The Short Tiles Category

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Abstract

Islamic geometric patterns can often be decomposed into a set of modular equilateral tiles, each decorated with a small motif, which assemble to form patterns. As the floodgate for tiles with additional side lengths was opened in the previous Bridges paper (Adapter Tiles Evolves the Girih Tile Set) in which the Φ-category was defined, it is time to examine another non-equilateral category – the Short category, which include a new edge length. While many historical 5-fold patterns can be tiled in two ways, only one way has been used to replicate them. With the introduction of Short tiles, the other way is now possible. I argue that the Short category provides a better way to replicate some of the historical patterns. Together with categories not yet published, the Short tiles play a crucial role when creating patterns, especially historical patterns.

Introduction

The common method of replicating Islamic geometric patterns is by hand, using compasses and a straightedge, but in recent years, the method of tiling (polygons in contact) has been gaining momentum, especially after Lu and Steinhardt popularized it in 2007 and coined the name "Girih Tiles" [14], and Bonner had his extensive book on the matter published in 2017 [2].

This paper is the second part of a series about non-equilateral sides in tiling. In the first part, *Adapter Tiles Evolves the Girih Tile Set* [12], I updated Lu and Steinhardt's Girih Tile set [14, 17] from the core 5 tiles to 14 tiles, introduced the Φ-category where at least one side has the length of Φ (phi) relative to the edges with unit length, and defined the non-equilateral tiles as Adapters, see Figure 1.

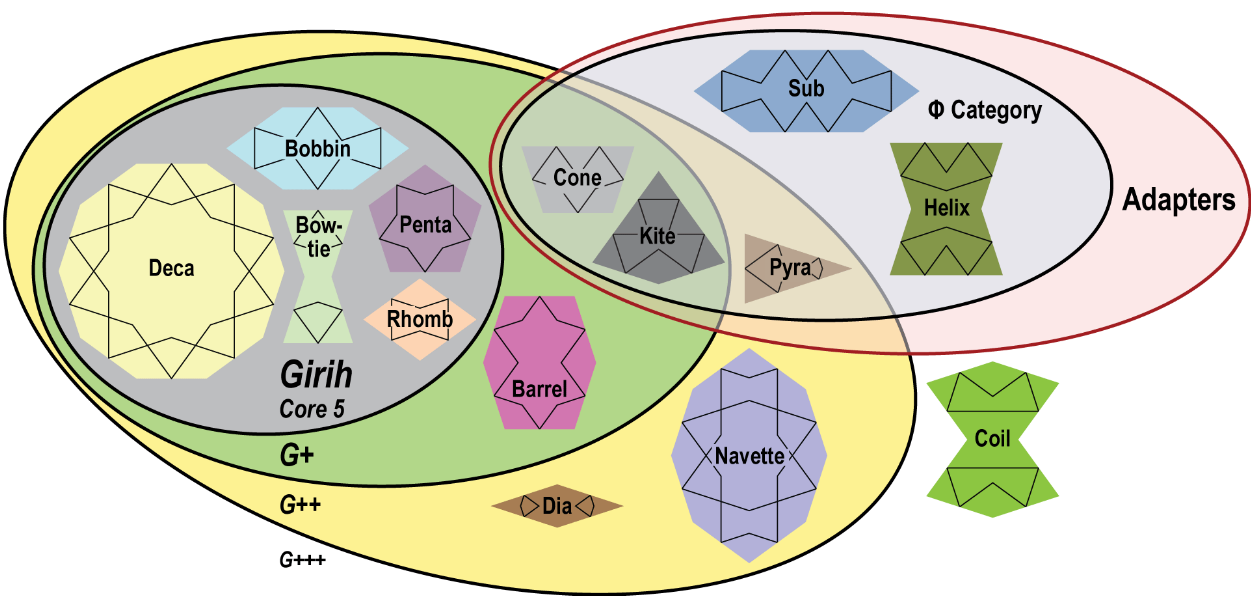

Figure 1: *The evolved Girih tile set, including the Φ-category adapter tiles.*

The Φ-category is one example of adapter tiles, but there are more. In this paper I present a new tile category based on tiles with a different side length: The **Short** category.

