A Periodic Sponge Surface Based on Truncated Octahedra

Yuki Kobayashi[1], Seiya Kirihara[2], and Chie Nara[3]

[1]Osaka City University, Osaka, Japan; kobayashi@osaka-cu.ac.jp
[2]Osaka City University, Osaka, Japan; a17tq011@ab.osaka-cu.ac.jp
[3]Meiji University, Tokyo, Japan; cnara@jeans.ocn.ne.jp

Abstract

We define a flexible periodic sponge surface T based on truncated octahedra (TOs), which can be folded flat. We provide a method of creating a model of T through drawing the development (net) of an element of T on a sheet of paper. We show that some parts of T can be fixed by filling some TOs in T, and we propose an architectural scale construction.

Introduction

We present an infinite structure composed of planar hexagons and squares that are connected by *hinges* at their *edges*, derived from the space packing of truncated octahedra (TOs). This surface is flexible but we explain how it can be made rigid by retaining one column of TOs (or equivalently, by locking the hinges along that column). We apply this idea to a transforming architectural design which begins flat then fills a room by inflating three pillars of TOs.

To understand our new structure, one should first be familiar with the well known fact that Archimedean truncated octahedra can be packed together to fill space without gaps or overlaps. This gives what is called the "bitruncated cubic honeycomb" (BCH) and is described in many references [1]. The BCH is a rigid structure with three polygons meeting at each edge: two hexagons and one square. The hexagons of the BCH are oriented in four different orientations. Our structure retains all the squares of the BCH but only the hexagons in one of the four orientations. A portion of the infinite structure (retaining only horizontal hexagons) is shown in Fig. 2, which we created by using a laser cutter.

Definition 1. [2] Let T be a subset of the surfaces (2-skeleton) in the tiling (BCH) consisting of all square *faces* and one set of parallel hexagonal faces. We call T a *periodic sponge surface* based on TOs.

It can be seen that T is connected and flexible. The complement of T is a connected volume, i.e., T does not divide space into two or more regions. The free edges of T (bounding two sides of each square) form a set of infinite triangular helices. In this paper, we discuss which panels can be secured after fixing some *panels* of a periodic sponge surface T.

Drawing the development of T and creating a model

We suggest a method to create a model of T by drawing the development (net) of an element of T on a sheet of paper. We call a *basic part* the figure obtained from a regular hexagon where the alternated three edges are attached with squares, as shown in Fig. 1a. We take three congruent basic parts and label the hexagons 0, 1 and 2 (Fig. 1b). For a basic part, we call it 0-type if it has the hexagon labeled 0 and its three edges, which are not adjacent to squares, marked by bold red line segments; 1-type if it has the hexagon labeled 1; 2-type if it has the hexagon labeled 2 and the three edges in squares, which are not adjacent to the hexagon, are marked by bold red line segments (Fig. 1b).

Using infinite congruent copies of the basic parts of the three types, we can tessellate a sheet of paper, where each basic part is attached to the basic parts of different types, so that it is surrounded by alternated different types and the bold red line segments are overlapped (Fig. 1c). Remove all triangles in the tessellation, and cut along all the segments marked with a bold red line. The resulting figure is then folded along edges so that all hexagons of 0-type (respectively, 1-type or 2-type) are included in a plane H_0 (respectively, H_1 or H_2), where they are parallel and H_1 is located in the middle of H_0 and H_2 (Fig. 2). This results in the achievement of an example of a flexible periodic sponge surface.

