Approximating Edge-Touching Regular Polygon Patterns Using Chain Maille

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Abstract

I have worked extensively with making geometric patterns using regular polygons. This document describes how this type of pattern can be interpreted as chain maille. I also present some alternatives to to using plain rings, followed by a brief discussion of how some other types of patterns can be obtained in a similar manner.

Introduction

For as long as I can remember, I have enjoyed creating patterns using regular polygons and exploring their properties. This became easier with the advent of graphics programs on personal computers. Starting with SuperPaint on a Mac Plus and progressing to Graphic on an iPad (while using various incarnations of CorelDraw along the way), I have been able to experiment with many ideas through the process of drawing them.

Along with the making of digital pictures, I had a desire to create physical representations. Last year, I wrote about translating my patterns to bead crochet [2], and since then, I have found that I can also interpret many of them using metal rings.

My earlier attempts to interpret polygon patterns with circular jump rings (metal loops whose ends touch but are not connected) were done with patterns consisting entirely of polygon rings. In those cases I could use one jump ring for each ring of polygons. However, many of my patterns have other components as well, so instead, I made the transition to interpreting each polygon as a separate jump ring, as if the ring was representing the polygon's outline. This is analogous to how polygons were represented as beads in [2].

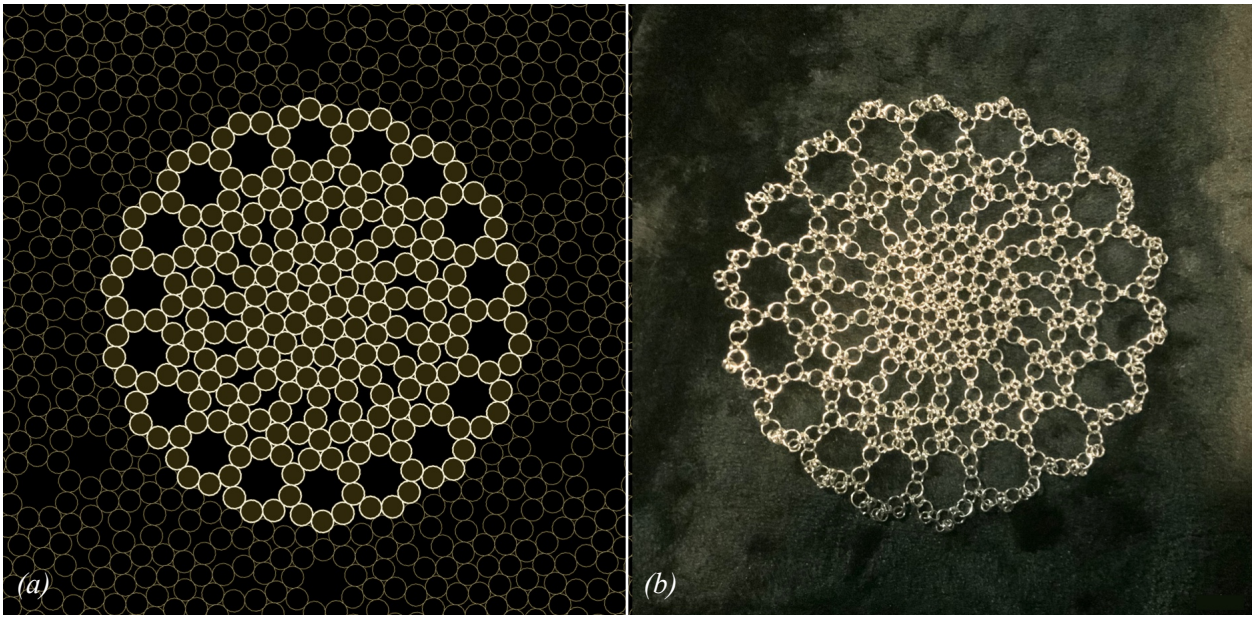

Figure 1: *Regular polygon pattern approximation using chain maille [3]: (a) pattern of edge-touching 18-gons, (b) corresponding chain maille piece.*

Pattern Conversion

Let us consider a pattern of edge-touching regular polygons. It makes sense that we would be able to spread the polygons apart by a uniform amount and still get a pattern which "works". It would basically just be increasing the size of each polygon in the pattern with a border. This is what happens when I represent the pattern using chain maille; the regular polygons are represented by the inner part of the main jump rings, and the extra bit (or border) is spanned by the inner diameter of the connector rings.

Using connector rings whose inner diameter is as small as possible (so that they will barely encircle two of the main rings) will result in a piece close to the original polygon pattern. Multiple gauges can be used for the main rings in the same pattern as long as they all have the same inner diameter and they fit within the connector rings.