Tiles vs. Shapes

What do I mean by “tile”, “tiles”, and “tiling”? When I read about tessellation of tiles, the word "*tile*" often refers to the ceramic tiles used to build architectural patterns, especially in Islamic geometric patterns. Each piece of the pattern is regarded as an entity of its own. In geometry, the word tiling means to cover a plane with these pieces (tessellation).

In contrast, we have Hankin’s “*polygons in contact*” approach, in which the pattern is the result of tessellation of polygons that interacts with each other by different *edge rules*. In his publication from 1925 [15], Hankin describes how lines are to cross the edge of a polygon:

> *”..through the centre of each side of each polygon two lines are drawn. These lines cross each other like the letter X and are continued till they meet other lines of similar origin.”*

His use of the word “polygon” left its mark in Bonner’s term for the same concept, the “polygonal technique” [1, 2]. As the word polygon is a general term, the word “prototile” is commonly used to specifically refer to these polygons. I prefer to leave out the “proto”-part and just use the word “tile”.

Here, **tiles** are the modular pieces that carry the motif, see Figure 2a. The motif continues over to other tiles by the guidance of *edge rules* (crossing point(s) and angle(s)). **Tiling** is the act of tessellating these tiles.

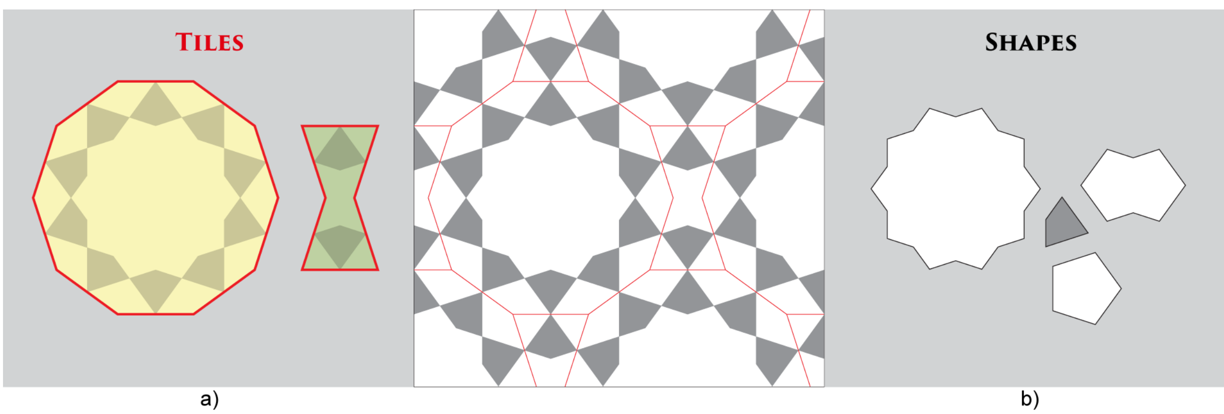

Figure 2: *The difference between Tiles and Shapes.*

For mid-point crossings the main *edge rule* is that the lines have to cross in a symmetrical way, that is, they have to have an equal angle on both sides of the crossing. When these lines continue and meet lines from other crossings, they form the **shapes**, see Figure 2b. The lines do not have to connect with only the first line; they can continue deeper into the tile and connect to the second or third line, giving the tile a different *depth* level. In Figure 2a the Deca tile has depth 2, and the Bow-tie tile has depth 1. The shapes here are visualized by gray and white color. They represent the binarity of the pattern, like a chessboard that have each shape connecting to a shape with the other color. Note that as I make a distinct difference between “tiles” and “shapes”, it might be confusing to read other papers where the use of “tiles” mostly is for “shapes”.

