How to Use Prime Numbers and Periodicity to Write a Poem

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Abstract

Participants in this workshop will read and experiment with writing poems structured by two poetic forms, each of which has a connection to mathematics. The first poetic form is of recent vintage, but it is based on an ancient mathematical result, *The Fundamental Theorem of Arithmetic,* which combined with careful word choices creates a pattern of repetitions that results in a poem with the musicality of tolling bells. The other is the classic and melodic poetic form, the villanelle, whose lines follow a strict meter and whose stanzas are formed by braiding elements of rhyme and refrain in a way that resembles the combined waves of sine and cosine functions in the plane. Both poetic forms are difficult to use successfully and require some adjustments of word choices throughout the process of creating the poem. The workshop will not assume prior knowledge of either the mathematics or the prosody involved. All Bridges participants are welcome! Please bring writing materials, that is, paper and pen.

Introduction

Among the similarities between composing a highly structured poem and providing a proof for a newly discovered mathematical result, is the necessity to create something new and beautiful under rigorous constraints. Mathematics has its axioms and highly structured poetry has its prosodic rules (which are often mathematical). The two poetic forms we will work with in this workshop are examples from the rich tradition of highly structured poetry [8, 10].

The Fundamental Theorem of Arithmetic

In the first part of this workshop we will learn how to use the mathematical result known as *The Fundamental Theorem of Arithmetic* as a structure for poems. *The Fundamental Theorem of Arithmetic* states that every positive integer greater than 1 is either a prime number or can be expressed in a unique way (up to order) as a product of distinct prime numbers. For example, consider the number 1200:

$$1200 = 2 \text{ x} 2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } 5 \text{ x } 5 = 2 \text{ x } 5 \text{ x } 2 \text{ x } 5 \text{ x } 2 \text{ x } 3 \text{ x } 2 = \ldots$$

The theorem says two things about this example: The first claim is that the number 1200 can be written as a product of prime numbers, in this case four 2s, one 3, and two 5s. The second claim is that, with the exception of reordering the primes, no other expression of 1200 into a product of prime numbers exists. In order to write positive integers as a product of prime numbers in only one way, we add the convention that multiple copies of the same prime number are expressed as exponentiation, and order the primes in increasing size from left to right. In this example, the only way to write 1200 as a product of primes with these added restrictions is:

$$1200 = 2^4 \text{ x } 3 \text{ x } 5^2$$

The theorem therefore implies that the prime numbers are the (multiplicative) building blocks of the integers. *The Fundamental Theorem of Arithmetic* was first proved in Euclid's *Elements* (circa 300 BC)

[3]. It is interesting to note that the famous Number Theorist, Andre Weil (1906 – 1998), pointed out that there is a gap in Euclid's proof of the uniqueness of expression in the product of prime numbers (up to order). This was remedied in a more modern proof of *The Fundamental Theorem of Arithmetic* given by Carl Friedrich Gauss in 1798 [12].

The Fundamental Theorem of Arithmetic can be used for the purpose of constructing a poem in the following way: One first decides on the length of the poem and numbers the poem's lines consecutively from bottom to top: 2, 3, 4, 5…. Then one chooses a word that stands for multiplication and a word that stands for exponentiation. The next step is to write the lines corresponding to prime numbers. Each line that acquires a prime number is a building block of the other lines much like the prime numbers build the positive integers.

Below is the poem, *13 January 2009*, which was constructed using this approach. Here is how the poem was built: Line 2 is: *Anuk is dying*, line 3 is: *The white of winter*, the word "in" stands for multiplication, and the word "for" stands for exponentiation. To construct, for example, line 12, we write $12 = 2^2 \times 3$, and replace the number 2 by line 2, the number 3 by line 3, multiplication by "in" and exponentiation by "for." This makes line 12: *Anuk is dying for Anuk is dying in the white of winter.*

13 January 2009

$12=2^2x3$	Anuk is dying for Anuk is dying in the white of winter
11	The coldest month
10=2x5	Anuk is dying in the falling snow
$9=3^2$	The white of winter for Anuk is dying
$8=2^3$	Anuk is dying for the white of winter
7	The drift of time
6=2x3	Anuk is dying in the white of winter
5	The falling snow
$4=2^2$	Anuk is dying for Anuk is dying
3	The white of winter
2	Anuk is dying
1	.

— Sarah Glaz [5]

Reading the lines from the bottom upward renders another poem. The echo created by the repetition of the prime numbered lines creates a bells-tolling effect. Showing the "scaffolding" of math equations in the left column is optional.

This poetic form was invented by Carl Andre, American minimalist sculptor and occasional poet. He wrote the first poem using this technique "On the Sadness" for an art exhibit at the Yale University Art Gallery in 1978 [1,4]. Carl Andre's "On the Sadness" does not show the numerical scaffolding. A handout of this poem will be provided at the workshop.