If we look at a line segments along the planes defined by the connectors, we can see a pattern similar to the stitches used when converting the pattern to bead crochet [2].

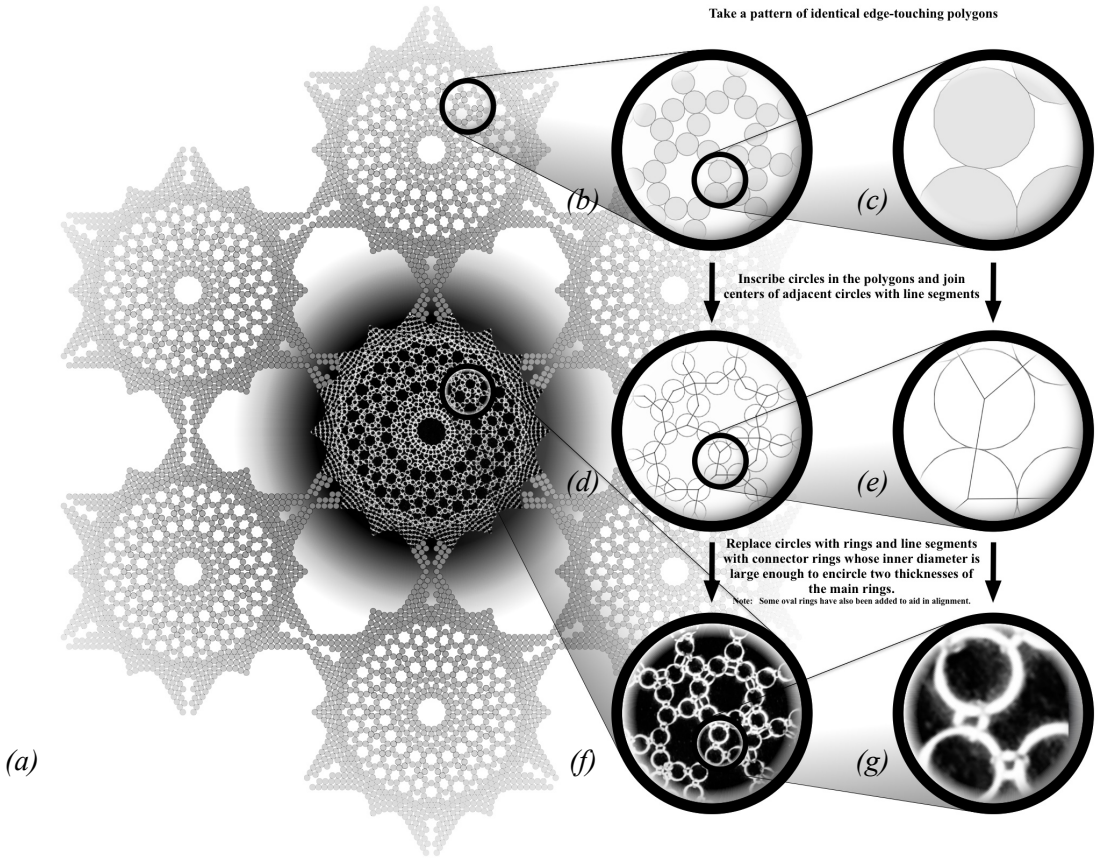

Figure 2: *Translating a pattern of edge-touching polygons for use with jump rings:*
(a) pattern of edge-touching 18-gons with central motif replaced by chain maille [3],
(b) magnified sample of pattern,
(c) further magnification of some 18-gons,
(d) the sample with circles inscribed in the polygons and centers of adjacent circles joined,
(e) further magnification of the same 18-gons with their inscribed circles,
(f) magnified part of chain maille piece corresponding to the sample,
(g) further magnification of the rings corresponding to the 18-gons.

