Reciprocal Polyhedral Forms using Strip Pairs

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Abstract

A framework is given for transforming one class of reciprocal structures into a larger structure using a splitting process. This process is illustrated for Platonic solids and several Archimedean solids.

Introduction

Linear components such as strips, sticks, and beams such as the one as shown in Figure 1 have been used to create reciprocal structures, where each component is connected with four others to build structures in which the components mutually support one another. The types of connections range from friction, to notches, to pins (as in this work). Leonardo da Vinci was one of the first to document this process applied to planar tessellations in his *Codex Atlanticus* notebook [1]. Illustrations of this pattern are shown in Figure 2. Roelofs expanded greatly on the work of da Vinci and demonstrated how polyhedral forms can be built using a reciprocal manner that uses sticks or long strips using a strict over-under-under-over pattern in a free-form manner [3].

Figure 1: *An example building unit. The equally spaced circles represent connection points, with the black and white circles differentiating between over and under connections.*

One can start with a regular polyhedron [2] or tessellation and construct a reciprocal structure. The vertices are simultaneously opened and the edges pinwheel about the original vertices and faces, resulting in a reciprocal structure as shown in the top right image of Figure 2. This will be referred to as the conventional reciprocal structure. The number of strips is equal to the number of edges in the base polyhedron or finite region of a tessellation. In this process, the vertices become small open spaces. The larger open spaces correspond to the polygons in the underlying tessellation or polyhedron. The interior connections for a given strip are from strips attaching from opposite sides.

Strip pairs

Each one of the strips in the conventional reciprocal structure can be split into a pair of parallel strips. Conceptually these can be thought of as the edges of the original strips. Thus, the interior connections to the new strips are from strips attaching from the same side. This process is illustrated in Figure 2. There are now three types of open spaces: "rhombic" spaces where the edges have been split, spaces from underlying vertices, and spaces from underlying polygons. The spaces coming from the vertices and polygons have equal edge lengths, unlike in the conventional reciprocal structure. Using this framework, one can start with a polyhedron with equal edge lengths and systematically construct a large class of reciprocal structures.

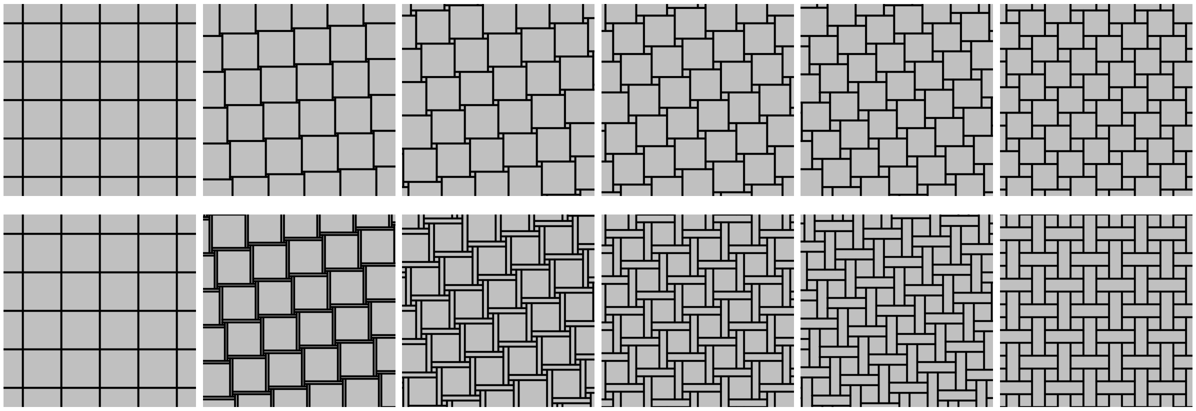
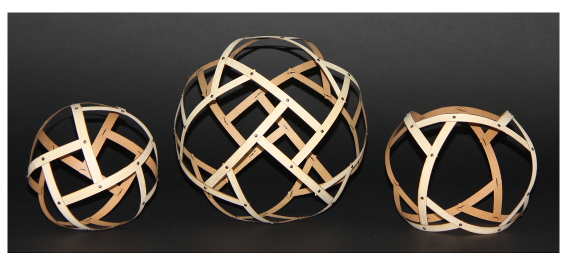