“Invertiles”

When you analyze an existing historical 5-fold pattern, with the purpose to find out how it can be tiled (tile analysis), you often end up with two ways of doing so. The pattern in Figure 3 [5] can be tiled in both ways, using the median 72º Deca and Bow-tie tiles or the obtuse 108º Deca, Penta, and Barrel tiles [12]. One of the angle alternatives has the dark shapes as main motif, while the other have the white shapes.

In a pattern the tile edges are not visible. When analyzing, you strive to find the hidden edges, which you find wherever two pattern lines intersect. Bisect each angle and you get a perpendicular line, forming a cross, marked in red in Figure 3. Each line of the cross indicates a tile edge, perpendicular to the other tile edge, and each represents a way to tile the pattern.

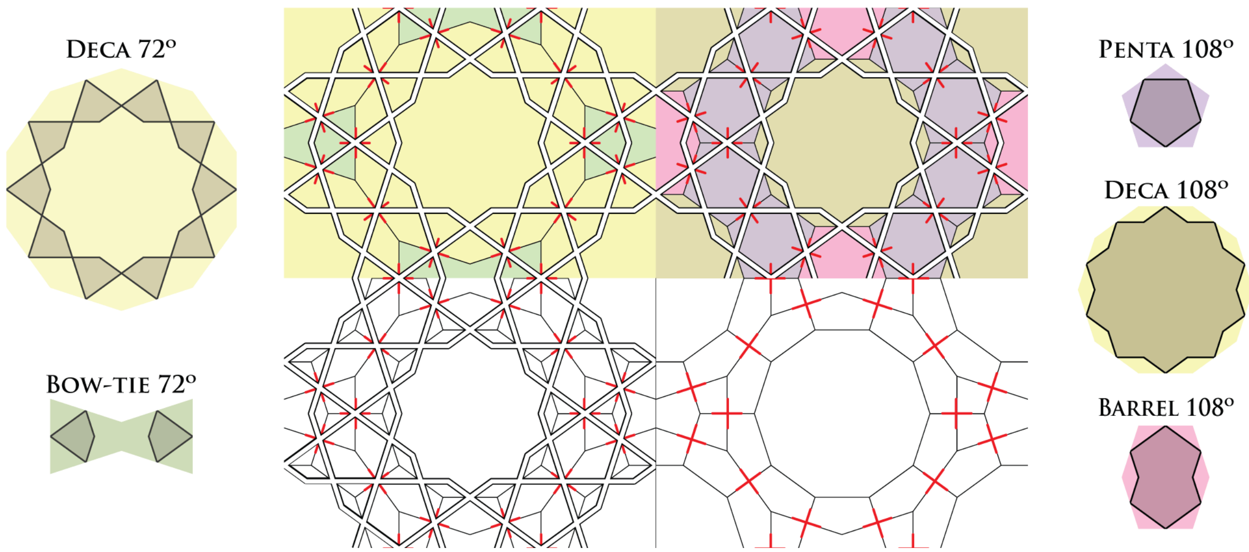

Figure 3: *Two ways of tiling a pattern.*

Far from all patterns can be tiled both ways, so when you set out to analyze which tiles a pattern has, you have to pick one way and test if it works. If it does not, then invert the angle. I use the term "*invert*" as it plays out nicely with the word "*tile*" in my made-up word "*invertile*", hence the name of the chapter. Invertiling shifts the polarity of the shapes, where the main motif shifts place with the areas outside the main motif (close to the edges). This phenomenon is not something new, but it is addressed from different point of views. In several papers [8, 9] Castera describes the concept of Positive and Negative shapes of the Kond pattern family, to which the two ways of tiling the pattern in Figure 3 correspond. It can also be interesting to compare this with Kaplan's "*rosette transform*", in his PhD thesis [17].

The Short Category

In his book from 2017 [2], Bonner describes only two side lengths for the 5-fold system, the unit side and the Φ-side. Cromwell does the same in his paper [11], but Castera depicted tiles with a third side in a workshop presentation [10]. Here, I define this new side length, and introduce a new category – the **Short** category – to the tile chart, see Figure 4.

