Double-layered Weaving of Infinite Bi-foldable Polyhedral Complexes

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Abstract

We present various weaving constructions, as applied to our previous work on infinite bi-foldable polyhedral complexes. These enable one to build doubly or triply periodic structures that mimic certain origami patterns.

Introduction

Weaving polyhedral complexes has been researched extensively by Rinus Roelofs. In [2], Roelofs explored the usage of interwoven multi-layered surfaces with holes to create polyhedrons such as rhombic dodecahedrons, snub cubes, rhombic triacontahedrons, etc. In [3], Roelofs discussed the creation of a "double" version of Coxeter's infinite polyhedron using a method he called "elevation". In [4], he further explored various ways to fold entwined double-layer polyhedral structures, and their elevated counterparts, based on various net designs. The weaving method discussed in this article is similar to the strips and rings discussed by Roelofs in which he gave a simple example of a double layered cube that is similar to a Borromean structure; the double-layered cube's 12 faces can be divided into 3 strips of 4 faces that can then be folded and interwoven into three connected strips. Alternatively, the cube can be understood as a generalized zonohedron [1], or a convex polyhedron bounded by parallelograms. Since a cube has three distinct edge directions, and every edge direction determines a zone of faces, therefore three different zones. Since each face of the cube has one edge that is equal and parallel to one of the zone's edges, and another edge that is equal and parallel to one of the other two edge directions, it can be understood that each face belongs to two zones which cross each other at the face (Figure 1). Using a method similar to this, this article aims to explore the various forms and expressions in woven infinite bi-foldable polyhedral complexes.

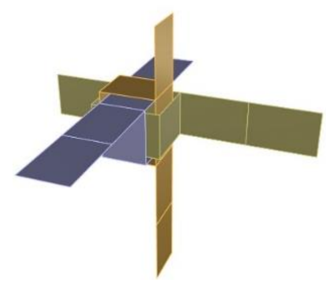

Figure 1: *A woven cube with its three zones unfolded.*

Infinite Bi-foldable Polyhedral Complexes

Previously in [5] we introduce a method to construct infinite polyhedral complexes that can be flat-folded or collapsed into two perpendicular planes. These infinite polyhedral complexes are based on the concept of a four vector star (Figure 2), in which vector *v1* and *v2* sit on *XZ* plane, vector *v3* and *v4* sit on *YZ* plane, and the XZ plane and YZ plane are perpendicular to each other and therefore called bi-planar. The infinite polyhedral complexes are constructed based on this four vector star. All of the edge vectors of the infinite polyhedral complexes lie in the same orientations as the vectors in this four vector star. There can be a total of six parallelograms that are spanned by these four vectors. For the infinite polyhedral complexes to be foldable in the two bi-planar directions, or bi-foldable, into either the XZ plane or the YZ plane, two facets, spanned by *v1* and *v2*, and by *v3* and *v4*, cannot be allowed in the construction. These missing facets allow the infinite polyhedral complexes to be

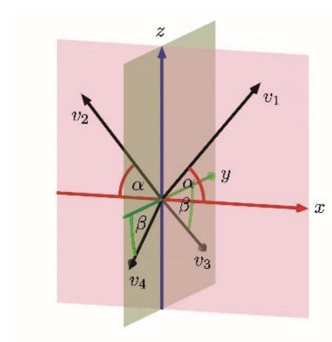

Figure 2: *Four-vector Star.*

collapsible. There is an abundance of infinite polyhedral complexes based on the four vector stars. In [5], we first describe a mathematical framework that explains the bi-foldability which allows for a very flexible construction method of general infinite bi-foldable complexes. After listing all fourteen types of polyhedral vertices that can occur in our construction, we construct a few basic examples, including a doubly periodic Miura Weave pattern, a triply periodic Butterfly pattern, and a triply periodic Dos Equis pattern that are discussed in this article. There are a few more examples we constructed in [5] that are not discussed here.

To construct a doubly periodic Miura Weave pattern, start with a vertex type of valency 8 (Figure 3a. Figure 3a also shows the two collapsed states of the vertex). This vertex is then mirrored in two directions to create an infinite doubly periodic polyhedral complex (Figure 3b). Figure 3c shows the two collapsed states in two perpendicular planes.

