Mathematics in the Poetry of Sefer Yetzirah

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Abstract

This paper offers my poem, "I witnessed the creation," along with a discussion on the inspiring poetic and mathematical source — the oldest and most mysterious Kabbalistic text, *Sefer Yetzirah*. The concluding section briefly highlights the place held by the mathematics of the ancient Hebrews among the mathematical developments that occurred in the region around the same time.

Introduction

Sefer Yetzirah: The Book of Creation in Theory and Practice, by Aryeh Kaplan [5], contains a translation from the Hebrew, along with extensive commentaries, of the oldest and most mysterious Kabbalistic text, *Sefer Yetzirah*, (*The Book of Creation*). *The Book of Creation* is so old its origins are no longer accessible, but it was attributed from early times to the Patriarch Abraham (2215 – 2040 BCE). Legend tells how Abraham, born in Mesopotamia and having lived for a period of time in Egypt, was initiated into all the astrological mysteries of his age. His teachings, transmitted down the generations in oral form and in great secrecy, were collected in writing at much later, unknown dates, in forms altered by the passage of time. Some believe it may have been put in writing as notes for his students or even authored by Rabbi Akiva (50 – 135 CE). The result is *Sefer Yetzirah*, a meditative and poetic text with strong magical overtones, which deals with letter permutations and various signs and incantations connected with letters, numbers and heavenly bodies. There had been many interpretations of *Sefer Yetzirah* through the ages, starting from the first century of the common era to the present (see [5] for an extensive list of references). Yet, the book still defies any definitive interpretation.

From the point of view of someone interested in the connections between poetry and the history of mathematics, *Sefer Yetzirah* offers a unique take on the story of the creation of the world where numbers and letters intertwine to form the basis of all existence. The modern reader can also enjoy the book's poetic expression, philosophical message, aura of mystery surrounding it, and the amazingly abstract and skillful number manipulations reminiscent of the work of Pythagoras (570 – 495 BCE). In Iamblichus' (240 – 320 CE) account of the life and teachings of Pythagoras [9], he mentions Pythagoras' travels. In particular, he states that Pythagoras learned geometry in Egypt, and number theory in Canaan. From Iamblichus' account it seems that Pythagoras came into contact with Hebrew number mysticism, with which he shared an abstract approach and a secretive and reverential attitude towards numbers and their properties. Iamblichus writes: "...he [Pythagoras] was transported without delay by some Egyptian seamen who had most opportunely anchored at the shore under Carmel, the Phoenician mountain where Pythagoras spent a good deal of time alone in sacred pursuits." Mount Carmel, situated in what is today the Israeli city of Haifa, is a well-known center for religious and mystical retreats [1, 8]. The oldest known retreat, said to have been founded by the Prophet Elijah (900 – 849 BCE), taught meditation, astrology and numerology. Several versions of *Sefer Yetzirah* are extant today, four of which are translated in Kaplan's book.

This paper starts by presenting my poem "I witnessed the creation," which was inspired by reading Kaplan's book. It then provides, both as background to the poem and as a topic of interest in its own right, a discussion on the mathematical ideas present in the poetry of *Sefer Yetzirah*. The concluding section extends the background beyond *Sefer Yetzirah* by highlighting the place held by the mathematics of the ancient Hebrews among the mathematical developments that occurred in the region around the same time.

The Poem: I Witnessed the Creation

My poem "I witnessed the creation," responds to and offers interpretations for five poetic passages from *Sefer Yetzirah*, which focus on ancient connections between mathematics and poetry — connections which are echoed in modern mathematical thought. Words in Italics are taken from Kaplan's translation of *Sefer Yetzirah* [5].

I witnessed the creation (*Yetzirah*)

When the world was uttered into being
it was also reckoned into being
Yah
the Lord of Hosts
the God of Israel
the living God
King of the universe
created the world with quality and quantity
Five dimensions spinning wildly for six days
and on the seventh — *Sabbath.*

Out of the cloud of my not yet becoming
I saw the twenty-two stones arranged in a circle of fire
The Circle oscillated back and forth
an ideal figure
fully round
before the orbit of this imperfect planet wobbled
and our world solidified
Lines were gates to a vision of heaven
231 Gates connected the stones on the circle —
a giant spiderweb pulsing in space
pinned to stone eyes
And my eye in the center
marking the time of waiting for a visible future
where *there is nothing in good higher than Delight (Oneg)*
and there is nothing evil lower than Plague (Nega).