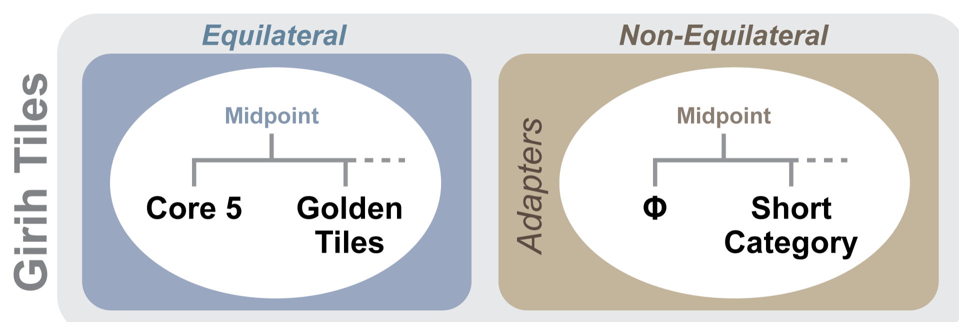

Figure 4: *The Short tile category in the Girih tile chart.*

The acute pattern in Figure 5b shows Jules Bourgoin's Plate 171 [4] depicted in his publication from 1879 [3]. The common way of tiling this pattern is to use tiles from the *core 5*, the Deca and the Penta tiles, together with the Cone adapter tile from the evolved Girih tile set [12], see Figure 5a. But *invertiled* it can also be tiled, with a different tile set – the **Short** category. It uses only two tiles, the **Petal** tile and the Short version of the Deca tile (depth 1) called the **Sun**, see Figure 5c.

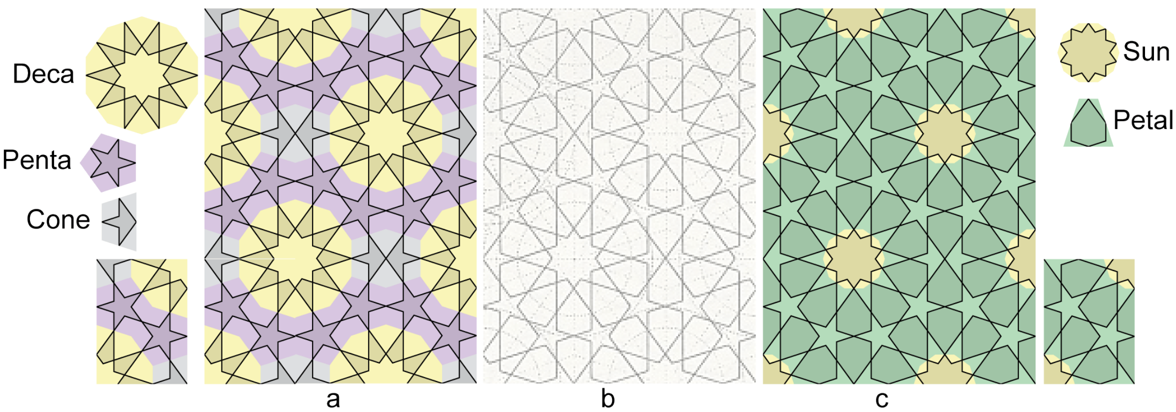

Figure 5: *Bourgoin, Plate 171, tiled in two ways.*

Definition

Take a Bow-tie from the core 5 and cut it in half, see Figure 6. You will end up with a tile with three unit-length sides and one shorter edge, hence the name Short. This is the **Petal** tile. A tile in the Short category is defined by having at least one side with the **Short** length.