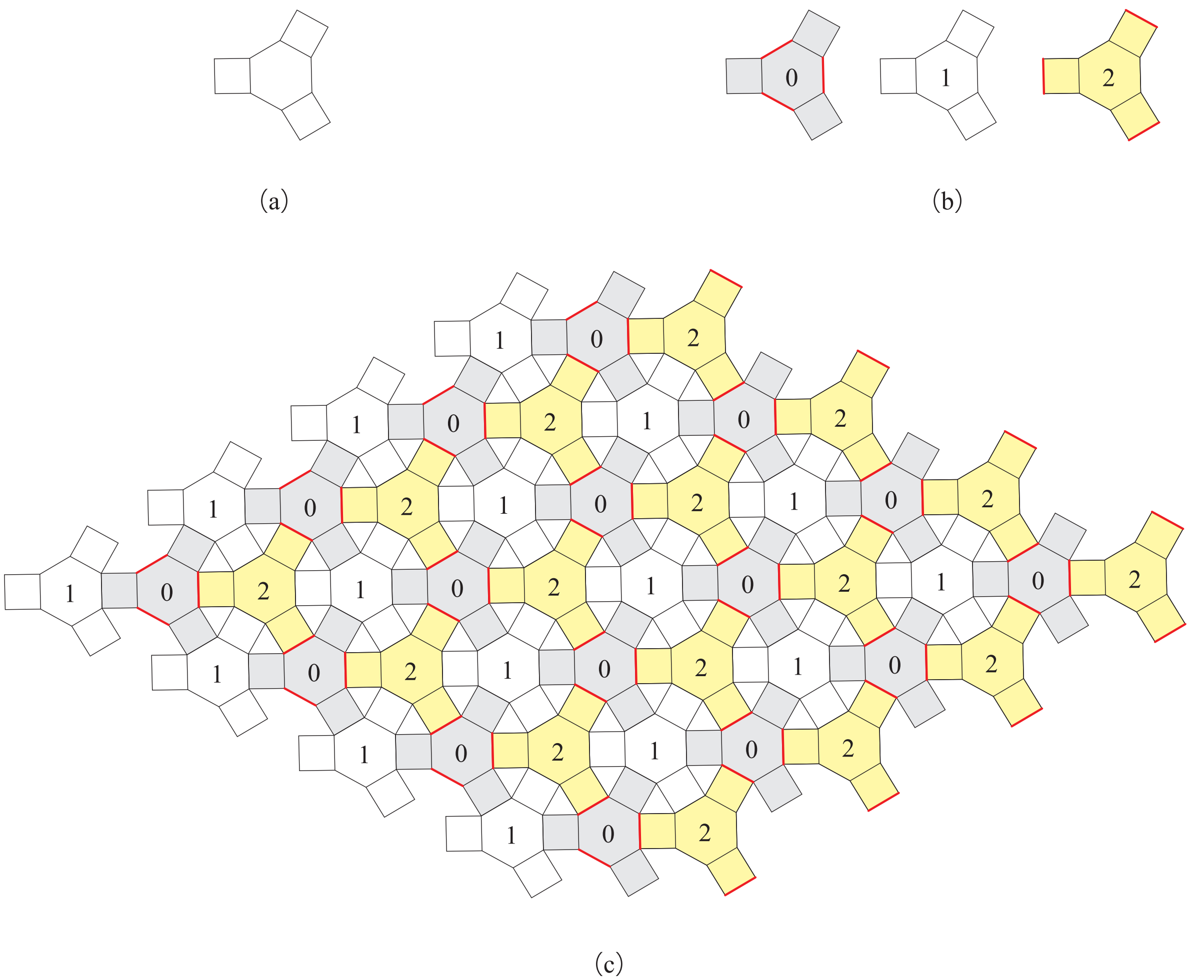
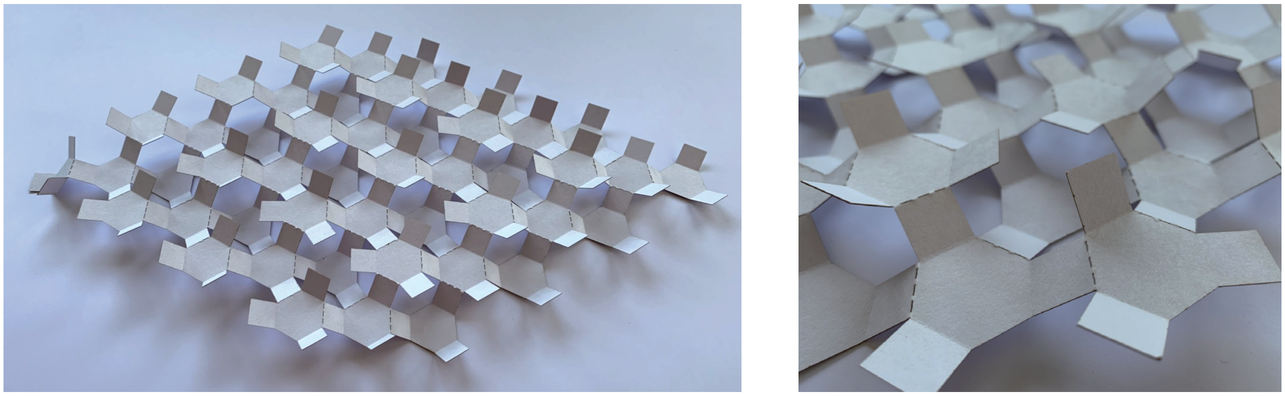


