# Weaving Paper into Star Patterns

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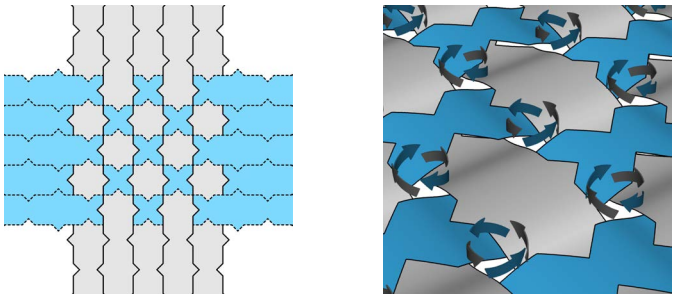
## Abstract

Several well-known tessellations, such as the triangular tiling and square tiling, the Archimedean tiling 3-6-3-6, or several ornaments appearing in girih tilings, can be two-colored. In this paper, we present patterns that can be woven from two sheets of paper and investigate what kind of tilings or ornaments are suitable.

## Weaving Two Sheets of Paper

The weavings considered in this paper are made of two colors or two sheets of paper, respectively, based on a two-colored tiling of the plane. According to the tiling, both sheets of paper will alternate between the lower level (back side) and the upper level (front side). At each edge of the tiling, one sheet of paper is cut to let the other color pass from the upper to the lower level.



**(a)** **(b)**

**Figure 1:** *Weaving without straight cuts in warp and weft. Alternating cyclic order of paper strips.*

The easiest way of weaving is having two perpendicular directions (warp and weft). Both sheets of paper are cut into strips. The strips in the weft direction alternately pass over and under the strips in warp direction; with straight paper strips this creates a checkerboard pattern. Changing the border line of the paper strips already creates a pattern with octagonal stars–see Figure 1a. Due to the construction, the front side and the back side show the same pattern but with reversed colors. In all figures, the cutting lines in one sheet of paper are depicted as solid lines, while the cutting lines in the second sheet of paper are depicted as dashed lines. At each corner of the tiling, a dashed line and solid line cross.

At each crossing point between cuts in weft and warp directions, all paper strips change from the lower level to the upper level, either in clockwise or counter-clockwise cyclic order. Along each cutting line, clockwise and counter-clockwise order alternates between neighboring crossing points–see Figure 1b. This is a necessary condition for alternately passing over and under the other sheet of paper.

Due to constructability, we do not consider tilings having more than two cutting lines crossing in one point. The condition to alternate in clockwise or counter-clockwise order, respectively, allows only tiles with an even number of neighboring tiles of the other color. In a periodic tiling, if all corners of each tile have valence four and all faces are even, the faces have to be quadrilaterals.

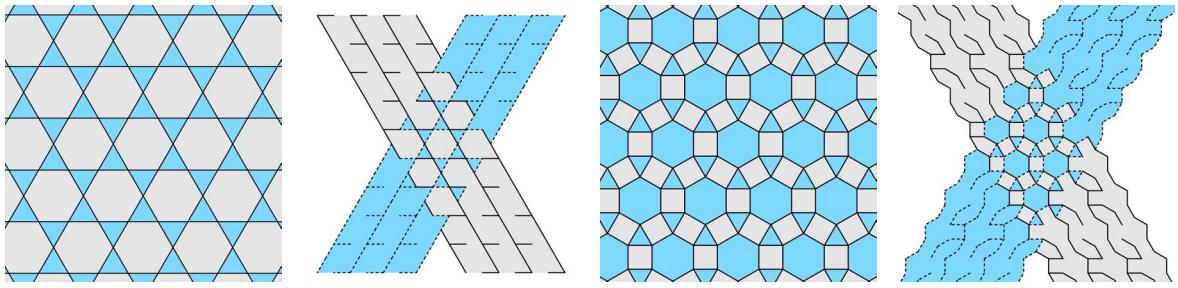


**Figure 2:** *Archimedean tiling 3-6-3-6 and Archimedean tiling 3-4-6-4 and cutting patterns.*

## Two-Colorable Tilings

In addition to regular tilings such as the triangular tiling or the square tiling of the plane, there are lots of tilings consisting of differently shaped tiles that can be two-colored such as the Archimedean tilings 3-6-3-6 or 3-4-6-4 (see Figure 2).