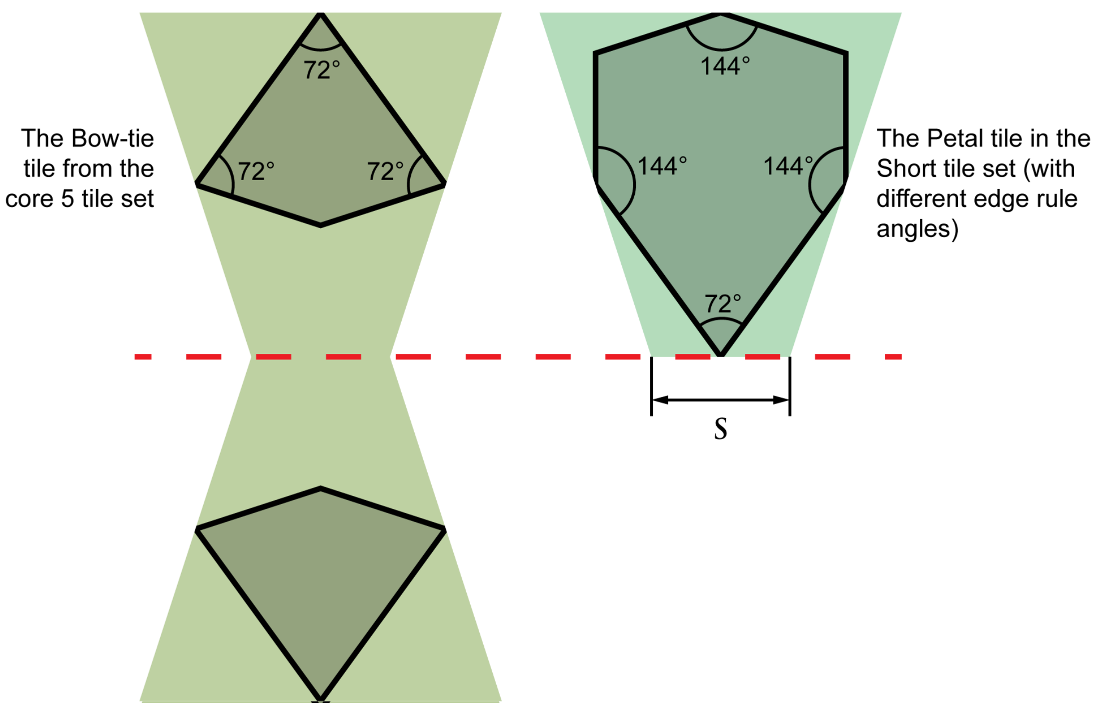

Figure 6: *Half a Bow-tie tile becomes the Petal tile with S as the short side.*

To calculate the Short side length (S) we can use the Cone tile as reference, see Figure 7. The difference between the Φ-side and the unit side is the same as for the difference between the unit side and the S side in the Petal, which gives this elegant formula, S = 2 – Φ.

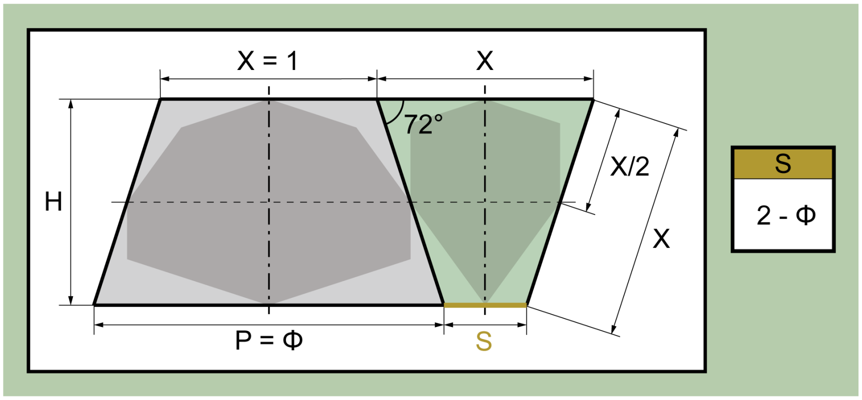

Figure 7: *The measurements of the Short side. The unit side X equals 1.*

The Core Short Category Tile Set – The Core S

There are more Short tiles than the Petal and the Sun tiles depicted in Figure 5c. Using half a sBobbin tile (see Figure 12) as cutting slice (yellow in Figure 8) to cut pieces of a Sun tile will create new tiles with unit length sides. The cut-away slice is the same for all, but different in number and angles throughout the set.

