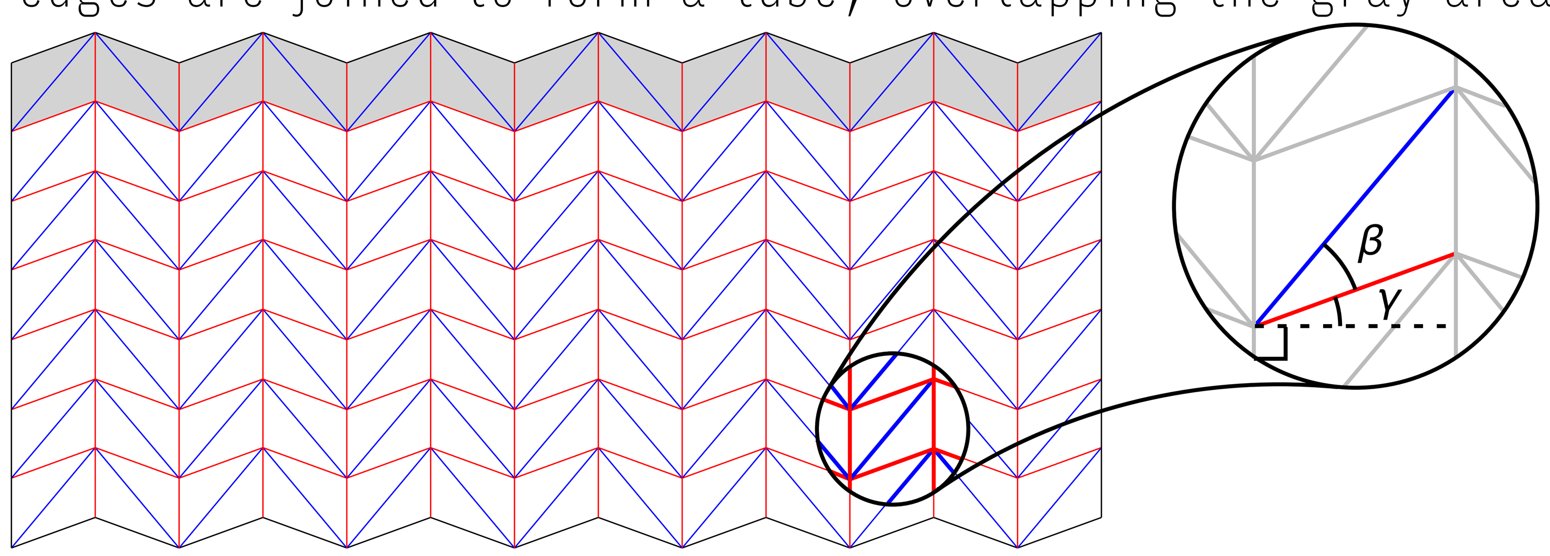
Force Tuning Bistable Origami Triangulated Cylinders

Uyen Nguyen, Marc Miskin, Huiyao Chen, Itai Cohen | Department of Physics | 2015

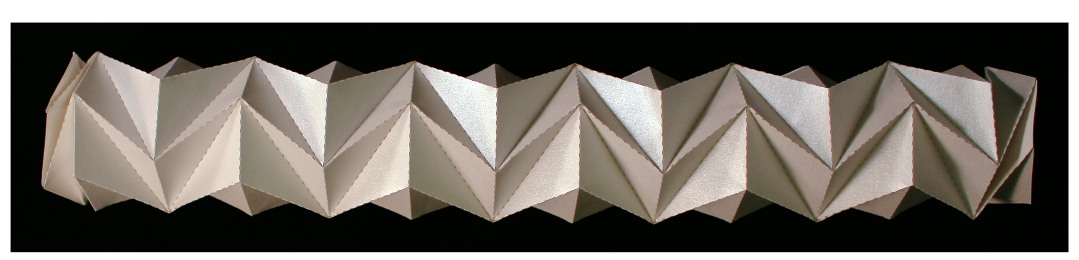
Presented is a preliminary study to characterize the mechanical behavior of origami triangulated cylinders under axial loading. These structures are compressed while force measurements are taken using a strain gauge.

Folding Pattern

To make an N-gon triangulated cylinder the crease pattern (below) should be N segments tall with angles $\beta = \pi/N$ and γ ranging from 0 to $\pi/2-\beta$. Red and blue lines represent mountain and valley folds, respectively. The top and bottom edges are joined to form a tube, overlapping the gray area.

