Markov Chains, Coptic Bananas, and Egyptian Tombs: Generating Tablet Weaving Designs Using Mean-Reverting Processes

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Abstract

Tablet weaving, also known as card weaving, is an ancient method of making strips of fabric that is still used by hobbyists and crafters today. One important difference from other sorts of weaving is that the threads are twisted as the cloth is produced, with different design elements producing different directions of twist. It is desirable for this twist to be balanced across the length of the strip, and this feature has inspired the use of a mean-reverting Markov process known as the Ehrenfest model to randomly generate tablet weaving patterns. I applied this process to the technique known as "Coptic Diamonds", with very good results. For perfectly balanced twist, however, some extra symmetry had to be artificially introduced into the patterns.

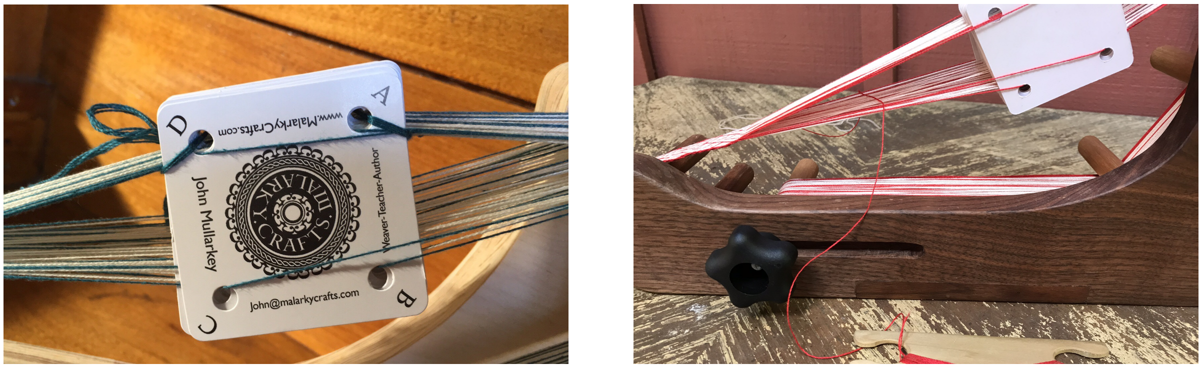
Tablet Weaving

Tablet weaving is a method of making strips of cloth using very simple equipment. Vertical (warp) threads are passed through holes in tablets or cards, as shown in Figure 1a. The warp threads are held under tension by a simple loom (as in Figure 2a), a even simpler pair of pegs, and/or parts of the weaver's body. The cards separate the warp threads into two batches with a space between them. A horizontal (weft) thread is passed between the vertical threads (Figure 1b), after which the card is turned. The weft thread is passed back in the other direction and the cards are turned again, in the same directions or different directions.

As the cards turn, the warp threads twist around each other and around the weft threads, which hold the warp threads in place and lock them together. The weft threads are rarely seen in the finished product, as they end up encased by the twisted warp threads. In addition, turning the cards brings different warp threads to the top of the fabric. If the different warp threads through a card have different colors, then turning the cards will produce colored patterns on the finished fabric. As each card can be turned individually, an exceptionally wide variety of different patterns can be produced. The final woven cloth can be as narrow as a shoelace or more than 70 cm wide in the case of a piece from Ethiopia [1, p. 179]. It can also be very long, as in the traditional Bulgarian sash, which is 20 to 27 feet long and wraps several times around the waist [2, p. 8].