Figure 1: *(a) A basic part. (b) Basic parts of 0-type, 1-type, and 2-type. (c) The arrangement of basic parts of three types to tesselate a sheet of paper.*

In this 3-level structure squares connect from hexagons at level 0 to level 1 and from level 1 to 2. Then level 2 connects similarly to level 0 of the next copy of the 3-level structure, as shown in Fig. 5a.

Regions that become rigid by adding TOs to T

We will show that to make up a loss by removing all the non-horizontal hexagons, if one infinite pillar of the TOs is retained, then the entire structure is rigid and cannot collapse. A rigid surface can be made by the following three operations.



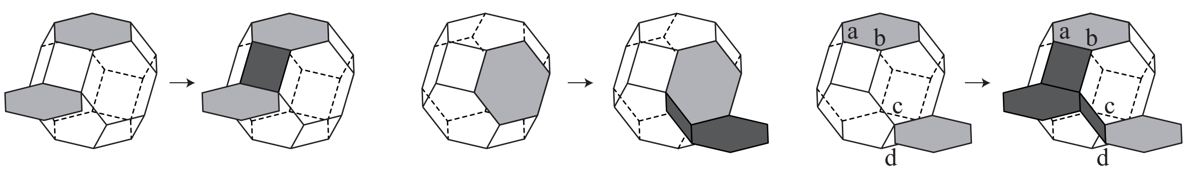
(a) (b)

Figure 2: *(a) A part of T obtained from the development in Fig. 1 (c), (b) its zoomed figure.*

Operation 1. Add one square panel to connect two already fixed hexagonal panels in T, as shown in Fig. 3a.

Operation 2. Add one square panel and one hexagonal panel, connected to each other, to an already fixed hexagonal panel in T, as shown in Fig. 3b.

Operation 3. Add two square panels and one hexagonal panel, connected to each other via a common vertex to two already fixed hexagonal panels in T, as shown in Fig. 3c.



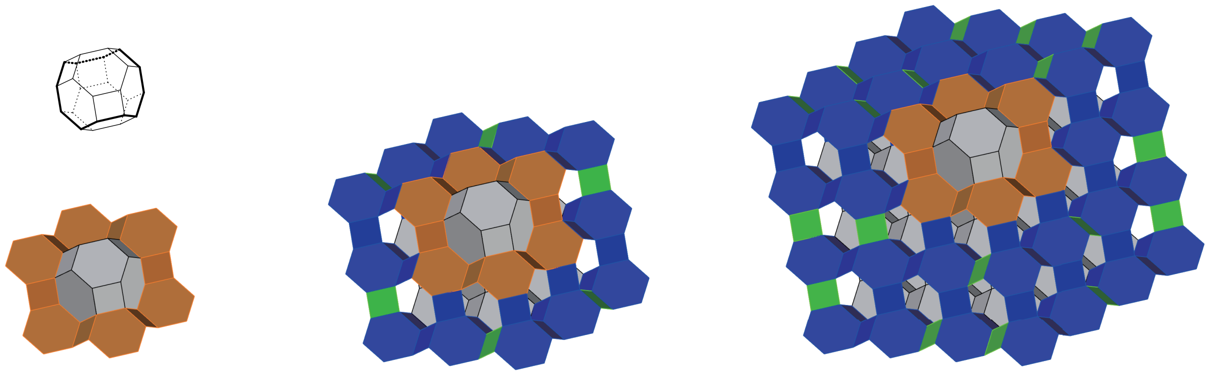
(a) (b) (c)

