Semi-generalizing Miura-Ori with Divots into Rotationally Symmetric Lampshades with Smooth-Curving Profiles

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Abstract

This article discusses using semi-generalized Miura-ori with divots added in order to create rotationally symmetric lampshades that have smooth-curving profiles.

Introduction

The Miura-ori tessellation is a geometric origami crease pattern made of repeated parallelograms arranged in zigzag formation. When folded, Miura-ori forms a corrugated surface that can be deployed and contracted rigidly in a plane with a single degree of freedom with no deformation of its parallelogram facets (Figure 1). Credited to Japanese astrophysicist Koryo Miura [2], Miura-ori has become well-known for its application in deployable structures, such as a solar array in a 1995 mission for JAXA, the Japanese space agency. Another way to work with a folded Miura-ori surface is to bend and stretch the surface in the direction transverse to the direction of corrugation to create rotationally symmetric forms, allowing the parallelogram facets to flex and distort. The distortion or the non-rigidness of the folding often happens in paper origami. In fact, this way of folding Miura-ori in paper was similar to a magic routine called "Troublewit" that was popular during the Victorian period in England (Figure 2). During a Troublewit routine, a magician would transform a paper folded rotationally symmetric object, such as a hat, into something else, such as a vase. The folding patterns used in the Troublewit designs are modified Miura-ori with repeated quadrilateral facets, or semi-generalized Miura-ori (SGMO), a term used by Robert Lang to refer to an approach for modifying a Miura-ori [1].

***Figure 1**. A Miura-ori and its folded forms.*

***Figure 2.** Examples of Troublewit paper origami changing forms from left to right. Photo credit: https://www.origami-resource-center.com*

In Lang's book [1], Lang detailed a construction process for using Miura-ori to generate any target profile that can be used to create a three-dimensional surface. Lang's construction method is fairly straightforward and requires little computational challenge. A crease pattern in an SGMO object is created by changing the angles and distances to create a non-periodic pattern in an origami strip (similar to the yellow part in Figure 3) to match a target profile design and then repeating the patterns periodically in another direction. Almost any targeted profile curve can be approximated and folded into a single strip of SGMO. The SGMO strip will then be repeated to create a origami tessellation that can be stretched and rotated to create three-dimensional surfaces. Many crease patterns of the popular Le Klint origami lampshades, for example, the original Le Klint by Tage Klint in 1943, can be designed using this method (Figure 3). However, Lang's simple SGMO requires further mathematical analysis as there can be limitations in SGMO, especially when used to create rotationally symmetric surfaces with smooth-curving profiles. This article discusses how the author, an artist, uses mathematics to understand SGMO, with divots added, in order to create rotationally symmetric paper lampshades with smooth-curving profiles.

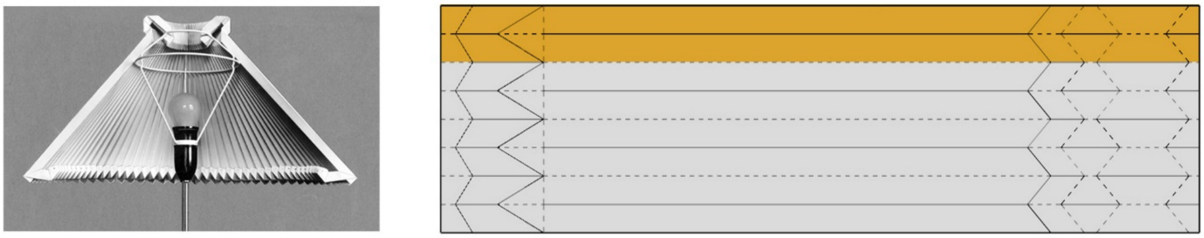


Figure 3. *The original Le Klint and an SGMO pattern that can be folded flat to match the profile as shown in the picture on the left. The SGMO pattern can then be repeated and folded into the rotationally symmetric Le Klint. Photo credit for the photo on left: Le Klint.*