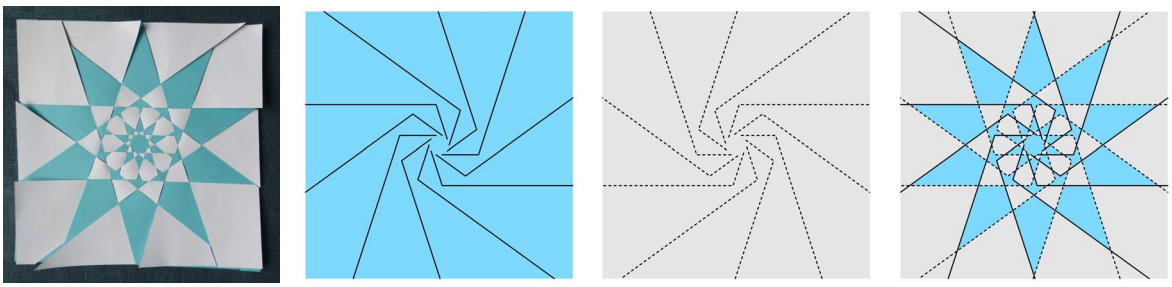
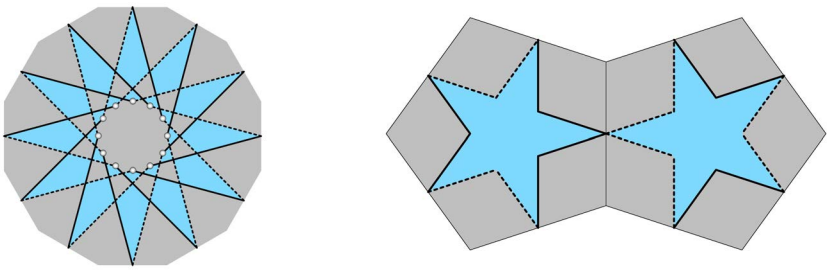


**Figure 3:** *A star woven from two square papers and the cutting patterns for the two sheets, paper model 16 cm × 16 cm.*

Disregarding the demand of regular tiles, the construction of *girih tilings* yields numerous beautiful patterns containing stars as shown in Figure 3. Additionally, because of their construction all girih tilings are two-colorable (see [2]).

As shown in Figure 2, most of the Archimedean tilings contain triangles which leads to the occurrence of nonalternating cutting lines. Each triangle has either two solid edges and one dashed edge or vice versa. As shown in Figure 4a, we can solve the issue of odd tiles by introducing additional points where the cut is changed to the other sheet. Alternatively, at a common corner of two tiles we can assign two noncrossing dashed lines; see Figure 4b.



