Discrete and Computational Differential Geometry for Functional Pattern Design

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Abstract

The realization of freeform shapes in contemporary architecture has sparked research on its mathematical and algorithmic foundations. This area, now often referred to as *Architectural Geometry* (AG), has close connections to *Discrete Differential Geometry* (DDG) [1]. The transfer of ideas and inspiration goes in both directions: Practical requirements in freeform architecture led to new concepts in DDG and known concepts in DDG turned out to have applications in architecture [5]. Here, we will discuss some patterns arising in architecture and design which are constrained by requirements on their fabrication and function.

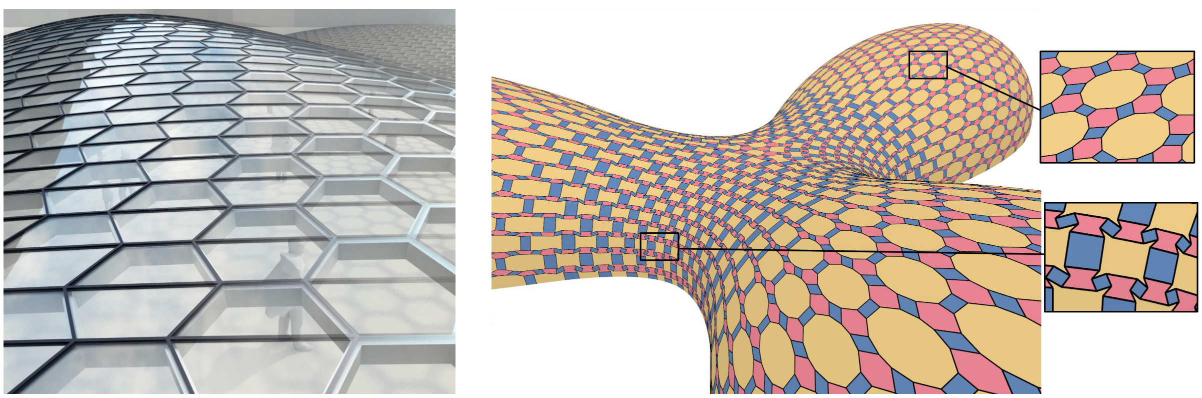

Figure 1: *Polyhedral patterns. Left: A quad mesh needs not look like a curved regular grid. Right: In positively curved areas, the pattern appears like the corresponding semi-regular pattern in the plane, but in negatively curved areas it changes significantly.*

Polyhedral patterns. A major stream of research in AG deals with meshes from planar quadriterals (PQ meshes). Under appropriate fairness requirements, these PQ meshes are discrete conjugate surface parameterizations, with discrete curvature line parameterizations as important special cases [1]. This leads to an understanding of the degrees of freedom when approximating a given shape with such a mesh [5]. However, the link to DDG is not valid anymore if the discrete structure is not resembling a good approximation of a smooth curve network. Such a structure is shown in Fig.1, left. The controlled zigzag polylines in one direction relieve the structure from its strong relation to the curvature behavior of the reference surface R, leading to an interesting and useful solution which for this surface R could not be achieved by a standard PQ mesh. This is an example of a polyhedral pattern, as it is formed by a pattern of planar polygonal faces approximating a surface. To obtain such polyhedral patterns, the *regularizers* in the optimization algorithm play a crucial role. They turn out to be certain symmetries, which can be found via local second order approximants of surfaces (paraboloids) [3]. Polyhedral pattern generation is not achievable by slightly adapting standard texture mapping algorithms towards planarity of faces.

Structural patterns. Naturally, if an architectural pattern is closely tied to a support structure, it needs to be structurally sound. This yields to the challenging task of generating patterns that are further constrained by the static properties of the support structure that may be aligned with the edges of the tiles. Quadrilateral meshes following principal curvature lines have advantages for manufacturing reasons, such as planar panels and simplified substructure connections [5]. On the other hand, for structural efficiency, it is convenient to ensure static equilibrium in the load bearing structure through axial forces only. Both of these goals can be reached for surfaces in membrane equilibrium where principal stress and curvature directions coincide [4]. To align these principal directions, one first has to minimally change the surface in an optimization algorithm and then re-mesh it with a planar quad mesh following the principal directions (Fig. 2, left). Here, the mesh combinatorics (pattern) follows from the geometry and cannot be influenced by the designer.

Patterns from repetitive elements. It is very difficult to achieve repetitive elements in panels of an architectural freeform skin (see e.g. [5]). Therefore, recent research by Eike Schling [6] focused on repetition in the original constructional elements which change their shape during construction, for example through bending. We will discuss only one instance of such a structure (Fig. 2, right). Its basic geometry already appears in a fascinating paper by S. Finsterwalder [2] which contains numerous ideas for further research in computational fabrication. The design starts with a minimal surface on which one computes a grid of asymptotic curves. On a minimal surface, these curves are orthogonal. Along an asymptotic curve c on a minimal surface S one can attach a developable surface strip which is orthogonal to R. Unfolding this strip, c becomes a straight line. Hence, one can fabricate such structures from planar straight strips of sheet metal, but has to observe further constructional details [6]. This research gives rise to several generalizations which provide again interesting links to DDG.

Figure 2: *Left: Pattern following aligned principal stress and curvature directions. Right: Curved support structure from repetitive elements (flat rectangular sheet metal strips) by Eike Schling.*

References

[1] A. Bobenko and Y. Suris. *Discrete differential geometry. Integrable structure.* Providence, Amer. Math. Society, 2008.

[2] S. Finsterwalder. “Mechanische Beziehungen bei der Flächendeformation.” *Jahresber. d. Deutschen Math.-Vereinigung*, vol. 6, 1899, pp. 43–90.

[3] C. Jiang, C. Tang, A. Vaxman, P. Wonka, and H. Pottmann. “Polyhedral patterns.” *ACM Trans. Graphics*, vol. 34, no. 6/172, 2015, pp. 1–12.

[4] D. Pellis and H. Pottmann.“Aligning principal stress and curvature directions.” *Proc. Advances in Architectural Geometry*, Gothenburg, Klein Publishing, 2018, pp. 34–53.

[5] H. Pottmann, M. Eigensatz, A. Vaxman and J. Wallner. “Architectural Geometry.” *Computers and Graphics*, vol. 47, 2015, pp. 145–164.

[6] E. Schling “Repetitive Structures.” PhD thesis, TU Munich, 2018.