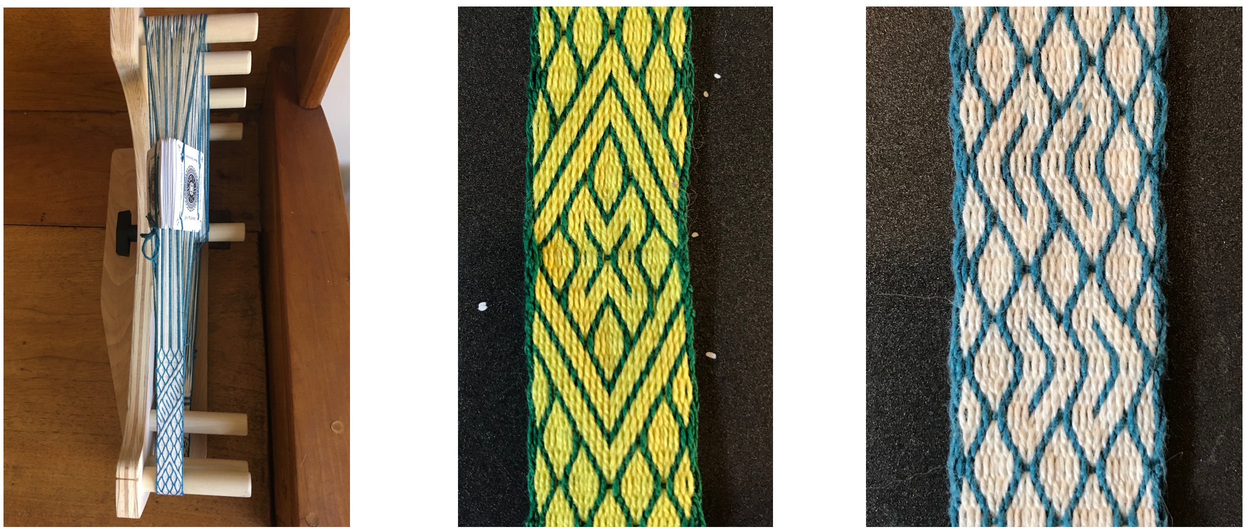
Tablet weaving is very old, dating back to at least the fourth century BCE and possibly earlier [1, p. 13]. Tablets used to weave cloth have been found in Europe, Asia, and Africa. As the reader might guess, a very large variety of different tablet weaving patterns have been developed in these different times and places. One well-known type of pattern is called "4×4 Tablet Weaving" or "Coptic Diamonds", due to its use in certain late Coptic bands [1, p. 112; 6, p. 11]. A pack of square cards are threaded with one thread of foreground color and three threads of background color in each card. The cards are arranged in groups of four and turned in such a way that the foreground color makes a diagonal stripe in either the Z (lower left to upper right) or the S (lower right to upper left) direction across a 4 by 4 block of threads. This block forms a rectangle with a ratio of 1.5 to 2 units in the warp direction for every unit in the weft direction. Common design elements in this technique include diamonds (Figure 2b) and "bananas" [6, p. 13] (Figure 2c). The bands in these pictures were each woven approximately 0.875 in. wide and 32 in. long; the details shown are 2.25–2.75 in. long.

One difference between tablet weaving and other types of weaving is that the threads from each card are twisted around each other as the piece is woven. If the equipment is not specifically designed to account



(a) **(b)**

Figure 1: *(a) A pack of tablet weaving cards. (b) The weft thread being passed between the warp threads.*



(a) **(b)** **(c)**

Figure 2: *(a) A tablet weaving loom. (b) Coptic Diamonds pattern, 0.875 $\times$ 2.75 in. detail. (c) Coptic "bananas" pattern, 0.875 $\times$ 2.25 in. detail.*

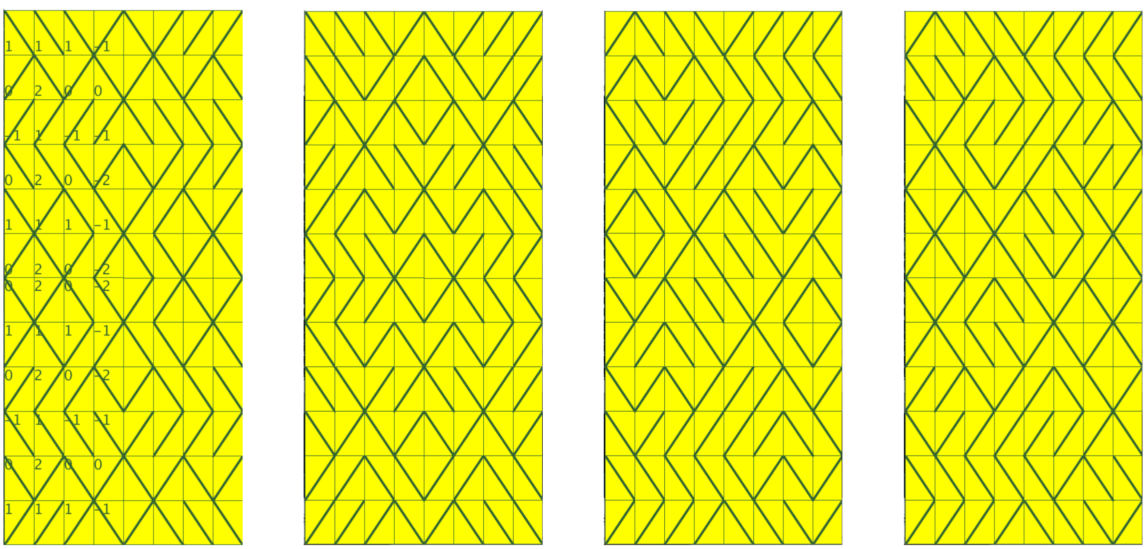
for this, it is important to design patterns such that the twist is more or less balanced throughout the piece. For Coptic Diamonds, the direction of twist is determined by whether the diagonal is in the Z or S direction. Therefore, it is important to keep the number of Z diagonals and S diagonals roughly equal for each vertical column of four threads. This inspired the idea of using a mean-reverting random process to generate random Coptic Diamond designs.

