Salsa Rueda Dancing and Mathematics

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Abstract

In this workshop, participants will experience salsa rueda dancing and ask questions about possible mathematical connections. In salsa rueda, couples dance the same moves while standing in a circle, often involving partner changes. We can represent some of the basic dance moves using star polygons which allows us to prove conjectures about the dancers' movements and partner meetings. We can also use results from graph theory to start answering difficult questions about dance movements new to salsa rueda.

Introduction and Goals

Many successful connections have been made between mathematics and dance, see for example [3], [9], [1], and [4] to name but a few, but not much research has been done in the area of salsa dancing specifically. In [5], Volker Ecke and I describe in detail how turns and arm movements in partner salsa dancing can be classified. In this workshop paper I study the movement patterns on several salsa rueda dancers simultaneously which leads to very different mathematical ideas. The activities were developed as part of the project "Discovering the Art of Mathematics" [2] and have been used for several semesters in general education classes at Westfield State University in Massachusetts, see Figure 1a. Westfield State is a small public university that serves many first generation students as well as students from lower socioeconomic backgrounds.

There are many goals for the activity described below. First of all, the dance activity allows students to ask their own mathematical questions about a topic that seems non-mathematical on the surface. This leads to students being motivated to answer their own questions as well as changing their beliefs about their identity as mathematicians. Learning the dance moves together also creates a sense of community and sets the expectation that students are welcome to make mistakes in this class without needing to feel embarrassed. In fact, we celebrate mistakes as an important aspect of learning. In addition, students will explore concepts like least common multiples, greatest common factors, star polygon patterns, graphs, and in general making conjectures and proving them.

(a) *Students Dancing Salsa Rueda* **(b)** *Representation* **(c)** *Student Conjecture*

Figure 1: *Students Engaging with Salsa Rueda*

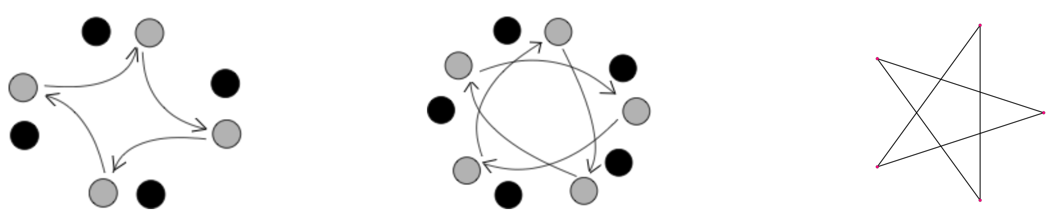
The Basics of Salsa Rueda

Salsa Rueda (also known as rueda de casino) was developed in Havana, Cuba in the 1950s. In Salsa Rueda couples are standing in a circle with the leader on the right side of the follower. In my class, the role of leader and follower doesn't depend on gender, students can choose or we assign roles randomly. One of the leaders is the "caller" telling the other leaders during every move which move is coming next. While it may be difficult for a group of students to quickly learn the correct steps for the dance, it is fairly easy to learn where their bodies are supposed to be positioned. So while I do show the correct steps, we focus on practicing the general movements. See [6], [7], and [8] for demonstrations of the dance.

In the beginning of the dance, the leader stands one the right side of the follower, holding hands, while both face the center of the circle. The basic step *Guapea* is performed in this position, see [8]. In *Dame*, pronounced dah-me which means "give me" in Spanish, the followers move to the next follower position to their left (clockwise) while the leaders stay in place, see Figure 2a. In *Dame Dos* the leaders first move to the next leader position to their right (counter clockwise) and then the followers move to the next follower position to their left (clockwise), which means that followers are now dancing with the second leader to their left (clockwise). To simplify the representation, we can think of dame dos as a movement that is done only by the followers while the leaders stay in place, see Figure 2b. The moves can be extended to *dame tres* and higher numbers. There is a limit to what is physically possible to dance but the mathematical ideas can of course be extended. Having practiced these moves, the students can think about questions they want to ask about the dance. For this paper we are are focussing on the following list of questions from the follower's perspective:

1. Do you meet up with your leader again? Why or why not?
2. Do you dance with all other leaders? Does this depend on the number of dancers?
3. How many times do you move around the circle before you return to your original leader? Does this depend on the number of dancers?

The first mathematical challenge is to represent the dance so that we can start making mathematical conjectures and arguments. Most students start out by representing both leaders and followers, which makes the move *dame dos* very difficult to show, see Figure 1b. Once students realize that you can think of the movements as just follower position changes, see Figures 2a and 2b, they can draw the dance as a *star polygon*, see Figure 2c.



(a) *(Simplified) Follower's Movements in Dame* **(b)** *(Simplified) Follower's Movements in Dame Dos* **(c)** *Star Polygon – 5 Couples Dancing Dame Dos*

Figure 2: *Dame and Dame Dos*

Conjectures and Proofs

All of the conjectures in this section are already known in the context of star polygons, but are new in the context of this dance. Notice that the proofs are accessible to students in our general education courses, in fact students generate the (kinesthetic) conjectures and proofs themselves. Some sample student work can be found in [6].

Conjecture 1. *Dancing only dame k with n couples, every follower will eventually dance with their original leader again.*

Proof. If we have n dancers and are dancing dame k, the couples will definitely meet again after a multiple of k meets up with a multiple of n. So after $\mathrm{lcm}(n, k)$ dame k's the original couples will dance together again. □

Notice that while this proof seems simple, it takes students some time to make this argument, for example using a number line instead of the circle representation.