I saw a host of flashing lights traverse the vastness —
plurality born of the singular light
of the Infinite (*Ein Sof)*
For the Master is singular
He has no second
And before One, what do you count?
Enumeration —
the ineffable given form through my body
Ten Sefirot of Nothingness
in the number of ten fingers
five opposite five
Finger opposite finger —
the concept of one-to-one correspondence
with which the future
will unravel the secrets
of transfinite cardinals.

22 stones — the Foundation Letters
10 digits — the numerals one to ten
Together they comprise
the *32 mystical paths of Wisdom*
I saw engraved
into the bedrock of existence at creation
which we are meant to discover and illuminate
with three books (Sepharim)
with text (Sepher)
with number (Sephar)
and with communication (Sippur)
My mind searched
my heart filled with yearning
Write
calculate
and speak
Share your story—
not one of us can live without others.

Down to earth
I learned to compute permutations
to gain peace of mind
and the patience to stay in one place
and construct.
Any given number of stones *n* can be permuted in *n!* ways
How many houses can we build from *n* stones?
Two stones build 2 houses
Three stones build 6 houses
Four stones build 24 houses
n stones build *n!* houses
To build houses that will hold the names of all the stars
in the observable universe
we need 22 stones.
22! = 1,124,000,727,777,607,680,000
A number which *the mouth cannot speak*
and the ear cannot hear
But we must.

The Background: Mathematics in the Poetry of Sefer Yetzirah

In this section we discuss the mathematical aspects of the poetic passages from *Sefer Yetzirah* that contributed lines to my poem, "I witnessed the creation."

Stanzas 1 and 4 of "I witnessed the creation," cite from and respond to the starting poem of *Sefer Yetzirah*:

1:1 With 32 mystical paths of Wisdom
engraved Yah
the Lord of Hosts
the God of Israel
the living God
King of the universe
El Shaddai

Merciful and Gracious
High and Exalted
Dwelling in eternity
Whose name is Holy —
He is lofty and holy —
And He created His universe
with three books (Sepharim)
with text (Sepher)
with number (Sephar)
and with communication (Sippur).

According to Kaplan [5], the "32 mystical paths of Wisdom" refers to the 22 letters of the Hebrew alphabet and the 10 numbers, numerals 1 to 10. The letters and the numbers are the most basic ingredients of creation: quality and quantity. The quality of any given thing can be described by words formed out of the letters, while all of its associated quantities can be expressed by numbers. The three books refer to God's divine emanations that form the basis of creation: text, number and communication, representing, respectively, quality, quantity and speaking. The three books may be viewed as corresponding to the three divisions of creation: space (Universe, World), time (Year, cycle, circle), and spirit (soul). Space has 3 dimensions, but represents one unity — the text (graphical representation). The 4^{th} dimension is time, represented by numerical calculations. The 5^{th} dimension is the spiritual dimension, representing relationships and meaning. All creation falls within the spectrum of continuum of these 5 dimensions [5]. We offer a complementary interpretation: One can consider the three books as corresponding to the three divine abilities that were imbued into the world at the time of creation, in latent form: text corresponds to writing, number to mathematics, and communication to verbal expression or language. Human beings were created with an innate capacity to discover and develop the three potentialities.

Stanza 2 of "I witnessed the creation," cites from and responds to a poem from the second chapter of *Sefer Yetzirah*:

2:4 Twenty-two Foundation Letters:
He placed them in a circle
like a wall with 231 Gates.
The Circle oscillates back and forth.
A sign for this is:
There is nothing in good higher than Delight (Oneg)
There is nothing evil lower than Plague (Nega).