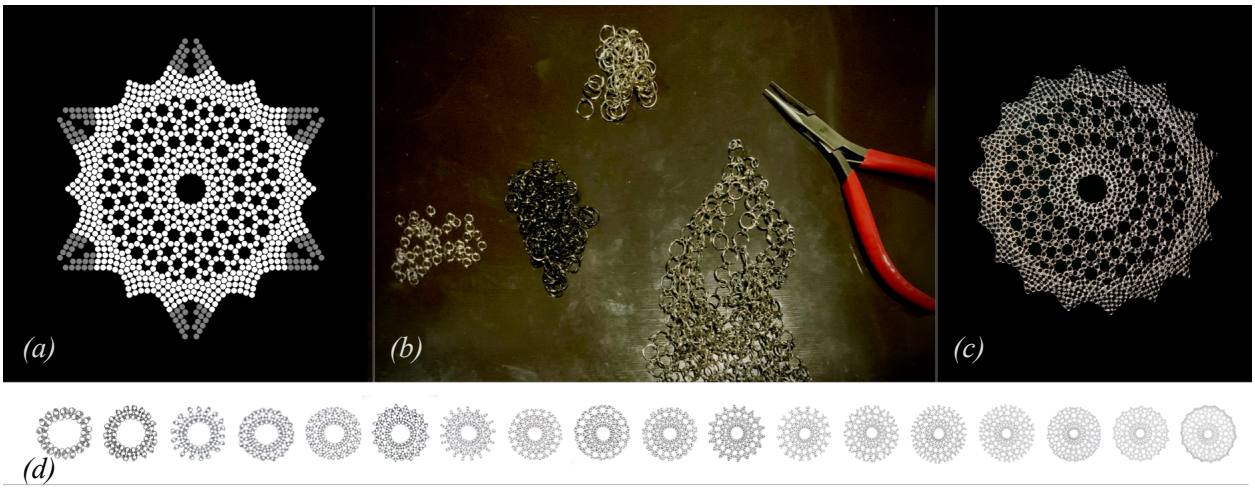

Figure 3: *Work in progress [3]: (a) pattern of 18-gons, (b) partially finished piece with jump rings and pliers, (c) mostly finished piece, (d) some steps along the way.*

The Rings

In some cases, I make the jump rings rather than purchasing them. To get precisely-sized inner diameters, I find that knitting needles provide easy-to-obtain shafts for winding the wire.

Before working a section, I open some of the larger main rings and close a number of the smaller connector rings. It is easier to hook the main rings in place and add a suitable number of connector rings to them rather than closing each of the rings in place (Figure 3b).

Display

The rings are free to move to some degree, so the resulting chain maille piece may not exactly match the original pattern, but there are a number of ways to work around this. One relatively easy option is to use steel rings (not stainless), so that the piece can be arranged on a magnetic sheet.

Another way involves adding additional larger rings behind or inside rings of polygons to keep them in a roughly circular arrangement. Or, additional rings of specific size can be added to draw parts together. In the chain maille portion of Figure 2, some oval rings were added to pinch together parts to match the original polygon pattern.

Of course, they could be sewed or pinned in place to a fabric or cork backer. But it also may prove to be aesthetically pleasing to make a hybrid arrangement where pieces of fabric or other material are attached behind rings of polygons and other interesting structures, leaving other areas open.

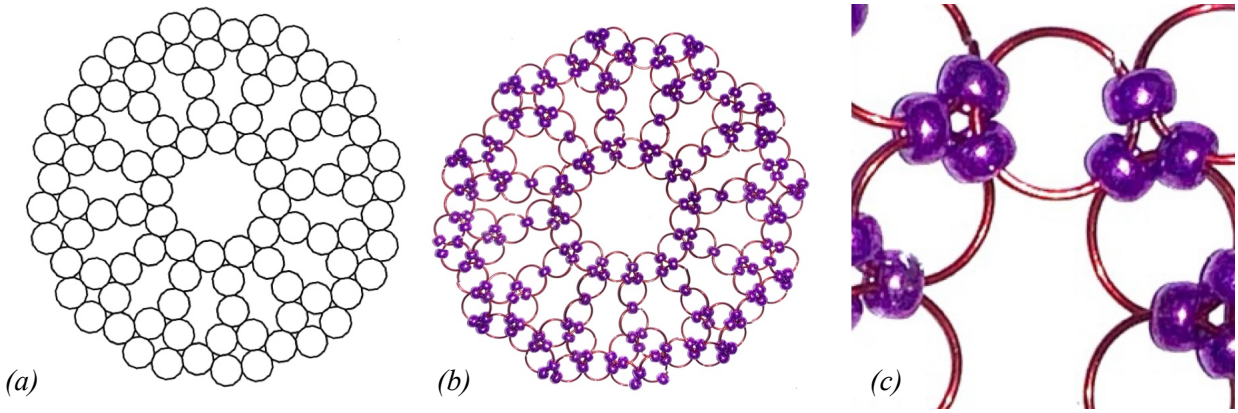

Figure 4: *Using beads as connectors: (a) polygon pattern, (b) interpretation with rings representing polygons and beads representing shared edges, (c) close-up of beads and rings.*

