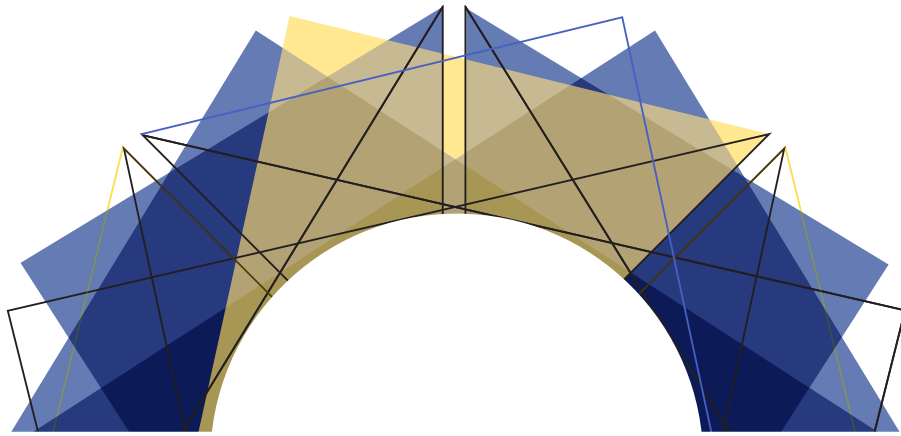


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Preface

Welcome to the 24th annual Bridges Conference! Once again, due to the uncertainties surrounding the coronavirus pandemic, we find ourselves sharing our innovations and excitement with each other online rather than in person. The continued commitment of the Bridges community to bring new ideas and to make connections between mathematics and art is encouraging and heartening.

We are committed to visiting Helsinki in person next year to experience the best possible exchange of ideas with peers. We look forward to the feedback we receive and the new collaborations that arise through informal interaction between participants. We have already begun preparations for Bridges 2022 in Helsinki and Espoo!

This year's Bridges Program Chair is David Swart. He coordinated an international program committee of over 65 experts, who provided extensive reviews and editorial comments on submissions. David also served as chair of the regular papers track, while Frank Farris chaired the short papers track, and Eve Torrence chaired the workshop papers track. Great thanks go to the program committee whose work enriches the field and makes the conference and proceedings possible. Special thanks to Art Exhibition chairs Robert Fathauer and Bruce Torrence, Poetry Reading chair Sarah Glaz, and Short Film Festival chair Bianca Violet. Thank you to Uyen Nguyen for designing the cover art and to Conan Chadbourne for preparing the Art Exhibition catalog. We are grateful to Randolph-Macon College in Ashland, Virginia for providing Zoom hosting for the conference.

The 2021 Bridges proceedings includes 32 regular papers, 34 short papers, and 5 workshop papers. Once again we are fortunate to have papers covering a wide range of topics. The papers showcase new ideas in fashion, music, dance, poetry, literature, visual art (both 2D artwork and 3D sculptures), and crafts such as beadwork, weaving, and origami. We see connections to an equally wide range of mathematical ideas including topology, symmetry, tilings, knot theory, polyhedra, optimization, and more. Readers are sure to find them engaging and captivating. The enormous range of creativity of the Bridges community continues to be astounding. This complicates the task of serving on the Program Committee. We are grateful for the meticulous service of those members who graciously assist their colleagues in polishing their written work.

An exhibition of mathematical art has been an annual feature of Bridges since 2001. Artists participate from around the world, representing diverse cultural backgrounds. A wide variety of artistic media are represented in the exhibition, including 2D and 3D digital prints, drawing, painting, beadwork, weaving, ceramics, woodwork, metalwork, quilting, and paper cutting and folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, number theory, and more. This year Robert Fathauer and Bruce Torrence served as co-curators of the exhibition. The jury considering the artworks consisted of Robert Fathauer, Bruce Torrence, Taneli Luotoniemi, and Katie McCallum. The art submission website was created and administered by Nathan Selikoff.

Running an online conference has its own share of unique challenges and we are extremely grateful to the organizers this year who made it possible for the Bridges community to connect with each other from locations all over the globe, something that would have been impossible a few years ago. Thanks to Henry Segerman for overseeing the creation of a virtual space for the 2021 conference.

For creating an atmosphere of goodwill, sharing, and working together to further the study of mathematics and art, we thank founder Reza Sarhangi, and for upholding those values, we commend the Bridges Community itself.

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