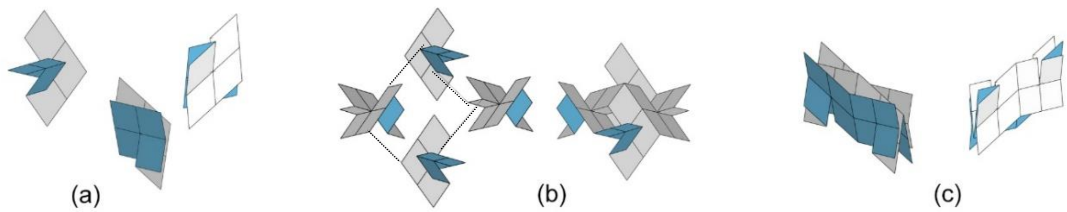

Figure 3: *Miura Weave construction process. The two colors show the front and back sides of paper.*

To construct a triply periodic Butterfly pattern, start with a vertex type of valency 8 that resembles a symmetrically balanced butterfly (Figure 4a). This vertex is translated in two directions, and then in a third direction, to create an infinite triply periodic polyhedral complex (Figure 4b-c). Figure 4d shows the two collapsed states in two perpendicular planes.

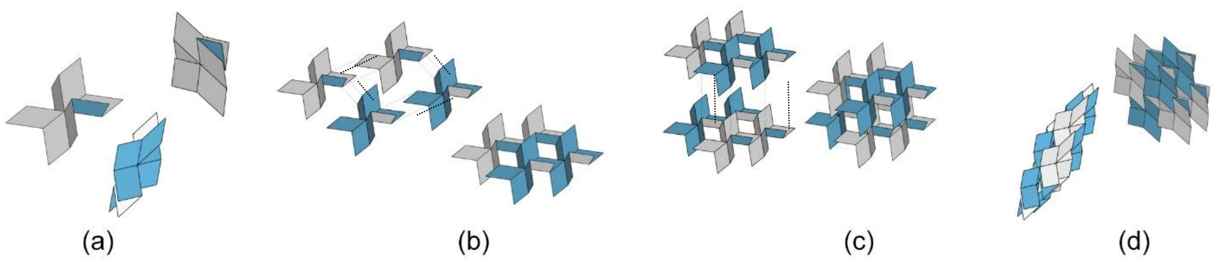

Figure 4: *Butterfly construction process. The two colors show the front and back sides of paper.*

To construct a triply periodic Dos Equis pattern (in Spanish, Dos means ‘two’ and Equis means ‘x’), start with a vertex type of valency 8 that resembles the image of an X (Figure 5a). This vertex is then translated and mirrored in two directions, and then in a third direction, to create an infinite triply periodic polyhedral complex (Figure 5b-c). Figure 5d shows the two collapsed states in two perpendicular planes.

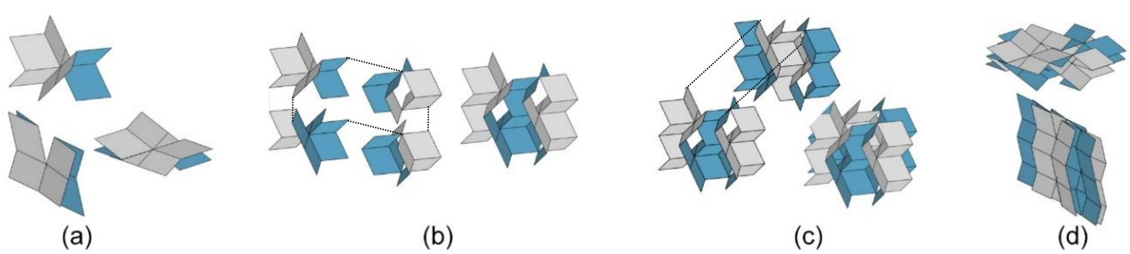

Figure 5: *Dos Equis construction process. The two colors show the front and back sides of paper.*