The Ehrenfest Process

Since the weaving pattern consists of discrete steps, it is reasonable to model it with a Markov chain, which is a random process where the probability of each event (in this case, the choice between an S and a Z diagonal) depends only on a discrete parameter describing the system (in this case, the total amount of twist). For simplicity, we model each vertical column independently, using a simple mean-reverting Markov chain known as the Ehrenfest model. This model was proposed by Paul and Tatiana Ehrenfest in 1907 for use in the field of thermodynamics, but it can also be used to model the motion of a particle traveling randomly but influenced by an elastic force [4]. If the particle is at the origin, it has an equal chance of traveling one step to the left or one step to the right. If it is not at the origin, it will move back towards the origin with probability $\frac{1}{2}\left(1+\frac{k}{R}\right)$, where k is the distance from the origin and R is the maximum allowable distance. Otherwise, it will move away from the origin. Since the particle is more likely to move towards the origin than away, the process tends to revert towards the mean. For our application, the position of the particle represents the total amount of twist in each column. When the particle is at the origin, there is no twist in the threads. In the current work each column, representing four warp threads, is modeled independently of every other column.

Randomly Generated Patterns

I have written a computer program (available at [3]) in the Processing language to generate random patterns according to the procedure defined above. After some trial and error, I determined that a maximum twist of 8 produced patterns that were interesting without building up too much twist on the threads. (Note that it is very unusual to actually achieve the maximum twist in this model [5]. Most patterns generated by the program have a maximum twist of 5 or less even after 15 to 20 steps, and many never reach more than 3. By comparison, Collingwood gives examples of 4×4 designs with as many as 39 steps [1, p. 187].) Figure 3a gives an example of such a pattern with the twist labeled in the first four columns. The numbers should be read as giving the twist as of the horizontal line they are closest to.



(a) **(b)** **(c)** **(d)**

Figure 3: *(a) and (b) Randomly generated patterns with two axes of reflection. (c) and (d) Randomly generated patterns with 180-degree rotation and glide reflection.*

I originally hoped that the mean-reversion property would frequently result in all columns achieving balanced twist simultaneously after a reasonable amount of time. This has not proved to be the case. In order to generate designs that can be repeated along a strip of fabric, the program reverses direction after a specified number of steps and generates the mirror image of the original pattern, as shown in Figures 3a and 3b. Since many tablet weaving patterns also have a line of symmetry along the length of the band, the program additionally mirrors the pattern across this line. This can be an exact mirror, as in Figures 3a and 3b, or a glide reflection, as in Figures 3c and 3d. If a glide reflection is desired, the user sets a variable in the computer program indicating how many steps to offset the left half of the pattern from the right. The corners are then filled in with a simple zigzag motif which preserves the balanced-twist aspect of the pattern. The exact symmetries of the final pattern are a 180-degree rotation and, if the filler is ignored, a glide reflection.

I wove several repeats of the pattern in Figure 3b in order to see how difficult it was to weave and how it looked. A detail from the resulting band is shown in Figure 4. The weaving was more challenging than the patterns in Figures 2b and 2c, but after some practice I could achieve nearly the same speed. Compared to the patterns in [1], I would estimate this as in the upper 75% of difficulty but by no means the most complex.

Summary and Conclusions

It is clear that this model can produce Coptic Diamonds patterns which are aesthetically pleasing while at the same time balanced in their twist thanks to the Ehrenfest process. There still remains work to be done, in terms of both mathematical analysis and design. There is much in the work in the literature dealing with the average time that it takes for an Ehrenfest model to reach a particular state [4; 5, for example]. However, it would be interesting to know that probability that the twist is less than some absolute value k after n steps. This could be useful for producing almost balanced designs without the necessity for mirror imaging.

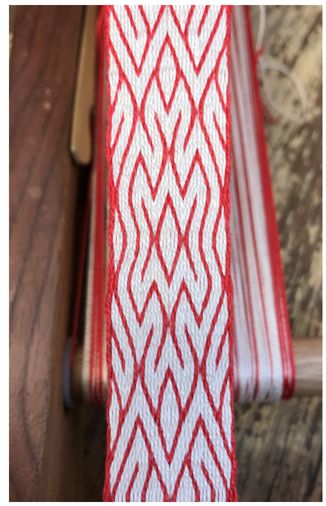


Figure 4: *Woven version of Figure 3b, 0.875 × 3.375 in. detail.*

Another useful analysis would be to estimate the average time that it would take for all the columns to achieve balanced twist at once. Even more useful, perhaps, would be to develop a model that would naturally induce faster reversion towards zero total absolute twist. Preliminary experiments involving an elastic force which depends on more than one column have proved unsuccessful as of yet.

Another well-known tablet weaving technique is known as "Egyptian Diagonals". The patterns produced resemble pictures of fabrics found in Egyptian tombs, although there is no conclusive evidence that this or any other tablet technique was used in ancient Egypt [1, p. 109; 2, p. 11]. I have tried applying the same generating technique to patterns in this style, but it seems clear that these patterns do not appear as structured as the Coptic Diamond patterns and are therefore less aesthetically compelling. Possibly the model can be adjusted to a different sort of fundamental block, perhaps a diamond shape, which will produce more aesthetic patterns. Or it is possible that a different mean-reverting model is necessary in this case.

Acknowledgements

I would like to thank Lana Holden for her invaluable aid with the photography, and Lee Trent and the anonymous reviewers for their helpful suggestions.

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