A Geometer Quite Acrimonious — a Limerick

Bjoern Muetzel and Nazarré Merchant

Department of Mathematics, Eckerd College, St. Petersburg, FL, USA;
muetzeb@eckerd.edu and merchann@eckerd.edu

Abstract

We present a math limerick about the famous problem of Apollonius and delve a bit into the history of math limericks and the problem.

The Limerick

A geometer quite acrimonious
played the drums in a manner felonious.
Though grim, but not dim,
with the size of each rim,
solved the problem of old Apollonius.

Math Limericks

The limerick is a verse of three long and two short lines rhyming *aabba* [12]. It's specialty is that it packs laughs anatomical into space that is quite economical. Though it bears the name of the beautiful town of Limerick in Ireland the clear origin of the limerick as well as the exact source of its name seem to be uncertain. The form clearly appeared in England in the early years of the 18th century. It was popularized by Edward Lear (1812-1888) in the 19th century. Lear, an English artist, illustrator, musician, author and poet was famous for his works of literary nonsense. An example of one of his limericks is

There was an old man with a beard,
Who said, "It is just as I feared!-
Two owls and a hen,
four larks and a wren,
have all built their nests in my beard!"

An early math limerick is attributed to the British wordplay and recreational mathematics expert Leigh Mercer (1893–1977):

$$\frac{12 + 144 + 20 + 3 \cdot \sqrt{4}}{7} + 5 \cdot 11 = 9^2 + 0.$$

It reads:

A dozen, a gross, and a score
Plus three times the square root of four
Divided by seven
Plus five times eleven
Is nine squared and not a bit more.

Our favorite recent limericks are the following: Ben Orlin's limerick about the number 17.

"How lonely a number I've been"
it said with a widening grin,
"but I should have known
I am not really alone:
I am not just a prime, but a twin!"

A geometric limerick about the surprising properties of the Moebius strip by an unknown author:

A mathematician confided
That the Moebius band is one-sided
And you'll get quite a laugh
If you cut one in half
'Cause it stays in one piece when divided.

Finally an example from complex analysis by Marion D. Cohen: "nth root of unity"

There are n of them sprawled on a wheel.
Among them at most two are real.
The others must go
half above, half below.
But they get paired off in the deal.

More equation limericks can be found at Stackexchange [10] and some more on Ben Orlin's website "Math with Bad Drawings" [8]. A collection of math limericks for different math courses was published recently [2] by Marion D. Cohen. Still, here a gap in current research reveals itself — the literature on the history of math limericks seems to be very poor and examples are hard to find.

The Problem of Apollonius

In Euclidean plane geometry, Apollonius' problem is to construct circles that are tangent to three given circles in a plane (see Figure 1).

Apollonius of Perga (ca. 262 BCE–ca. 190 BCE) posed and solved this famous problem in his work "Tangencies". Though this work has been lost a report of his results by Pappus of Alexandria from the 4th century CE has survived. Three given circles generically have eight different circles that are tangent to them. A constant source of inspiration over the centuries, a rich repertoire of geometrical and algebraic methods has been developed to solve Apollonius' problem [3] which has been called the most famous of all geometry problems.

A special case is the case where the three given circles are mutually tangent. In this case Apollonius' problem has five solutions. Three solutions are the given circles themselves, since each is tangent to itself and to the other two given circles. The remaining two solutions correspond to the inscribed and circumscribed circles. These are called Soddy's circles (see Figure 2).