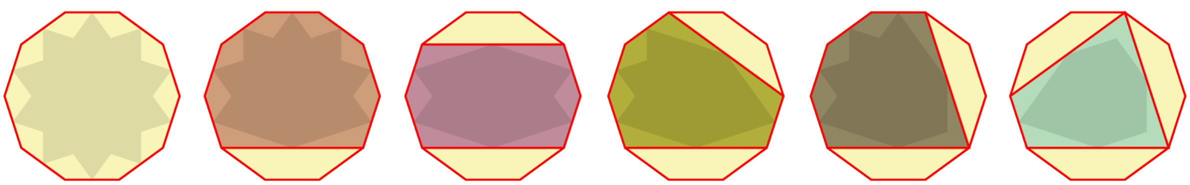

Figure 8: *Cutting up the Sun.*

With all possible cuts we get six tiles, one without cuts (the **Sun** tile), one with one cut (the **Sunrise** tile), three with two cuts (the **Burger**, the **Shield**, and the **Shovel** tile), and one with three cuts (the **Petal** tile). Together, they are the core set of the Short tiles – the **Core S** tile set, see Figure 9.

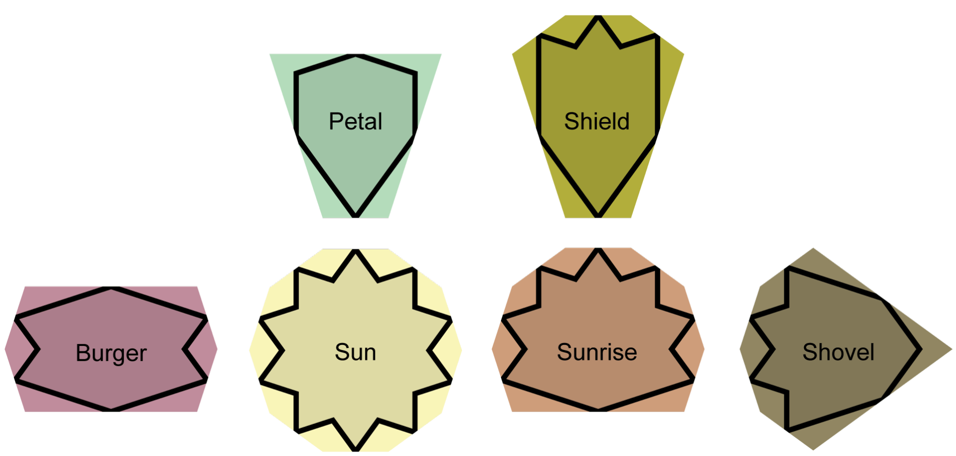

Figure 9: *The Core S tile set.*

"Kond" and "Tond" are two types of Persian patterns [16]. Castera uses the terms "Starry" and "Floral" patterns [9]. Figure 3 is an example of a starry pattern and Figure 5 is a floral pattern (as it has the petal shape). The edge rules of the motifs in the Core S tile set are 144º for the unit side and 72º for the Short side. The motifs in floral patterns correspond to Castera's **positive** and **negative** shapes of the Tond family, see Table 1.

Table 1: *Mapping of tiles to Castera's positive and negative shapes.*

	Kond/Starry		Tond/Floral	
Positive	G+	108º	G++	36º/108º
Negative	Core 5	72º	*Core S*	144º/72º

The floral pattern in Bourgoin's Plate 188b [6] (see its unit cell in Figure 10b) can be tiled in both ways. the Short tiling (see Figure 10c), four tiles from the Short category are used. (Worth mentioning is that Castera's "X-tiles" [7, 8, 9] offer another way to tile this pattern.)

