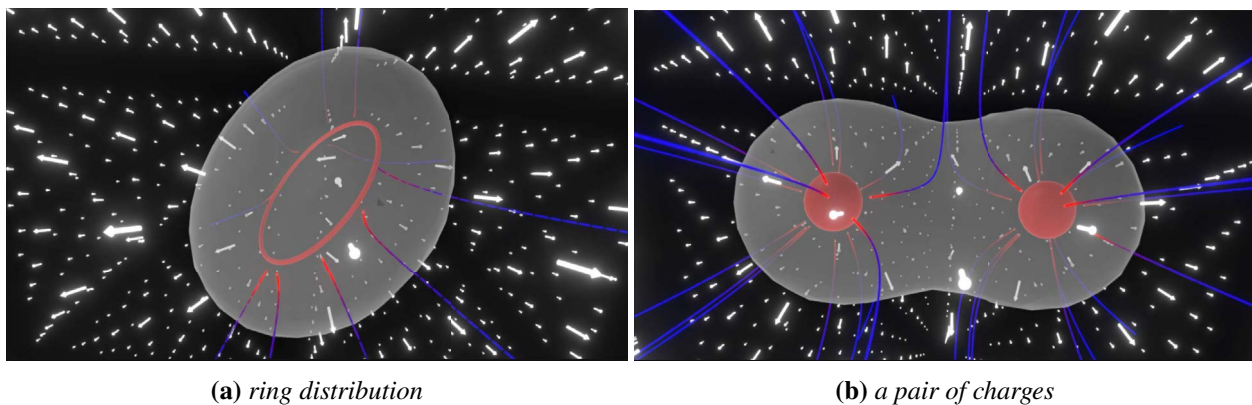


Figure 5: *Isosurfaces.*

Vector calculus requires that we not only perform operations such as addition or multiplication on a vector field, but we also need to take derivatives of it. In \mathbb{R}^3 , the three elementary differential operators are:

- *Gradient:* For the scalar field $\phi : \mathbb{R}^3 \rightarrow \mathbb{R}$, the gradient $\nabla\phi : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a vector field that measures the direction and rate of change of ϕ .

- *Divergence*: For the vector field $\mathbf{v} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, the divergence $\nabla \cdot \mathbf{v} : \mathbb{R}^3 \rightarrow \mathbb{R}$ is a scalar field that measures the source of \mathbf{v} at every point.
- *Curl*: For the vector field $\mathbf{v} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, the curl $\nabla \times \mathbf{v} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a vector field that measures the rotation of \mathbf{v} in a given direction.

To illustrate these operators, we need to visualize scalar fields as well as vector fields. Given a scalar field ϕ , we plot its *isosurfaces*. These are surfaces in \mathbb{R}^3 where ϕ is constant. We use the Marching Cubes algorithm to construct such surfaces. This is a computer graphics algorithm that is used to construct meshes of an isosurface from a three dimensional scalar field [5]. In order to update these surfaces in real-time, as shown in Supplementary Video 3 [4], we use a multi-threaded implementation of the Marching Cubes algorithm. In Figure 5, the we plot the isosurfaces for two electric potentials ϕ , where $\mathbf{E}(\mathbf{r}) = -\nabla\phi(\mathbf{r})$.

Conclusion & Future Work

We plan to incorporate more activities into the vector derivatives module aimed at increasing student intuition of vector derivatives. For example, we can evolve surfaces in \mathbb{R}^3 according to the gradient of a scalar field. Additionally, we would like to marry students' mathematical intuition with physical concepts. The curl shows up prominently in magnetostatics as well as in vortices formed in inertial fluids. In the future, we will present several of these to students in courses on multivariable calculus and introductory electromagnetism to explore the efficacy of "Visualizing Virtual Vector Fields" as a learning tool.

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