Figure 2: *Reciprocal structures based on the tessellation by squares. The vertex of the tessellation is opened (top, from left to right) and the edges simultaneously pinwheel about the original vertices and faces, resulting in a conventional reciprocal structure (top right). Each edge can also be split into pairs (bottom) to form a paired strip reciprocal structure. The rightmost images show the strips with two equally spaced connection points along the interior of the strip.*

Constructions with flexible strips

Constructions were made using laser-cut flexible strips of paper-backed bamboo veneer with dimensions 20.3 cm by 1.2 cm and four equally spaced holes as shown in Figure 1. This material is lightweight, flexible, strong, and resistant to humidity changes over time. It has a long parallel tight grain structure that makes it very suitable for this application. Split pin brad fasteners were used to connect the strips. Conventional reciprocal structures and paired strip structures were made for the Platonic solids and several Archimedean solids. The conventional and paired strip reciprocal structures based on the tetrahedron are shown in Figure 3a; note the paired structure has pyritohedral symmetry. For the cube and octahedron, which are duals, the conventional reciprocal structures have differing sized vertex/face openings whereas the paired structure is identical for both as shown in Figure 3b. Similar structures are shown for the dodecahedron and icosahedron in Figure 4, the truncated tetrahedron in in Figure 5, the cuboctahedron in in Figure 6, and the the icosidodecahedron in in Figure 7.

(a) *The conventional reciprocal structure with 6 strips (left) and the paired strip reciprocal structure with 12 strips (right).*



(b) *The conventional reciprocal structures with 12 strips for the cube (right) and the octahedron (left), and the paired strip reciprocal structure with 24 strips (center).*

Figure 3: *Reciprocal structures based on the tetrahedron, cube, octahedron. Note that the interior edge connections are from opposite directions in the conventional reciprocal structures and the same directions in the paired strip reciprocal structure.*

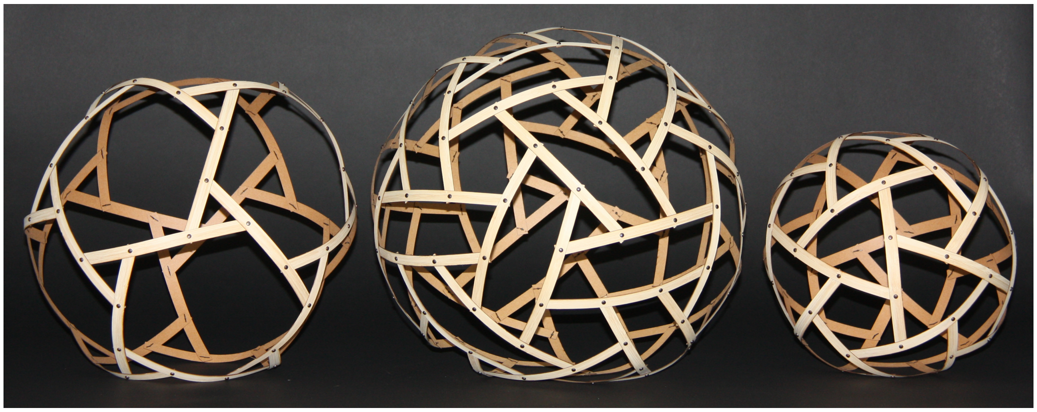


Figure 4: *Reciprocal structures based on the dodecahedron and icosahedron. The conventional reciprocal structures with 30 strips for the dodecahedron (left) and the icosahedron (right), and the paired strip reciprocal structure with 60 strips (center).*

Figure 5: *Reciprocal structures based on the truncated tetrahedron. The conventional reciprocal structure with 18 strips (left) and the paired strip reciprocal structure with 36 strips (right).*

Figure 6: *Reciprocal structures based on the cuboctahedron. The conventional reciprocal structure with 24 strips (left) and the paired strip reciprocal structure with 48 strips (right).*