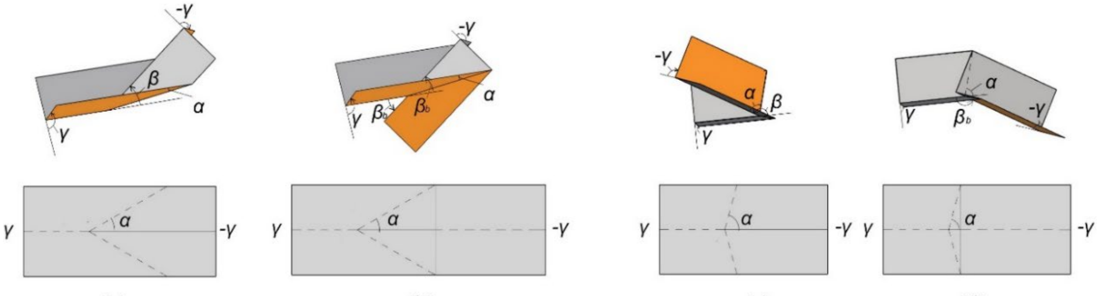
Comparing Semi-generalized Miura-Ori and Semi-generalized Miura-Ori with Divots

To understand the limitation in SGMO in creating smooth-curving profiles, let's take a closer look at its vertices. In an SGMO, there is only one type of vertex: a degree-4 vertex (Figure 4a and 4c), or a Miura-ori vertex that has bi-lateral symmetry. (The bi-lateral symmetry is a necessary condition to allow the tessellation to be flat-folded.) Three parameters in Figures 4a or 4c showing a Miura-ori vertex can affect the result of SGMO: the sector angle α, the folding angle along the corrugation crease γ, and the bending angle along the corrugation crease β. The relationship among them (proved by Lang in [1]) can be expressed as:

$$\alpha = \mathrm{ArcTan}\left(\frac{\tan\left(\frac{\beta}{2}\right)}{\sin\left(\frac{\gamma}{2}\right)}\right) . \qquad (1)$$

If γ is constant, decreasing the bending angle β will decrease the sector angle α. If a target profile requires a more shallow bending profile (smaller β) as shown in Figure 4a, it will result in smaller sector angle α. If a target profile requires a sharper bending profile (larger β) as shown in Figure 4c, it will result in a larger sector angle α.

A Miura-ori with divots added is defined here as adding a degree-4 vertex with sector angle $\alpha=\pi/2$ to each of the Miura-ori vertices. For every Miura-ori vertex (Figure 4a and Figure 4c), there exists a corresponding Miura-ori vertex with a divot added (Figure 4b and Figure 4d) and a corresponding bending angle β_b in which $\beta_b = \beta$. As shown in Figure 4a and 4b, the same small sector angle α will result in a small bending angle β and a shallow bending profile in a Miura-ori vertex, a small bending angle β_b but with a sharp bending profile in a Miura vertex with a divot added. As shown in Figure 4c and 4d, the same large sector angle α will result in large bending angle β with sharp bending profile in a Miura-ori vertex, but a large bending angle β_b with a shallow bending profile in a Miura-ori vertex with a divot added.



(a) (b) (c) (d)

Figure 4. *Comparison of bending pofiles in two Miura-ori vertices and two corresponding Miura-ori vertices with divots: (a) a Miura-ori vertex with a small sector angle α and a shallow bending profile, (b)*

the corresponding Miura-ori vertex with a divot added with a small sector angle α and a sharp bending profile, (c) a Miura-ori vertex with a large sector angle α and a sharp bending profile, (d) the corresponding Miura-ori vertex with a divot added with a large sector angle α and a shallow bending profile.

Folding Semi-generalized Miura-Ori with Divots

Figure 5 shows a target profile that approximates a smooth curve. To fold the smooth-curving profile using SGMO with a very small bending angle β, the sector angle α needs to be very small, thus requiring pleat width to be very small as well (as a wider pleating width with the same small sector angle α will result in bigger distances between the Miura-ori vertices, causing the profile to be less smooth [1]). The resulting crease pattern is nearly impossible to fold (Figure 5a). To fold the same curve using Miura-ori vertices with divots added, the pleating width w can be wide enough so that the crease pattern is easy to fold (Figure 5b). In a Miura-ori vertex, $\alpha = \frac{\beta}{2}$ when $\gamma = 180°$, and in a Miura-ori vertex with a divot added, $\alpha = (\pi - \beta_b)/2$ when $\gamma = 180°$.