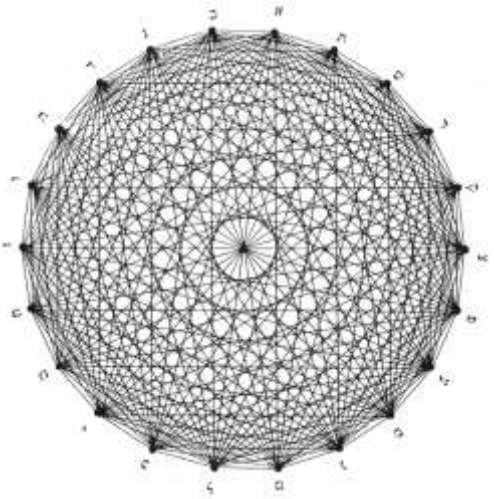

Figure 1. *The 231 line segments connecting the 22 letters of the Hebrew alphabet [5].*

According to Kaplan, The Foundation Letters are the 22 letters of the Hebrew alphabet. In *Sefer Yetzirah*, the letters of the Hebrew alphabet are frequently called stones. The Kabbalists say that they are "stones quarried from the great Name of God." If a number, *n*, of points are placed on a circle, the number of connecting lines between these points can be calculated by the formula $n(n–1)/2$. Arrange the 22 letters of the Hebrew alphabet as points on a circle. The number of "gates," meaning "connecting lines," is $(22 \times 21)/2 = 231$. It is unlikely that this formula was known to the ancient Hebrews except perhaps as a calculating pattern. The calculations were most likely caried out one number at a time, a practice designed to induce a meditative trance. The mystical significance of this particular mathematical calculation and the meaning of the mysterious last two lines are unclear, although the last line, "nothing evil lower than the plague," resonates with our feelings today during the current pandemic. Mathematically speaking, the ancient computations in this passage are the seeds that will grow into several modern areas of mathematics. The formula mentioned above is nowadays an elementary part of Algebra, Combinatorics, Number Theory, and other areas; while the image of the 231 line segments connecting the 22 points on the circle is that of a complete graph on 22 vertices, anticipating Graph Theory, and bringing to mind the relatively modern use of graphs to create string art.

Stanza 3 of "I witnessed the creation" cites from and responds to lines from two poems from the first chapter of *Sefer Yetzirah*:

1:3 Ten Sefirot of Nothingness
in the number of ten fingers
five opposite five
with a singular covenant
precisely in the middle
in the circumcision of the tongue
and in the circumcision of the membrum.

1:7 Ten Sefirot of Nothingness
Their end is imbedded in their beginning
and their beginning in their end
like a flame in a burning coal
For the Master is singular
He has no second
And before One, what do you count?

The "10 mystical paths of Wisdom" consisting of the 10 numbers, 1 to 10, are also manifested in the Ten Sefirot, which according to Kabbalists are the most basic concepts of existence. The Hebrew word *Sefirah* (singular of *Sefirot*) means "counting" or "enumeration." It is distinct from the word for number. Numbers could not be defined until there existed a concept of plurality in creation. The Creator Himself is singular, the absolute unity, the indivisible Infinite (Ein Sof). Plurality came into existence at creation in the form of the Ten Sefirot. The Ten Sefirot are "of Nothingness," meaning purely ideal concepts, without any substance. Unlike letters which have form and sound, the Sefirot have no physical properties [5]. In Kabbalah, the Ten Sefirot have names that are not mentioned in *Sefer Yetzirah*, but are well known from other classical Kabalistic texts. They are usually represented pictorially in a three-column vertical arrangement called "The Tree of Life" [10]. It is interesting to note that the Ten Sefirot pictured as dots or circles in the "Tree of Life," are connected by precisely 22 line segments, which are often labeled by the letters of the Hebrew alphabet. A full discussion about the meaning of the Ten Sefirot is beyond the scope of this paper. We offer a mathematical interpretation: The Ten Sefirot may be considered to be the abstract idea of enumeration embodied in the concept of one-to-one-correspondence. Kaplan notes that a possible arrangement of the Ten Sefirot is on the fingers of two hands. The hands are presented with the fingers "five opposite five", that is, they are positioned to show the one-to-one correspondence between the two sets of five fingers.