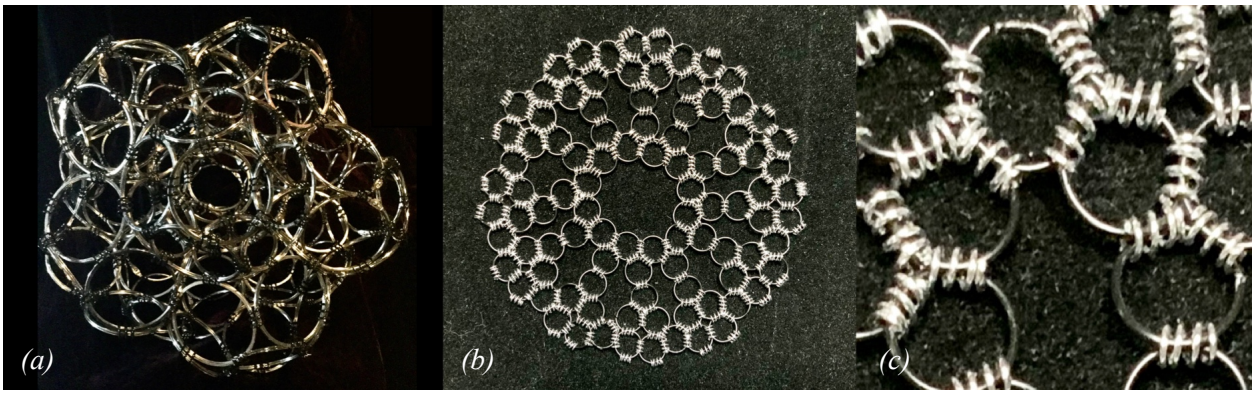

Figure 5: *Coils and springs as connectors: (a) a 3D shape made of binder rings and cut binding coils, (b) pattern from Figure 4 using rings as polygons and springs as shared edges, (c) close-up of Figure 5b.*

Some Variations

It is not necessary to use circular connector rings; oblong or rectangular rings should also work well. I can even replace the connector rings with one- or two-holed beads and still make the pattern work (Figure 4). For beads with only one hole, the gauge of the wire must be fine enough to go through the hole twice. Consequently the pattern of beads (edges) can dominate over the pattern of rings (polygons). If I work with very small beads, the resulting piece may be too delicate to handle roughly.

The gauge of the wire can be varied to help emphasize a pattern. Small springs or coils could be used in place of connector rings if their opening and length were sized appropriately, in much the same fashion as I have made three-dimensional figures using large binder rings and binding coils (Figure 5).

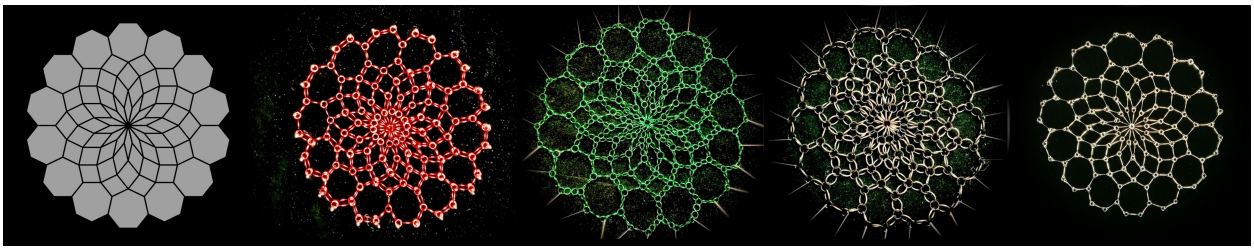

Figure 6: *Adapting to patterns with equilateral segments using various types of links.*

Other Notes

When the connector rings are small relative to the main rings, one is constrained by how closely polygons or circles can pack around one of their own. I have been exploring how altering the height and width of links can allow me to represent patterns involving only equilateral segments, by essentially shrinking the main rings to become vertices and expanding connector rings to become edges (Figure 6).

To interpret my polygon patterns, I have used chain maille in a very simple form. However, chain maille often involves much more complexity in how the rings (or links) weave together. Geometers may be interested in Frank Farris's use of nonplanar links to form wallpaper patterns using chain maille [1].

References

[1] F. Farris. "Wallpaper Patterns from Nonplanar Chain Mail Links." *Bridges Conference Proceedings*, Online, Aug. 1-5, 2020, pp. 183-190. http://archive.bridgesmathart.org/2020/bridges2020-183.html

[2] R. Sunder-Raj. "Approximating Edge-Touching Regular Polygon Patterns Using Crocheted Bead Lace." *Bridges Conference Proceedings*, Online, Aug. 2-3, 2021, pp. 281-284. http://archive.bridgesmathart.org/2021/bridges2021-281.html

[3] R. Sunder-Raj. "Chain Maille From Patterns of Regular Polygons." 2021. https://twitter.com/i/moments/1498453083180908548