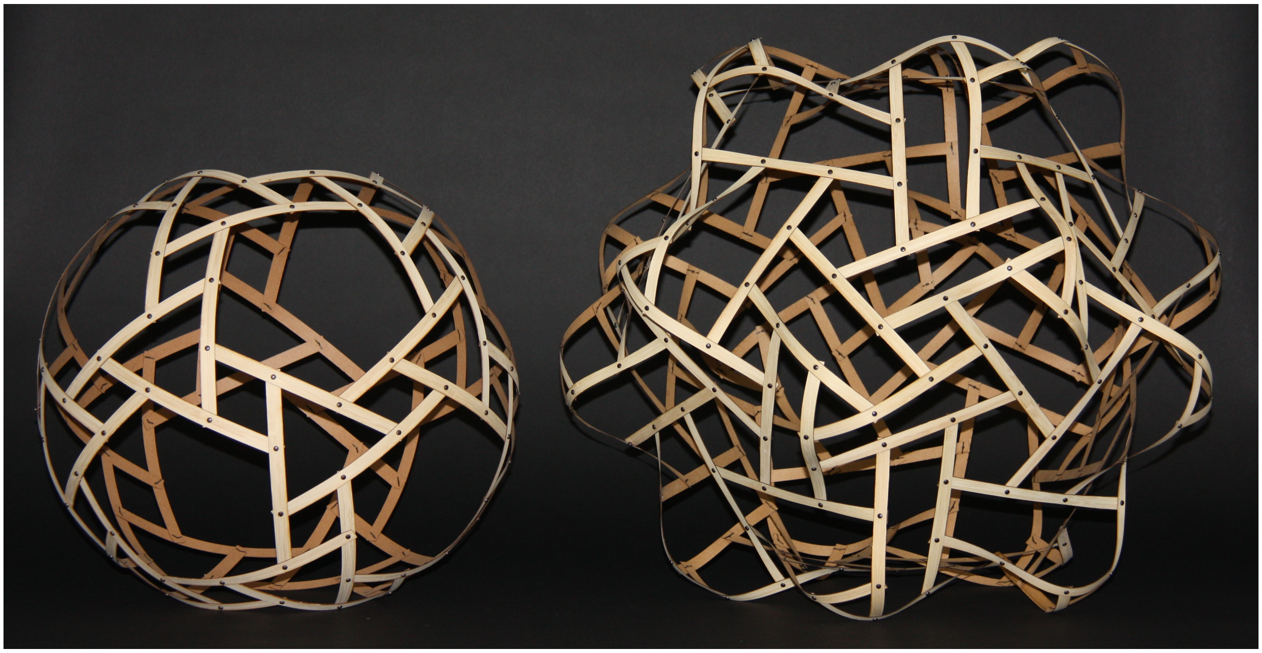


Figure 7: *Reciprocal structures based on the icosidodecahedron. The conventional reciprocal structure with 60 strips (left) and the paired strip reciprocal structure with 120 strips (right).*

Discussion

A framework is given for transforming one class of reciprocal structures into a larger structure using a splitting process. This can help conceptualize a large class of reciprocal structures. Flexible building units are required in many cases. However, it seems plausible that structures with less curvature differences could be made using strips of differing lengths or by varying the hole spacings.

An interesting feature of a paired reciprocal structure is that each strip in the pair can be associated with a half edge of the original polyhedron. Thus one can create a more symmetric structure based on the Platonic solids using an asymmetric building unit than one can with the conventional reciprocal structure. For example, each strip in central object of Figure 4 could be replaced with a directed arrow leading away from the pentagonal regions and the symmetry would be unchanged. In the other objects in Figure 4, each strip is adjacent to two pentagonal regions, thus a directed edge would force a change in the symmetry.

References

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[3] R. Roelofs. "Leonardo da Vinci's Bar Grids." *ISAMA-BRIDGES Conference Proceedings*. e. a. Javier Barrallo, Ed. Granada, Spain: University of Granada, 2003. pp. 229–234.