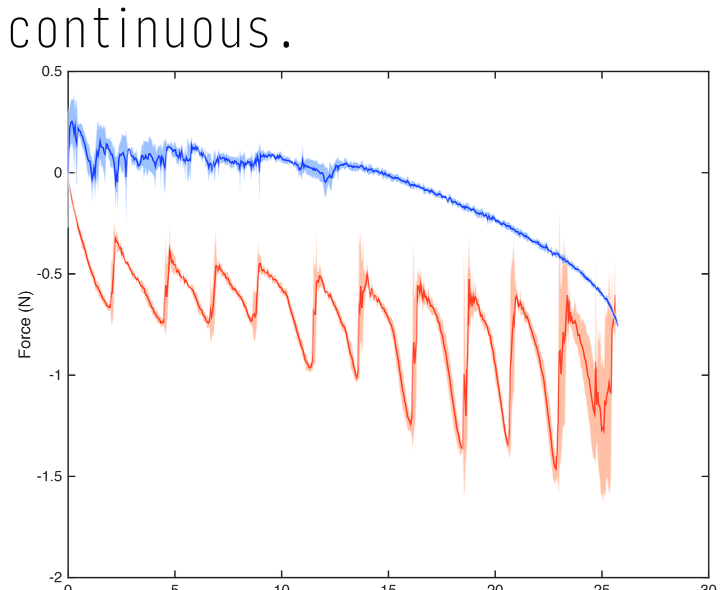


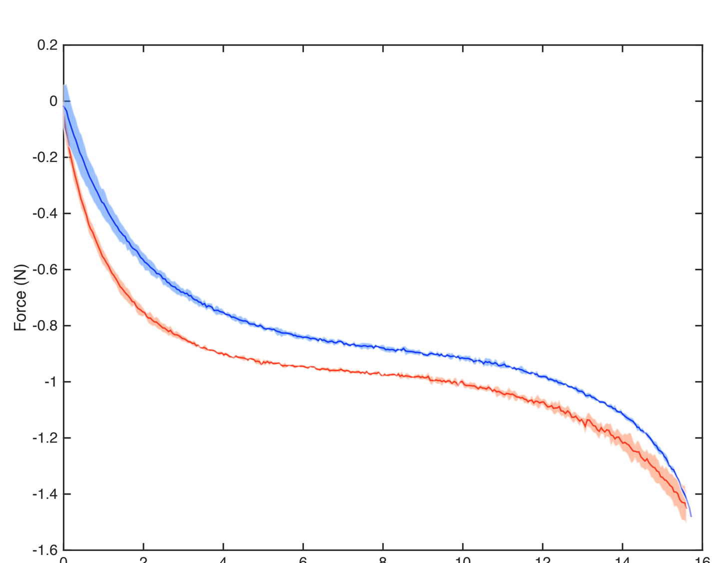
Once folded, the structure can exist extended (left) or collapsed (right), as pictured below.



Force Measurements

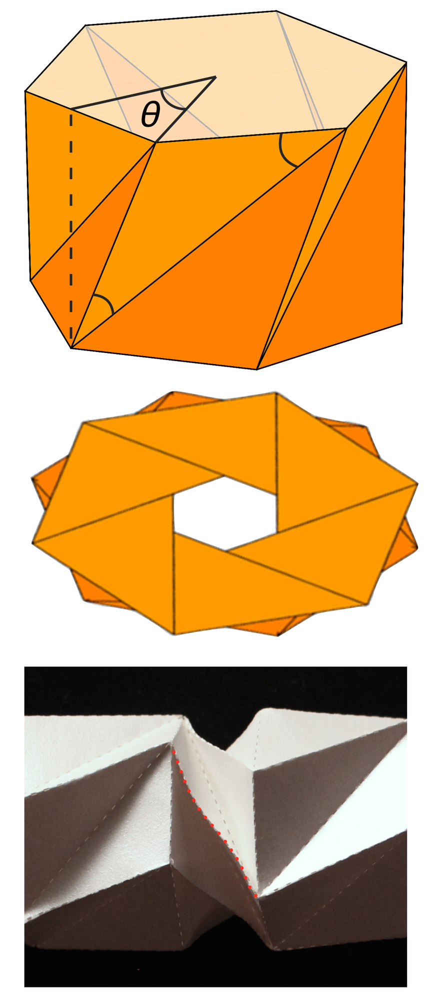
To explore their mechanical behavior, the triangulated cylinders were axially compressed (red) and extended (blue). Data was collected for structures with different N and γ. Below is the loading curve for two pentagonal columns, one with $\gamma = \pi/12$ (left) which exhibits snapping behavior, and one with $\gamma = 5\pi/36$ (right) whose loading curve is continuous.

