# Paper Hyperbolic Sculptures

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## Abstract

Using circles, squares, and octagons, I have developed three different types of paper sculptures inspired by hyperbolic forms. This paper discusses the properties of each and provides instructions for artists to create these forms and incorporate their own 2-D artwork onto these 3-D shapes.

## Introduction

Building on my experience producing Artist's books featuring my mathematical drawings, I have developed 3 paper forms that have a hyperbolic structure. The first is made up of n >=2 copies of a slitted circle rotated and glued 45 degrees successively with respect to each other, giving a total angle sum of n x 315 degrees, since for each circle, 45 degrees are duplicated by another circle. The second form is made up of slitted squares with sub squares rotated and glued at 90-degree angles. The third form is made up of non-regular octagons with equilateral triangles as sub polygons and glued together after rotating by 60 degrees along those equilateral triangles.

Each of my sculptures are created from a set of identical geometric paper cutouts with patterning on both sides. The pattern on each piece needs to be designed with a specific symmetry so that they can be glued together to have the appearance of continuous surfaces. The following sections will show how to create each of these forms.

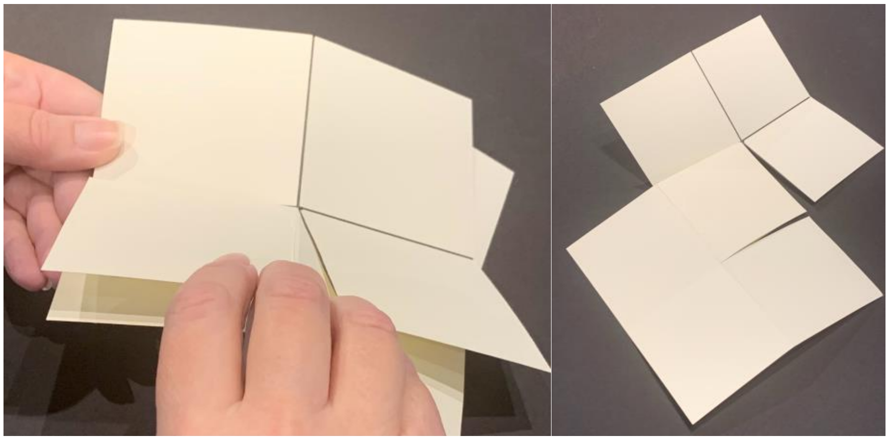
## Circle Hyperbolic

(a) (b)

**Figure 1:** *Circle Hyperbolic: (a) construction of a circle hyperbolic, (b) example: "Chaos Circle Hyperbolic", 2022.*

To build a circle hyperbolic sculpture you need to use at least two identical circles with the same pattern on all sides. I used a pattern with order-8 rotational symmetry. For each circle, cut a slit from the same point on the perimeter of each circle to the center point. Then, stack the circles and rotate them, so that the top circle overlaps the bottom circle by 45 degrees (Figure 1a). Next, glue down the overlap in a similar fashion to building a helix. Note that for all the sculptures, I would suggest double adhesive sheets. Next, "un-spiral" the two unattached ends from the slit and bring them together to form a saddle. Finally, overlap these ends 45 degrees and glue the pieces together. To create Chaos Circle Hyperbolic (Figure 1b) I used three stacked circles.

## Square Hyperbolic



(a) (b)

**Figure 2:** *Square Hyperbolic: (a) stacked construction, (b) rotated construction.*

After working with the circle hyperbolic form, I decided to explore incorporating the concepts of non-Euclidean geometry creating similar sculptures that can be folded up, more along the lines of a book form. The circle sculptures allowed me to render a circular form with more than 360 degrees (2 circles together 630 degrees, 3 circles together 945 degrees). Using squares, I can have a shape with more than four 90-degree vertices.

Starting with a set of identical squares, fold each in half both horizontally and vertically. For each square, cut a slit from the center point of one side to the center point of the square. The process for attaching the squares is like the technique for circles. Overlap two squares by a quarter (Figure 2a). Once you have stacked and connected the desired number of squares, fold them accordion style.

To create a sculpture that opens in a more linear format, I designed a second gluing position, allowing me to attach two stacks of these stacked and folded squares after rotating 90 degrees (Figure 2b). To make this series of twisted ruffles look cohesive it is important to preserve the rhythm of the accordion folds. The pattern on each of the four small squares that make up the larger paper square must be identical and feature 4-fold rotational symmetry.

My book "Quadrilateral Lace Hyperbolic" (Figure 3) features algorithmically generated square drawings. [1].