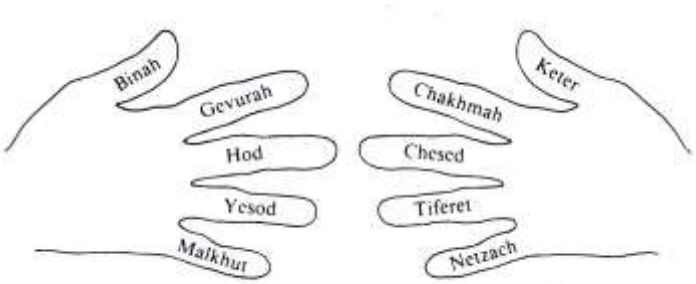

Figure 2. *Polarizing the ten Sefirot through the ten fingers [5].*

Before the counting numbers acquired names, that is, before humans acquired the ability to count, the most likely way to assess quantity had been through one-to-one correspondence. For example, if a large cattle area had a number of individual cattle pens and when the herd entered the area each cow occupied exactly one pen and no pen remained empty, we say that the pens and the cows are in one-to-one correspondence, and so clearly, without actually counting either, their numbers are equal. This early way of assessing quantity acknowledges plurality before counting. The notion of one-to-one correspondence was dramatically revived by Georg Cantor (1845 – 1918 CE) when he used it as a technique to compare the "sizes" of various infinite sets. In particular, it was used to prove that the infinite number (cardinality) of all the real numbers is larger than the infinite number (cardinality) of the counting numbers 1, 2, 3,…. With this, the concept of one-to-one correspondence became a tool in set theory, which was instrumental in the development of the accurate mathematical treatment of transfinite cardinals [3].

It is not clear what was the notation for numerals and how were the numerical calculations appearing in *Sefer Yetzirah* carried out. The book is believed to have been in existence long before the invention of the Hindu-Arabic numerals along with the base 10 place value number system, both of which were invented in India circa 600 CE. There are speculations that numbers were denoted by tally lines, and the Ten Sefirot were represented in the form of the Tetractys [7], a symmetric triangular arrangement of tally lines with 4 tally lines at the bottom, 3 on the row above them, 2 on the next higher row and 1 on top. The total number of tally lines in such a tetractys is $1 + 2 + 3 + 4 = 10$, hence the 10 Sefirot. The Tetractys, or "holy fourfoldness," later became the symbol on which the members of the Pythagorean Brotherhood swore their oath of allegiance [2].

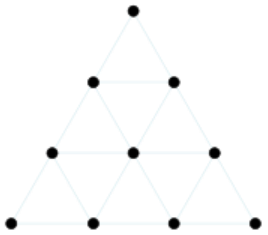

Figure 3. *The Tetractys*

There is a possibility that numbers were represented not by tally lines, but by small stones, as it was the custom in the Mediterranean region in ancient times, and from which the name Calculus is derived (calculus — Latin for stone). If true, this will make another interesting connection between numbers and the letters of the Hebrew alphabet, which were also called stones. In general numbers and letters were intertwined in numerous ways, and Mordell claims that the notation for numbers came first and gave birth to the symbols representing letters [7]. Regardless of priority, at some point in antiquity the Hebrews assigned numerical values to the letters of the alphabet. The numbers 1 to 10, corresponded to the first ten letters; the numbers 10, 20, … 100, corresponded to the next ten letters, and so on (see Table 1).

Table 1. *Each Hebrew letter is given a numerical value between 1 and 400, the five final-letters (end-of-word letters) are given their own values, ranging from 500 to 900.*

Number	Name	Letter	Number	Name	Letter	Number	Name	Letter
1	*Aleph*	**א**	**10**	*Yod*	**י**	**100**	*Koof*	**ק**
2	*Bet*	**ב**	**20**	*Kaf*	**כ**	**200**	*Reish*	**ר**
3	*Gimel*	**ג**	**30**	*Lamed*	**ל**	**300**	*Shin*	**ש**
4	*Dalet*	**ד**	**40**	*Mem*	**מ**	**400**	*Taf*	**ת**
5	*Hey*	**ה**	**50**	*Nun*	**נ**	**500**	*Kaf (final)*	**ך**
6	*Vav*	**ו**	**60**	*Samech*	**ס**	**600**	*Mem (final)*	**ם**
7	*Zayin*	**ז**	**70**	*Ayin*	**ע**	**700**	*Nun (final)*	**ן**
8	*Het*	**ח**	**80**	*Pey*	**פ**	**800**	*Pey (final)*	**ף**
9	*Tet*	**ט**	**90**	*Tzadik*	**צ**	**900**	*Tzadik (final)*	**ץ**

Although not a place value number system, the number 10 acted as sort of a base. It is interesting to note that the first letter of the Hebrew alphabet, **א**, is the symbol Cantor had chosen to denote (with various subscripts) infinite cardinalities. The shared symbolism between numbers and letters led to an interweaving of meanings between numerical values and words, which resulted on one hand in esoteric mystical practices, and on the other hand it provided the impetus for performing marvelously skillful number manipulations heralding modern mathematics.