Figure 3: *(a) Operation 1, (b) Operation 2, and (c) Operation 3.*

We have found that if the framework is rigid, the resulting framework obtained by Operation 1, 2, or 3 is also rigid. The first case is shown by the fact that the four corners of the new square panel are fixed. The second case is shown by the fact that any trihedral connection of three polygons together make a rigid structure even though the individual edges are flexible like hinges. In the third case, denote by ab and cd the two edges of the already fixed hexagonal panels, as shown in Fig. 3c left. By realization of T in $\mathbb{R}^3$, as a part of the surfaces (2-skeleton) of the BCH, it can be shown that the distance of a and d is exactly twice the diagonal length of a square. Hence, the common vertex should be located in the middle of a and d, and uniquely determined. This implies that the new hexagonal panel is fixed by the two edges parallel to ab or cd.

When one TO (W_1) is added to T, the 12 panels adjacent to W_1 become rigid by applying Operation 2 six times (Fig. 4a). When one more TO (W_2) is added to Fig. 4a, applying Operation 2, Operation 3, and Operation 1 subsequently, the 54 new panels become rigid (Fig. 4b). When three TOs are added to T, that is, we add one more TO (W_3) to Fig. 4b, we can show that the 120 new panels become rigid (Fig. 4c). By continuing the process above, we can show that one infinite pillar of the TOs is sufficient to make the structure T rigid.

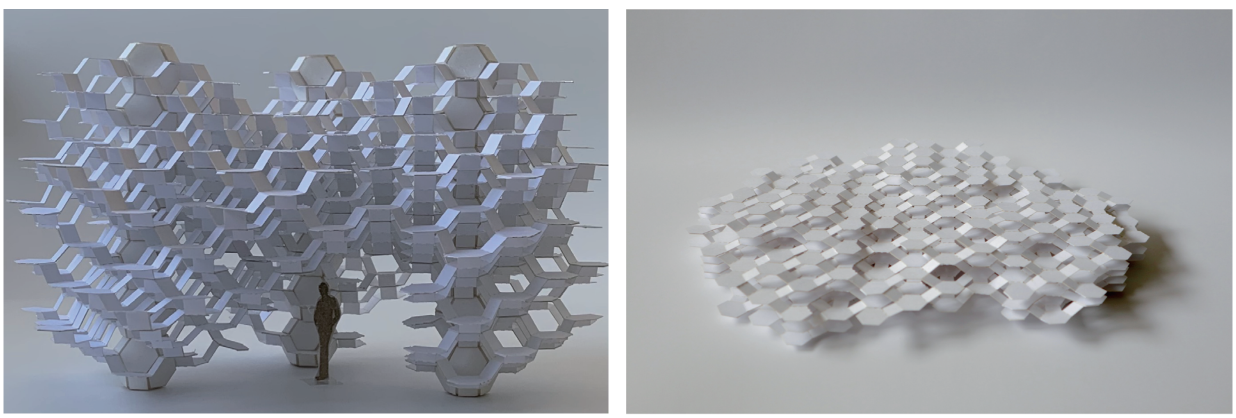
We created a model based on T that can be made rigid by adding TOs. We stack TOs in a column or pillar. For example, if we consider three pillars, the periphery becomes rigid, and the end panels can be



(a) (b) (c)

Figure 4: *(a) Rigid panels when one TO is added. (b) Rigid panels when the second TO is added. (c) Rigid panels when the third TO is added.*

removed as needed. Let M be an example with three pillars. Remove some panels from M to create a space that a person can stand under, as shown in Fig. 5a. If the pillars of TOs can be fabricated using inflatable structures, the entire structure can be laid flat by removing the air from the pillars (Fig. 5b).



(a) (b)

Figure 5: *(a) Example of a model obtained by removing some surfaces from T, with pillars, to create a space under which a person can stand. (b) The model of (a) folded flat by removing the pillars.*

Summary

We have presented a sponge surface derived from TOs and shown that one infinite pillar of the TOs is sufficient to rigidize it.

References

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https://en.wikipedia.org/wiki/Bitruncated_cubic_honeycomb

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