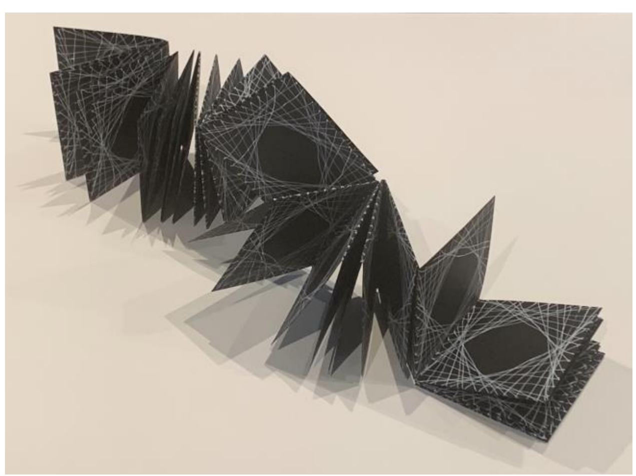


**Figure 3:** *Quadrilateral Lace Hyperbolic*

## Octagon Hyperbolic

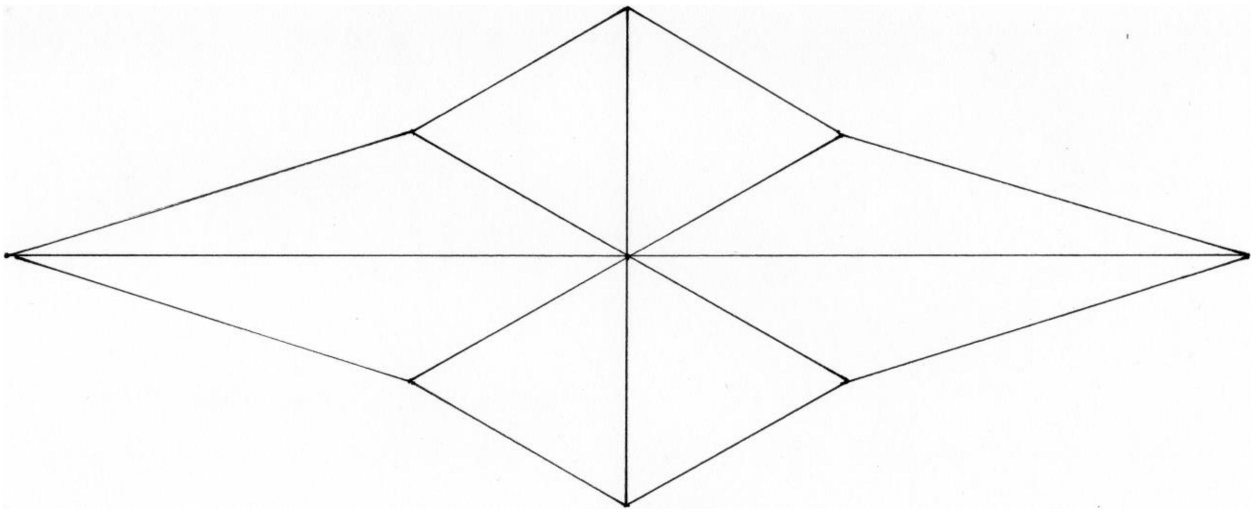
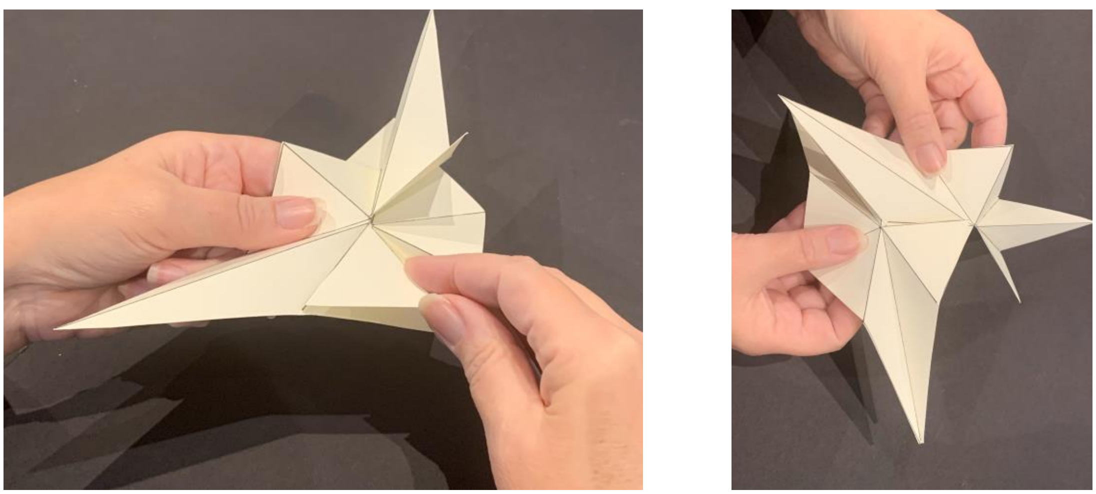


**Figure 4:** *Concave Octagon Pattern*



(a) (b)

**Figure 5:** *Octagon Hyperbolic: (a) stacked construction, (b) rotated construction.*

After making sculptures with squares, I started to use other geometric shapes, such as hexagons, trapezoids, and octagons. To have a set of paper octagons connect to form a series of twisting ruffles, it needs to have two adjacent equilateral triangles. After a bit of experimenting, I came up with the concave octagon pattern in Figure 4.

Score the lines from the center to the vertices to create the eight triangles. Then, stack and attach three sets of three octagons (Figure 5a). After accordion-folding each stack to connect the stacks, rotate them 60 degrees before gluing (Figure 5b). I call this general form an "Exploding Octagon". To preserve the continuity of the structure at the connections, the equilateral triangles in the octagon need to have a pattern with order-3 rotational symmetry.

**Figure 6:** *Exploding Octagon Moebius*

I went one step further and connected the equilateral triangles at each end while still preserving the accordion folds. The resulting shape "Exploding Octagon Moebius" (Figure 6) has the same topology as a Moebius strip. Each time I attached two stacks, I rotated 60 degrees, 180 degrees in total. Despite its wild complicated appearance "Exploding Octagon Moebius" has only one side.

## Conclusion

What started out to present my algorithmically generated drawings in 3-D forms, has turned into a series of projects that I hope can be used by other artists. Each of the sculptural forms discussed in this paper require only one type of 2-sided geometric building piece, but there is the possibility for artists to create their own versions. These sculptures are well suited for artists whose work is printed digitally in multiples. I plan to keep working on developing new accessible ways to present drawings on sculptural forms inspired by non-Euclidean geometry.

## References

[1] S. Happersett. "Quadrilateral Lace", Bridges Conference proceedings, 2020, pp. 411–412.