**(a)** **(b)**

**Figure 4:** *Handling of tiles with odd number of adjacent tiles. (a) Additional corners, (b) noncrossing lines.*

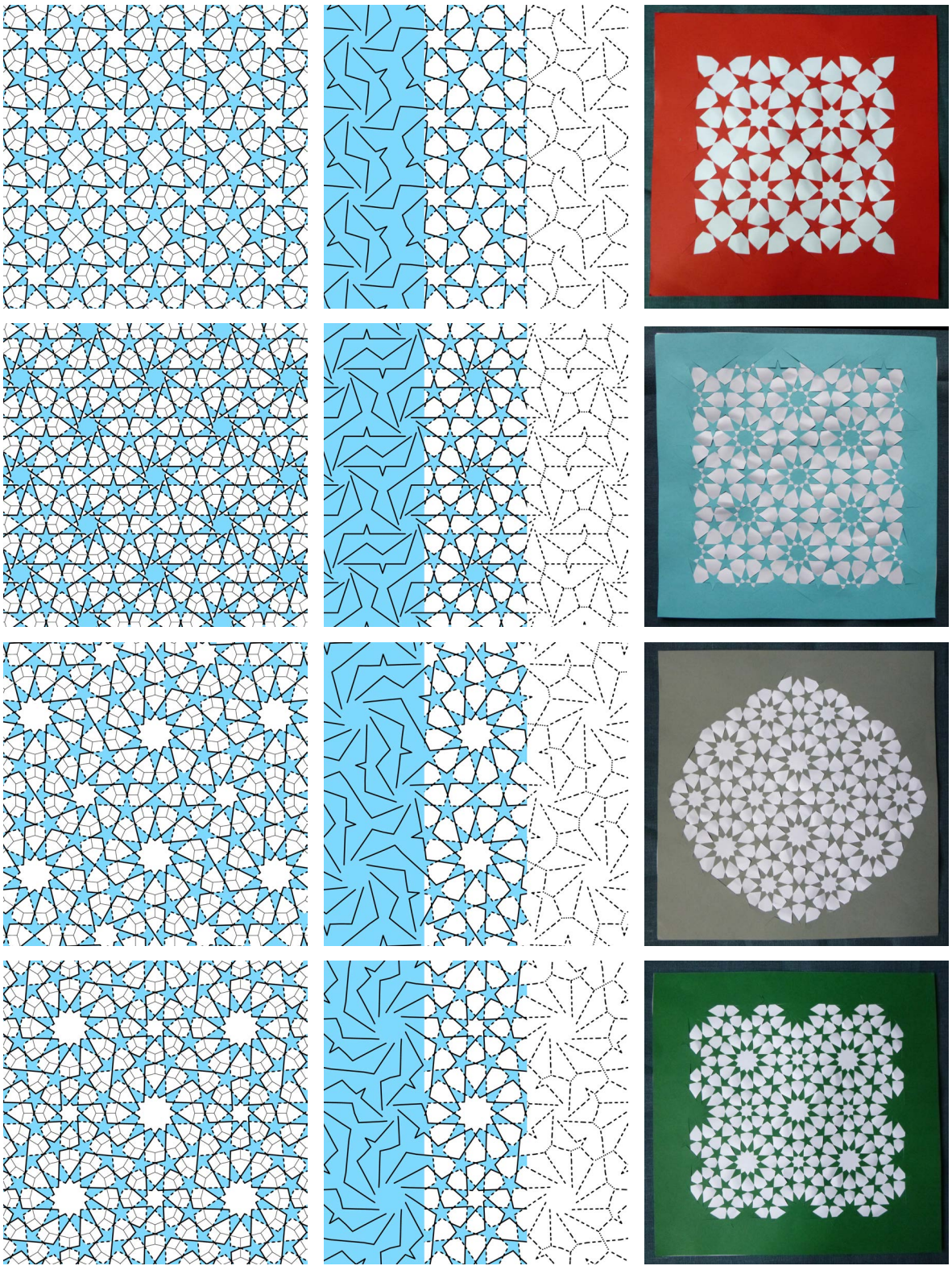


**Figure 5:** *Examples of woven star patterns. From left to right: girih tiling with construction lines and cutting lines; cutting pattern (dotted lines on the white part mark necessary additional cutting lines); woven paper realizations, 30 cm × 30 cm each. All cuts are made using pen knife and ruler. From top to bottom: star patterns as in [4] Fig. 29, [4] Fig. 32, [7] panel 42 (described in [1]), [3] Fig 118.*

## Construction of Star Patterns

Given a polygonal tiling of the plane, girih tilings arise by drawing two crossing lines at the midpoint of each edge of the poylgonal tiling. These lines are made longer until they meet with the lines drawn at the midpoint of some other edges (see [4]). The intersection points and the midpoints split the original polygonal tiling into smaller polygons. Here, no edges of the original polygonal tiling are used, only the newly constructed edges. Examples made of paper constructed using the lines of the star patterns are presented in [5] and [6].

## Notes on Examples

The star patterns shown in Figure 5 were all created using the girih construction described in [4]. All patterns in Figure 5 contain tiles having an odd number of neighboring tiles such as the five-pointed stars or kites adjacent to stars with more than five tips. The pentagonal stars pair as shown in Figure 4b, while the kites with only three neighboring faces are handled as shown in Figure 4a.

Since by this construction the cutting lines always end at stars with more than five tips, each of the two sheets of colored paper is not simply connected. Therefore it is not possible to weave the two sheets without additional cuts. In the shown examples, the white sheet is cut into simply connected components such that each component has a bigger star in the middle, which are then woven into the other sheet (see Figure 3).

The weaving gets easier if the cuts are made a bit longer. Because of the symmetry of the patterns both sheets of paper are equally cut and one is afterwards provided with the additional cuts needed to weave it into the other one. After choosing one color distribution for the upper level, the pattern itself tells how is has to be woven.

## References

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