The Villanelle

In the second half of this workshop, we're going to learn to write villanelles! What is a villanelle, and why is it worth learning to write one? How is it related to mathematics? The answer is hidden in the depths of periodicity, which is symmetry in time – but we know that time goes ever, ever, on and never returns. What can we mean by symmetry in time? The structure of a villanelle, and the effort and inspiration of writing one, helps to answer these questions. We'll also learn something about the history of this form, how it showed up in the first place, on the street and in the courts and then as a diversion and task for poets [9]. As a bonus, we'll provide free copies of the small new anthology, *Love Affairs at the Villa Nelle* [11].

A villanelle has six stanzas, which all have three lines, except the last which has four. The first line of the first stanza is repeated as the last line of the second and fourth stanzas, and the third line of the final stanza; the third line of the first stanza (which rhymes with the first line) is repeated as the last line of the third and fifth stanzas, and the fourth line of the final stanza, so it ends the poem. Each three-line stanza has the same rhyme scheme, a b a, and the last stanza has the rhyme scheme a b a a. Using capitals for the refrains and lowercase letters for the rhymes, the form could be expressed as: *A1 b A2 / a b A1 / a b A2 / a b A1 / a b A2 / a b A1 A2*.

A beautiful villanelle in English, by W. H. Auden, is "If I could tell you" [2]. Here is a poem written a few years ago using the same rhyme scheme as Auden, about saying goodbye in airports.

Holding Pattern

We can't remember half of what we know.
They hug each other and then turn away.
One thinks in silence, never let me go.

The sky above the airport glints with snow
That melts beneath the laws it must obey.
We can't remember half of what we know.

His arms are strong and warm, his breath is slow;
She holds him close, not knowing what to say.
One thinks in silence, never let me go.

Time silts the rivers, ravaging the flow
Of wave on wavelet, and suspends the day.
We can't remember half of what we know.

This holding is agreement to forego,
This flight another strategy to stay.
One thinks in silence, never let me go.

The silver trees spring back to life, although
Their roots are gilded by the leaves' decay.
We can't remember half of what we know,
One thinks in silence. Never let me go.

— Emily Grosholz [6]

The interesting thing about the structure of the villanelle is that it requires a pattern of rhyme (and by implication meter), but also, in song-like fashion, the repetition of whole lines, like refrains. This confluence sets up interesting superimpositions of different kinds of periodicities: the repetition of sound (phonemes), of the formal line (defined by meter, like iambic pentameter), and of meaningful grammatical units (the phrases that are repeated).

The mathematical schema for periodic pattern is the sine wave: a given sine wave has a certain amplitude (how big or intense it is) and a certain frequency (how quickly its peaks pass by a given fixed point). A cosine wave has the same shape, but with a phase shift. When two sine or cosine waves of different amplitude and frequency are superimposed, they create a new pattern, with especially high peaks at points when the high peaks of the original waves happen to reinforce each other, especially low peaks when two low peaks are superimposed, and a complex but regular mixture of amplitudes in between. Any periodic function, no matter how apparently irregular, can be represented by a (possibly infinite) sum of sine and cosine functions, which is called its Fourier transform.

How can poets use the magic of this superposition? One obvious example of this is the difference between end-stopped lines, and lines that exhibit weaker and stronger kinds of enjambment (the continuation of a sentence without a pause beyond the end of a line). The lineation of poems establishes a formal periodicity; but grammar has its own periodicity, signaled by the completion of a sentence when a noun and a verb are coupled properly, and in Western languages by a capital letter at the beginning, a period at the end, and a space before the next sentence. These two kinds of periodicity may coincide, as in carefully end-stopped lines, or in the formulae chosen over centuries by the bards of oral traditions. However, they may not. The grammatical structure of enjambed lines overflows and violates the boundaries set by the poetic line, setting up a tension between the thought expressed and the form, like a river articulated and deflected by boulders but still rushing over them. Conversely, the boundaries set by the poetic line may interrupt the grammatical structure in ways that reinforce and emphasize words or phrases, or ironically undermine and analyze them. So we will play with that [7]!

Of course, people who read periodicity as repetition cannot ever forget the difference that nuances their sameness. Natural systems are not aware of their own dissolution; the solar system does not rue the day when the sun will grow large and red, devour its children-planets, and then sink into darkness. We are able to use periodicities to make ourselves at home in the world, but only because we know and represent them; and the shadow side of knowledge and representation is death. The circles of periodicity are really spirals, stretched out along the arrow of time that flies only in one direction, and sooner or later brings down every creature — periodicity is symmetry in time.

Workshop activities

In this workshop the time devoted to each of the two poetic forms will be equal (45 minutes each). Each half of the workshop will start with preparatory exercises designed to familiarize the participants with the technique and generate good choices for poem topics. These will include, for example, reading and analyzing poems constructed with these techniques, practicing strategies for choosing poem topics that have connections to mathematics, practicing writing lines or stanzas (for villanelles) or poems with just a few lines (for the *Fundamental Theorem of Arithmetic* poems). Handouts or booklets for the readings will be distributed at the workshop. We will then practice constructing poems employing the techniques. We will use various form-specific strategies to facilitate the composition of poems, such as, for example, structure worksheets or templates (which will be provided at the workshop). Depending on the number of workshop participants, the work on the composition of poems may be carried out in small groups rather than individually. After completion, draft poems will be shared with, and receive feedback from, all the workshop participants.

References

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