The theorem that gives the relationship between the radii of these circles was stated and proven by Descartes in 1643 and bears his name. A proof can be found in [4], Chapter 1. It was rediscovered again independently in 1826 by Jakob Steiner [11] in 1842 by Philip Beecroft [1] and in 1936 by the polymath and Nobel Prize winner Frederick Soddy. To close the circle we would like to mention that Soddy published his findings in the scientific journal *Nature* as the poem, "The Kiss Precise" [9]. This was rounded out by

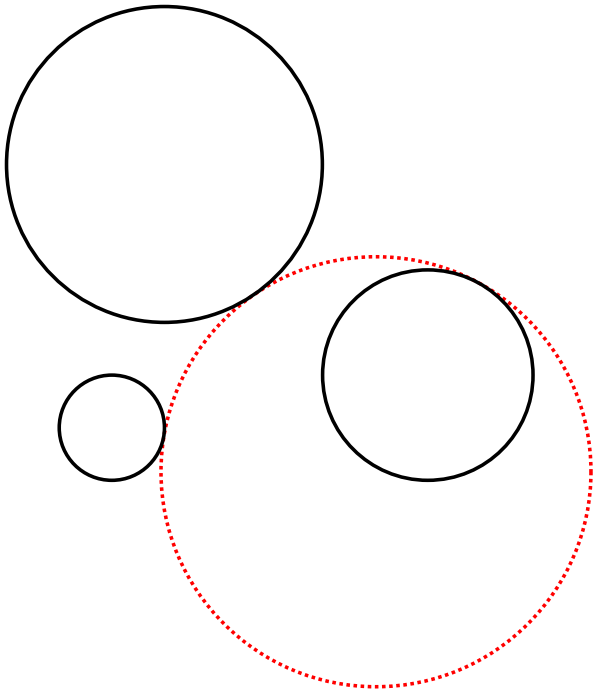

Figure 1: *Three given circles in black with a solution to Apollonius' problem in red / dashed.*

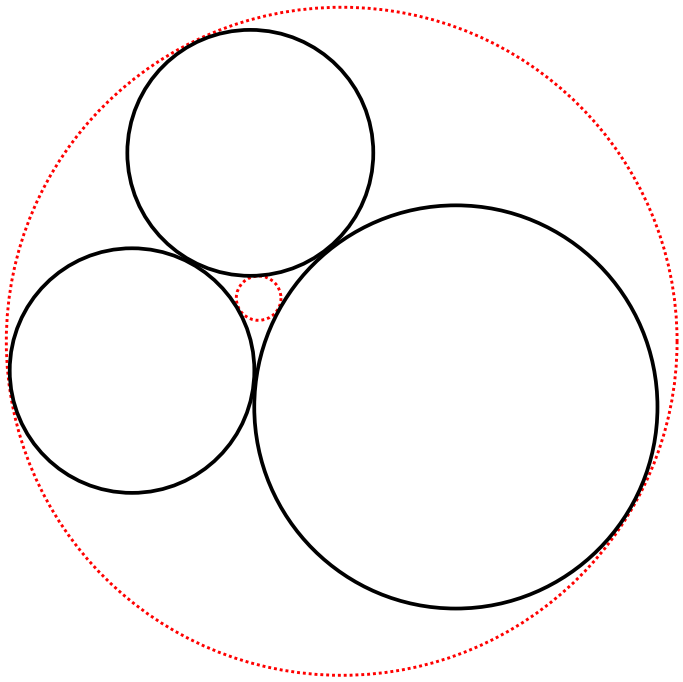

Figure 2: *Three given tangent circles with the two inscribed and circumscribed circles in red / dashed.*

Gosset, an English amateur mathematician in a following issue, where he added a fourth verse generalizing the theorem to hyperspheres in dimension n [7]. A thorough discussion of this poem can be found in Chapter 3 of Martin Gardner's book *Mathematical Circus* [5].

Summary and Conclusion

There are quite a number of witty and fun math limericks, however the literature on the history of math limericks seems to be very poor. Together with other poetry inspired by mathematics [6] these could be a great device to make math classes more entertaining and connect to other disciplines. The problem of Apollonius has a long and interesting history. The given math limerick can be an entry point to discuss this problem in the classroom. Here the circles from Figure 1 could be cut out and used to represent the drums, then shifted in the right configuration to reveal the solution to Appolonius' problem. The limerick about the Moebius strip is equally suited for a surprising demonstration.

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