The last stanza of "I witnessed the creation" cites from and responds to a poem from the fourth chapter of *Sefer Yetzirah*:

4:16 Two stones build 2 houses
Three stones build 6 houses
Four stones build 24 houses
Five stones build 120 houses
Six stones build 620 houses
Seven stones build 5040 houses
From here on go out and calculate
that which the mouth cannot speak
and the ear cannot hear.

These lines discuss the number of permutations possible with a given number of "stones," that is, letters of the Hebrew alphabet. For two stones this number is 2. For three stones this number is 2 x 3 = 6. For four stones this number is 2 x 3 x 4 = 24. In general, for *n* stones the number, called *n* factorial, is *n!* = 2 x 3 x 4 x....x *n*. Thus 5! = 120, 6! = 620, 7! = 5040. Continuing the computation yields larger and larger numbers. For 8 stones, we get 8! = 40,320. This number, in itself, is not impossible to pronounce. According to Kaplan [5], what is beyond human ability to "mouth" or "hear" is the articulation of every letter in all the 40,320 permutations of 8 letters, a total of 40,320 x 8 = 322,560 letters, chanted like a mantra to induce altered states of consciousness.

Subsequent chapters of *Sefer Yetzirah* speak of twelve "elemental letters." Twelve letters can be permuted in about half a billion ways, which would take 63 years to pronounce. There are about sextillion (10^{21}) possible permutations of all the 22 letters of the Hebrew alphabet, which according to some sources is close to the total number of stars in the observable universe [5, 11]. Thus, among all the permutations of the letters of the Hebrew alphabet one can find "housed" the names of all the stars in the observable universe. Permutations of letters for mystical and meditative purposes were transformed over thousands of years into the mathematical area of Group Theory, and naming all the stars in the observable universe is still a favorite practice of astronomers and poets alike.

Concluding Remarks

The ancient Hebrews lived in the territory situated around present day Israel. Located on the eastern shore of the Mediterranean, it was one of the cultures in the region that included several major centers of mathematical learning and activities. These centers were: Mesopotamia (situated around present day Iraq), Egypt, and Greece along with its colonies located in Egypt (Alexandria) and in southern Italy (Crotona, Sicily, etc.). Mathematics was developed to respond to both practical and spiritual needs and although many mathematical developments occurring around that time were common to all cultures (like, for example, a working knowledge of the so-called Pythagoras Theorem), each culture also made unique contributions to the foundations of the discipline. Briefly outlined, the specific contributions of the cultures in the region were: the first place value number system in the world (sexagesimal) was invented in Mesopotamia, as was the ability to perform basic algebra and the beginnings of a scientific approach to astronomy [2,4]; Egyptian mathematics contributed advanced plane and space geometry [2]; while the Greeks contributed the notion of proof within an axiomatic framework, which is the cornerstone of mathematics, as well as numerous innovations in geometry, number theory, and physics [2,3]. Building the foundations of mathematics was not the work of any single culture alone, it was through cross cultural exchanges facilitated by travel that the shared body of knowledge grew into the discipline we know as mathematics.

What is the unique contribution of the ancient Hebrews to the shared body of knowledge that built the discipline? *Sefer Yetzirah* provides an answer: The Hebrew culture had an abstract bent that was new in the ancient world. It manifested itself in the first monotheistic religion with the belief in a god who has no earthly image, and it is also evident in the mathematics, as well as in the philosophical message, contained in the poems from *Sefer Yetzirah* discussed in this paper. In particular, the abstract number manipulations, independent of any practical uses, which heralded modern ideas and areas of pure mathematics, made their first appearance in the poetry of *Sefer Yetzirah*.

Interested readers may find additional sources of mathematics in ancient Jewish texts as well as further references on Jewish mathematics in [6].

Acknowledgements

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