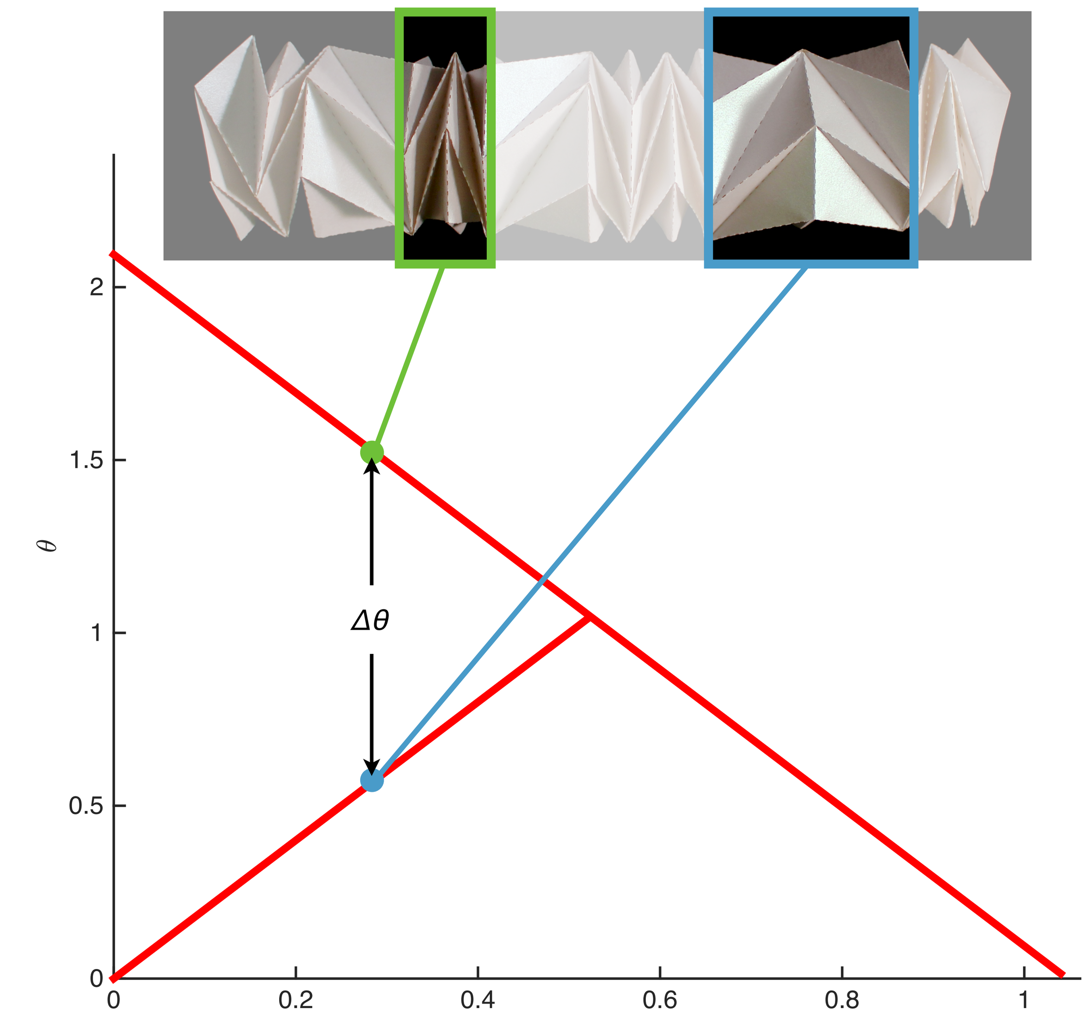




Snapping Behavior Arises from Geometry

The stable configurations of the triangulated cylinder occur where there is no bending of the facets, thus the segment lengths and folding angles of the folded model should remain consistent with that of the flat crease pattern. The parameter θ is the rotation of one polygon relative to its