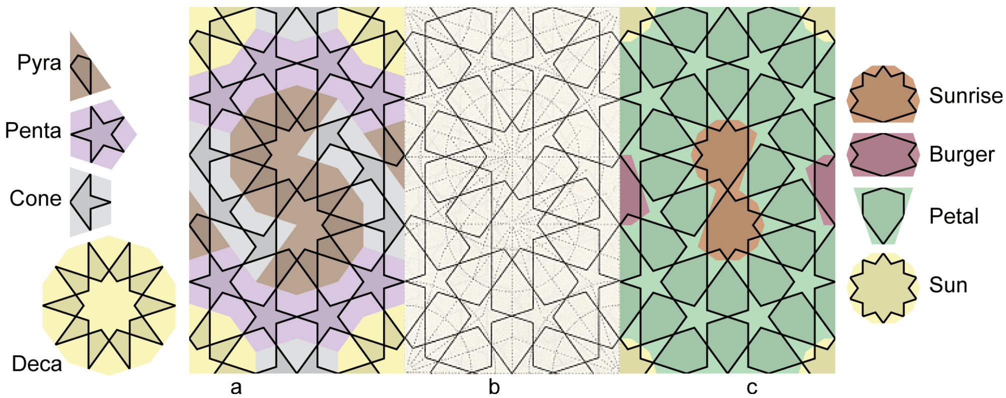

Figure 10: *Bourgoin, Plate 188b, tiled in both ways.*

In the Mirza Akbar architectural scrolls [18], there is a floral pattern that is of particular interest, see its unit cell in Figure 11b. It is more complex and when tiled it uses all six tiles from the Core S tile set, see Figure 11c.

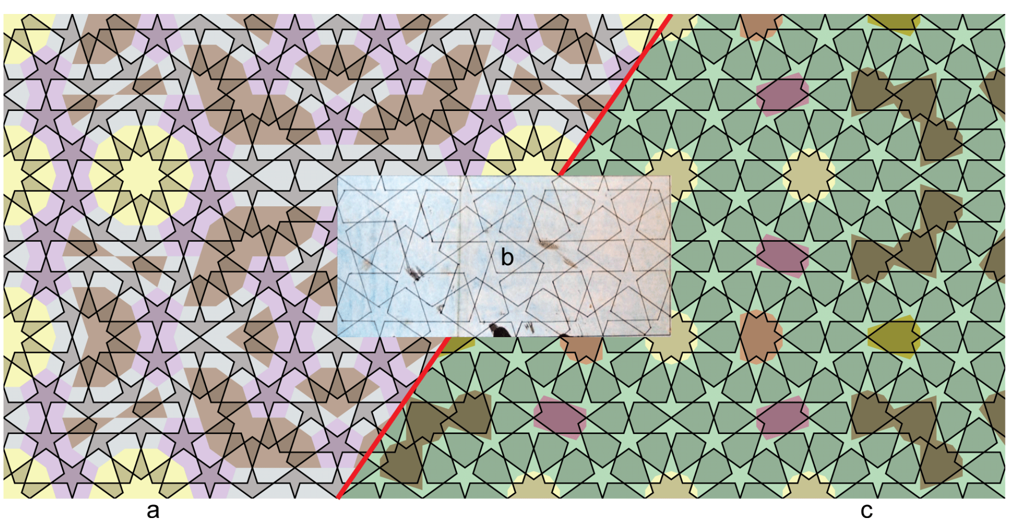

Figure 11: *Two ways of tiling a pattern from a drawing in the Mirza Akbar scrolls (b in the center).*

Short Core 5

As the name Core S implies there are a vast number of other possible Short tiles. One sub-category is already represented within the Core S – the decagonal tile Deca from the Core 5 tile set. When populated in the Core S set the difference is, besides the scale, the depth of the lines of the motif. The rest of the Core 5 tile set can be similarly adjusted to become Short tiles, see Figure 12.