Conjecture 2. *Dancing only dame k with n couples, the followers will dance with* all *leaders if and only if the greatest common factor of n and k is equal to 1. To find this conjecture it is helpful to record data in a table and look for number patterns. See Figure 1c.*

Proof. Let $\mathrm{gcf}(n, k) = a > 1$, then a divides both k and n. This means that you can divide the number line into hops of a, see Figure 3. So the follower starting at zero will definitely miss places consistently before coming home to the original leader again. You can also use the equation $\mathrm{lcm}(n, k)\mathrm{gcf}(n, k) = nk$ but most general education students will not be familiar with this fact. □

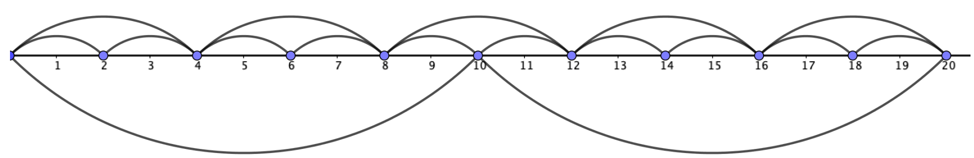


Figure 3: *10 pairs dancing dame 4*

Conjecture 3. *Dancing only dame k with n couples, every follower dances* $\mathrm{lcm}(n, k)/n$ *many times around the circle of leaders.*

Proof. Notice that the circle of leaders turns itself in space since the leaders also move during dame k. We will ignore this and just focus on the position of the followers with respect to the leaders. Looking back at Figure 3 we can see that moving along the circle of leaders corresponds to one hop of n on the number line. Follower and leader meet again at $\mathrm{lcm}(n, k)$ so we need to know how many groups of n are in $\mathrm{lcm}(n, k)$. This gives you the answer. Notice that, using $\mathrm{lcm}(n, k)\mathrm{gcf}(n, k) = nk$, you can also express the answer as $k/\mathrm{gcf}(n, k)$. □

Using the above ideas, we can invent new salsa rueda moves, for example a $(1, 2)$-dame in which the follower first skips one, then skips 2 leaders in the circle, and then repeats the pattern. In Figure 4 you can see the beginning of the path – until the followers dance with their original leader again. Similar to the conjecture above, we can see that the followers will eventually dance with their original leader again. It is harder to determine if the followers will dance with *all* leaders or how many times the followers dance around the circle of leaders. Notice that with dame 3 it takes 3 orbits before the followers are back with their original partner. With dame (1,2) however it takes only one orbit before the follower is back for the first time.

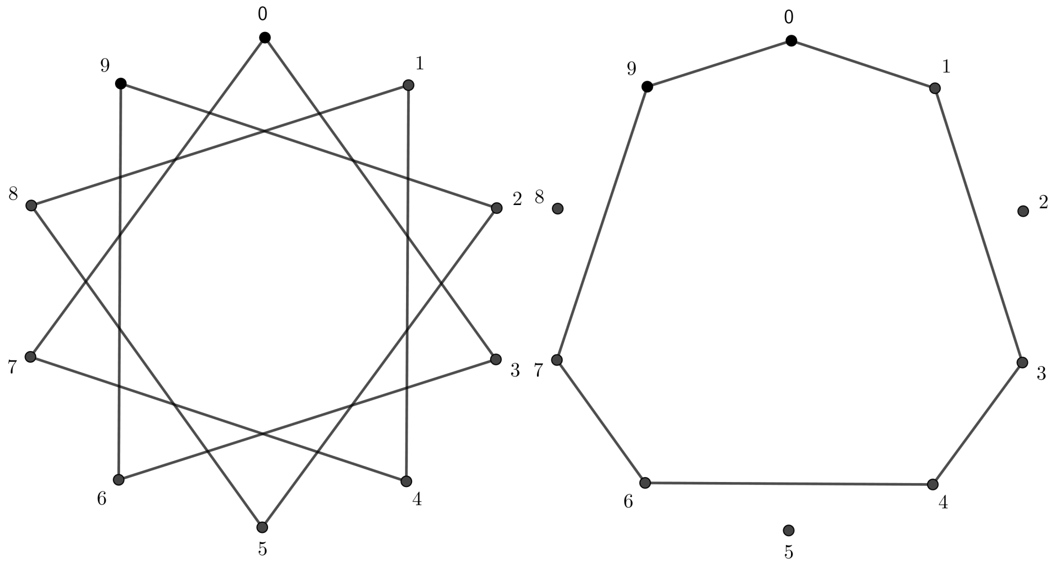


Figure 4: *10 pairs dancing dame 3 versus dame (1,2)*

We can connect the $(1,2)$-dame questions to other concepts in graph theory: In a forthcoming paper, Gee-Choon Lau, Sin-Min Lee, Karl Schaffer, and Siu-Ming Tong prove that not all cycles are (a,b)-step Hamiltonian paths. In particular, for the case $(1,2)$ their results show that the followers would not dance with all the leaders exactly once during repeated $(1,2)$-dames. We could also generalize this question to triplets of numbers, for example $(3,1,2)$-dames. It is unknown in which situations the followers would be dancing with all the leaders (or in graph theory language: which cycles are (a,b,c)-step Hamiltonian.)

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