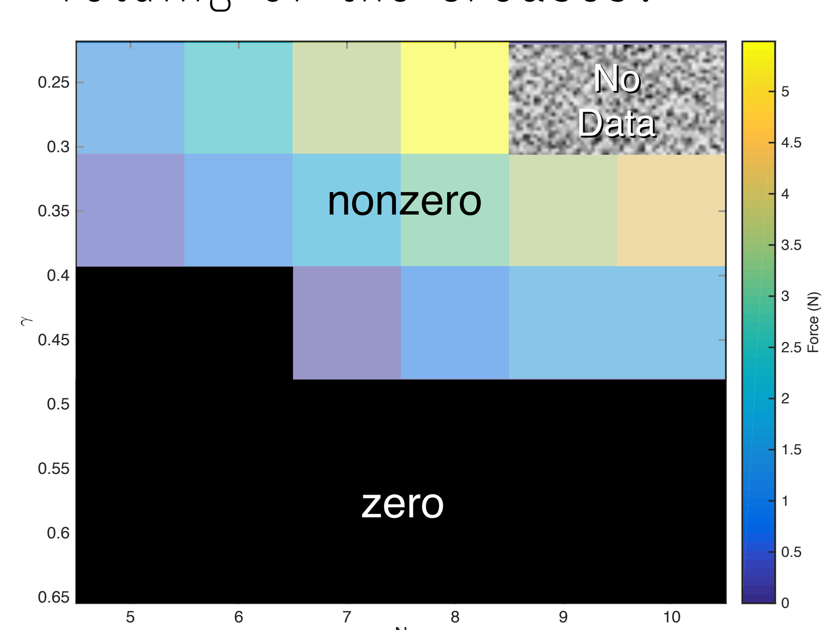


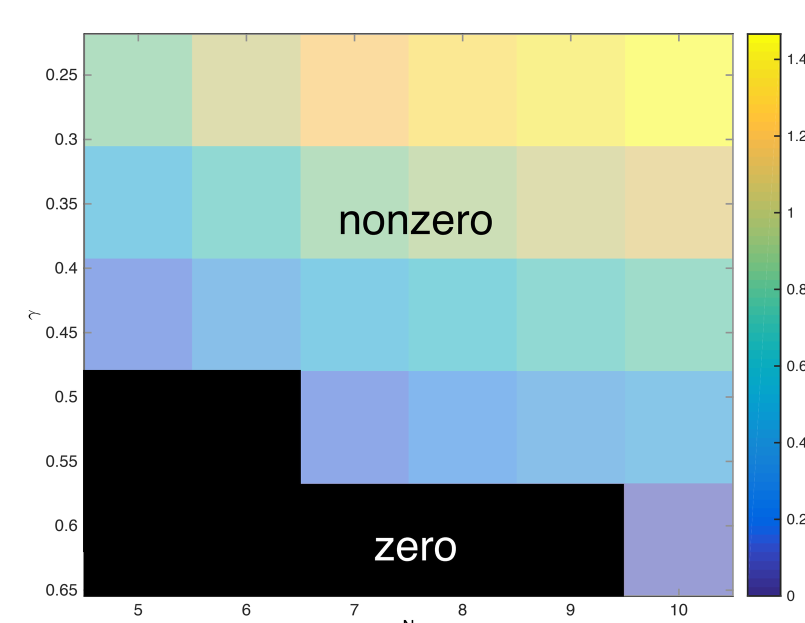


adjacent polygon (above, upper left). The graph (above, right) shows all the calculated values of θ for a hexagonal column which satisfy conditions for stability. The bifurcation point for N = 6 occurs at $\gamma = \pi/6$. All hexagonal columns with γ values to the right will not exhibit snapping behavior, while in theory, those to the left would, undergoing facet bending (above, bottom left) when transitioning between stable states.

Theory vs Experiment

In reality, the transition from bistable to monostable behavior occurs at a lower γ value than the calculated bifurcation point. The discrepancy is due to the use of a purely geometric model, which does not account for spring forces within the paper that have a tendency to resist folding of the creases.