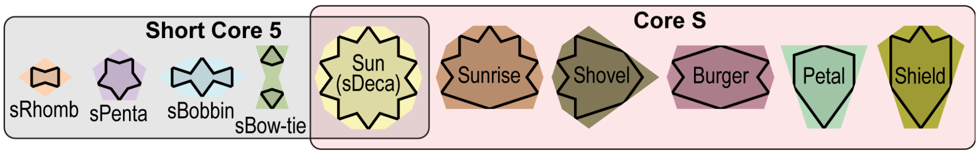

Figure 12: *The Short Core 5, and the Core S, tile set within the Short category.*

Figure 13 shows an original floral pattern, tiled by the author. It includes the Short Bow-tie (sBow-tie) tile. Even though such combinations generate well-known shapes, the Short Core 5-induced shapes appear in different scales, and the result is not common in Islamic geometric pattern.

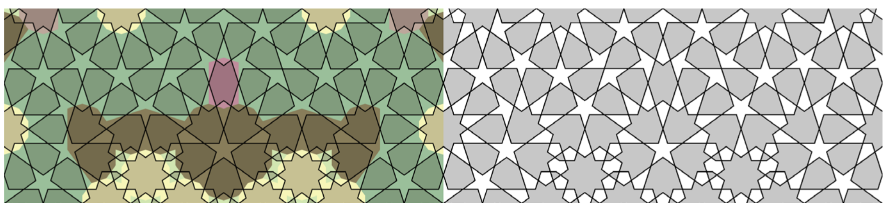

Figure 13: *The sBow-tie tile gives a unique look (as pentagonal shapes are unusual in floral patterns).*

Pattern Families in a Non-Equilateral World

Bonner's definition of *pattern families* (acute, median, and obtuse) [2] is based on a general perception of the angles in the whole pattern (36º, 72º, and 108º or 144º), not specifically on the edge rule angle of the tiles. The pattern in Figure 5 is considered acute (36º), even though the pattern also consists of 144º angles. For the core 5 tiles, the family angle does correspond to the tile's edge rule angle, as do the edge rule angle of the unit sides for adapters from the Φ-category. For the Short tiles, the mapping of the tile edge rule angles (144º and 72º) to the pattern's family definition of acute angle (36º) is not applicable.

Summary and Conclusions

Every historical pattern that I have tiled with Short tiles can also be constructed with the evolved Girih tile set (G++). It seems this would make the Short tile set a bit superfluous, but it does have its merits. When including Short Core 5 tiles the pattern cannot be invertiled with only the evolved tile set, at least not until further categories or additional Short sub-categories have been introduced.

I argue that Short tiles provide a *better visual overview* during the design of a new pattern, verification of historical pattern, and analysis of pattern structure. The interaction between these new categories and the Short tiles will further enable the replication of historical patterns. The reason is the focus on the core visual information of the pattern, and how it interacts with us humans. (This requires that the tiles have a unique background or motif coloring. Without it, both ways are equally challenging.) It follows the basic rules of Gestalt psychology [13]. The principle (or law) of Similarity states that elements that are similar to each other tend to be perceived as a unified group.

Applied to patterns that can be tiled with Short tiles (Tond patterns), the petal shape is often more frequent than the other shapes combined. Using tiles that carry this shape would be a better solution for the ability to read and decode the visual stimuli of the pattern, which the Core S tile set (negative Tond shapes) does. During the design phase, or when a pattern undergo tile analysis, it is easier to spot the nonpetal shapes as they stand out from the background of petal shapes. G++ tiles (positive Tond shapes) carry the three more insignificant shapes as main motifs, which makes the petal shapes form only when tiled, and therefore not directly visible during tiling.

In this paper the Core S tiles interact only with themselves and tiles from the Short Core 5 tile set. For interaction with the evolved Girih tile set or the all-Φ-sided Golden tiles, another category will have to be introduced – the non-midpoint **Flat** (F) category. This addition could open up ability to tile a wider range of historical patterns. In the next level of tile category exploration, the focus will lie on non-midpoint crossings.

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