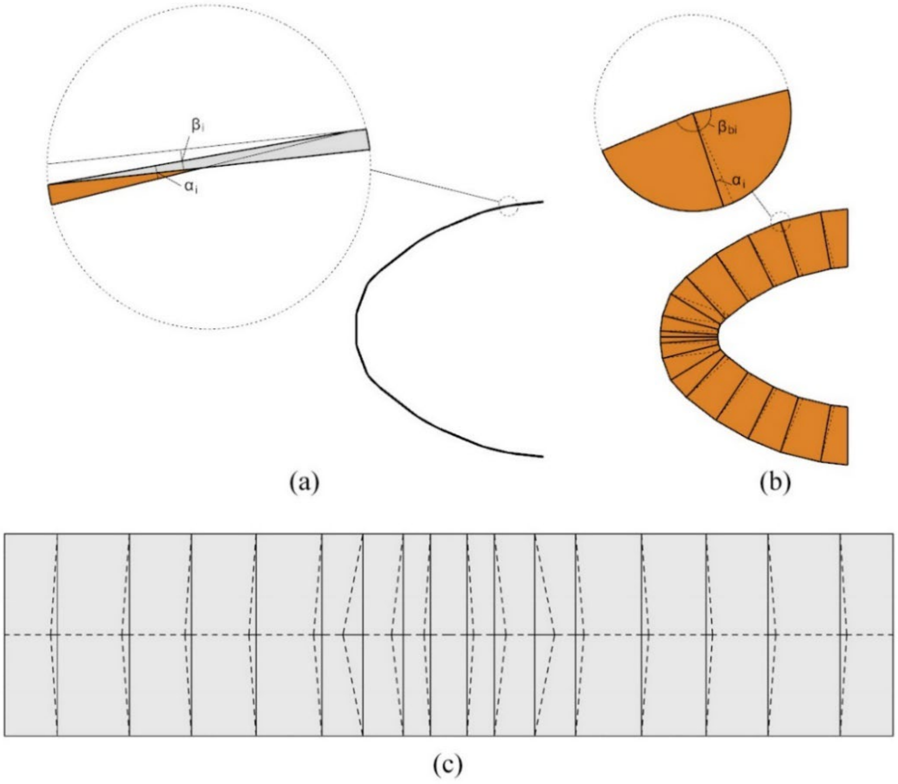


Figure 5. *Comparison between two profiles folded using SGMO and SGMO with divots added: (a) an SGMO profile, (b) an SGMO with divots added profile, (c) resulting strip from (b) using SGMO with divots added.*

The SGMO with divots added generated from the profile in Figure 5b can then be stretched to form a rotational three-dimensional surface that can be used as a lampshade at the expense of distorting and bending the paper. With rotationally stretched pleats, the bending angle β varies based on the changes in folding angle λ, the radial distance from the axis of rotation, the number of repetitions used in the rotational form, and the distortion and bending of the paper [1]. However, the smooth and graceful profile that is calculated when $\gamma = 180°$ can be used as an approximation. Figure 6 shows a lampshade design that is based on an SGMO with divots added. It has graceful curves and characteristics of semi-rotational symmetry based on an ellipse. The design can be folded from one single piece of paper (if the paper is large enough and if the digital cutting tool is large enough to cut the paper). The changing sector angle α and the divots together create harmonious gradational effects of light and shadows. Figure 7 shows how the lampshade in Figure 6 can be compacted and folded flat.

Figure 6. *An example of smooth-curving rotational symmetric lampshade using SGMO with divots.*

Figure 7. *A partially folded profile from Figure 6.*

If the profile curve that is folded using Miura-ori with divots is shaped similarly to a sine wave and then arrayed and stretched into a rotational surface, it will create a double-curved surface that has nonzero Gaussian curvature. Since pleating width is independent of sector angle α, bending angle β, and folding angle γ, the pleating width can then be varied to create more expressive designs. Figure 8 and Figure 9 show such a rotationally symmetric design that produces an interesting and aesthetically pleasing three-dimensional surface.

Figure 8. *An example of a smooth-curving rotational symmetric lampshade design using SGMO with divots added and variant pleating width w.*

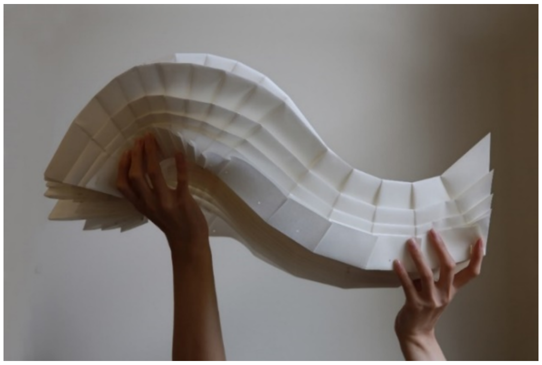


Figure 9. *A partially folded profile from Figure 8.*

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