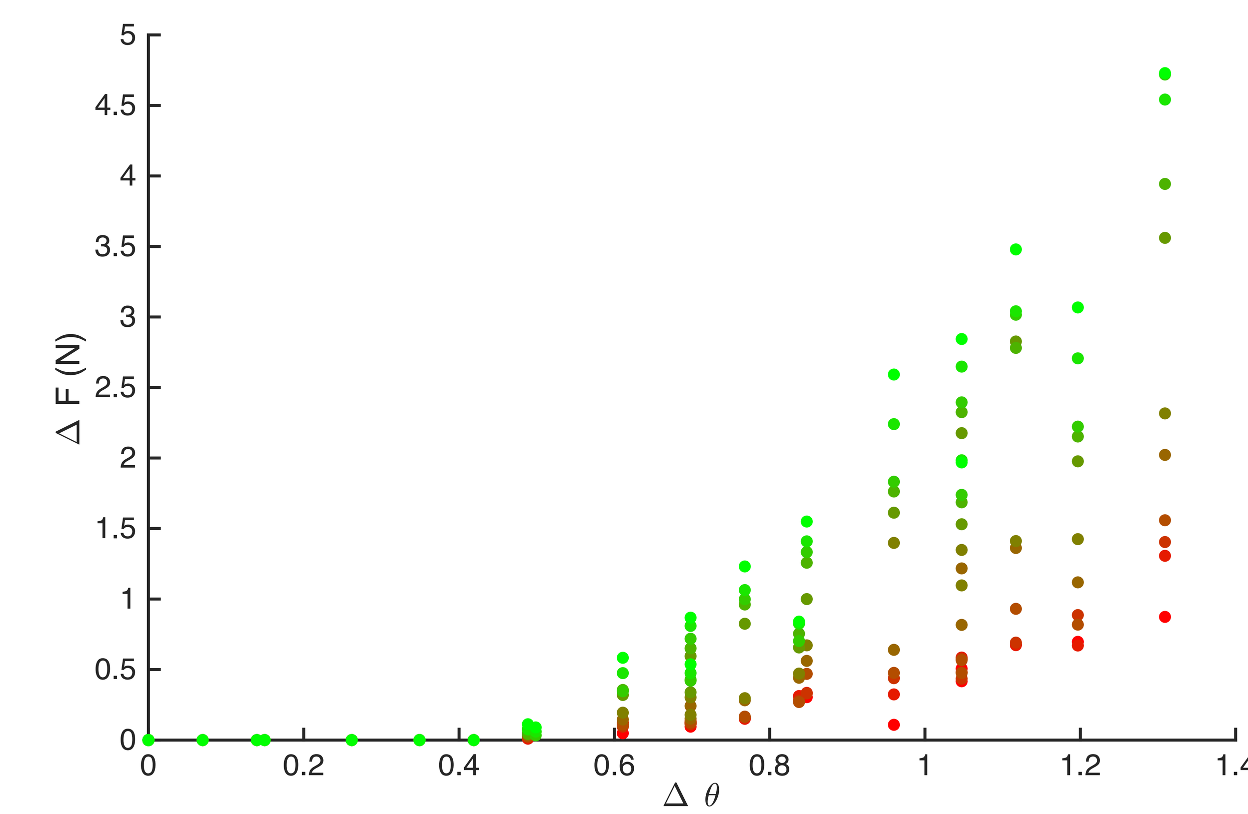




The above left plot shows the force magnitude of the last snap for columns with various values of N and γ. For N = 9 and 10 at $\gamma = \pi/12$, no data exists because the snapping force exceeded the force required to tear the paper. The right plot shows all the values of $\Delta\theta$ for columns with the same parameters. The zero region of the force plot is greater than the zero region of the $\Delta\theta$ plot due to the aforementioned unaccounted spring forces.

Designing the Response

The graph below shows snapping force as a function of $\Delta\theta$ with red corresponding to the first snap and green corresponding to the last snap during the compression of columns with constant γ. Using this data, a crease pattern with varying γ can be reverse engineered to produce a desired force response. However, given that a cylinder of uniform γ throughout each segment results in an increasing force with each snap, it can only accurately produce curves that increase in slope. That is, it cannot produce a concave down response.



Using data from the graph above, the crease pattern below was designed to produce a force curve whose snap magnitudes exhibit exponential growth. The subsequent graph shows the input force used to generate the CP in blue, and the response of that CP under axial compression in green.