Weaving of Infinite Bi-foldable Polyhedral Complexes

We explore using weaving techniques to fabricate bi-foldable infinite polyhedral complexes using sheet material such as paper, plastic, and metal. Similar to a cube, the infinite polyhedral complexes can also be understood as generalized zonohedrons [4] that are bounded by parallelograms. Since all the faces are parallelograms, each of the edge directions determines a zone. If n is the number of different directions in which edges of the parallelograms occur, the infinite polyhedral complex will have n zones. Since there are four edge directions in the group of polyhedral complexes discussed in this article, these polyhedral complexes, or zonohedrons, have four zones. Similar to the cube in Figure 1, it can also be understood that each face of the polyhedral complexes belongs to two zones which cross each other at the face and again elsewhere. Therefore, we can visualize each of the infinite polyhedral complex as faces that are the result of the interweaving of four zones, each represented by one color. Furthermore, each face has two overlaying colored zones and its neighboring faces also have two overlaying zones. Figure 6 shows the weaving constructions of three polyhedral complexes. Each of the four colors used in the weaving constructions of a Miura Weave pattern (Figure 6a), a Butterfly pattern (Figure 6b) and a Dos Equis pattern (Figure 6c) represents a zone.

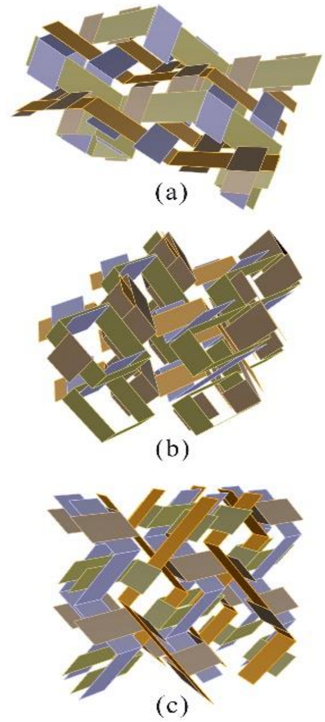

Figure 6: *Weaving constructions.*

The first example is the doubly periodic pattern Miura Weave. Figure 7 shows a stylized construction of the Miura Weave surface in various deployed and folded configurations made with tinted Mi Teintes paper. The stylization creates interesting patterns of colors and shadows. The second example is the triply periodic Butterfly. Figure 8 shows a stylized constructions of the Butterfly surface, again in their various deployed and folded configurations. The third example is the triply periodic Dos Equis. Figure 9 shows a stylized construction of the Dos Equis surface, again also in its various deployed and folded configurations. Each zone, using only two unique unit patterns, is folded and interwoven with other zones to create the structure. Notice that the four colored zones, embedded with its two unit patterns, and its under or over weaving alternations, create a total of sixteen design variations for the quadrilateral faces.

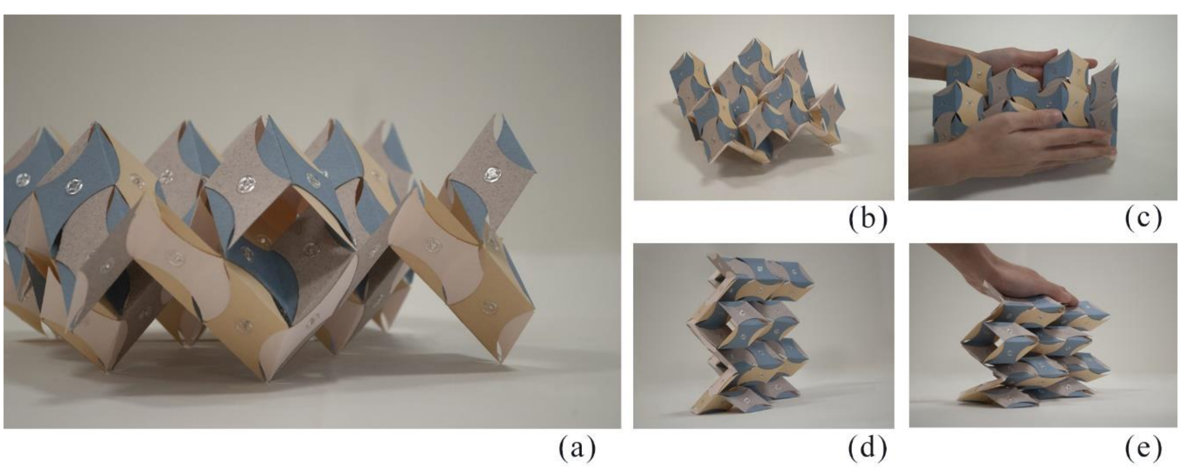

Figure 7: *Weaving of Miura Weave surface using Mi Teintes paper: (a) Front view, (b)&(c) in deployed and folded stages in the first direction, (d) (e) in deployed and folded stages in the second direction.*

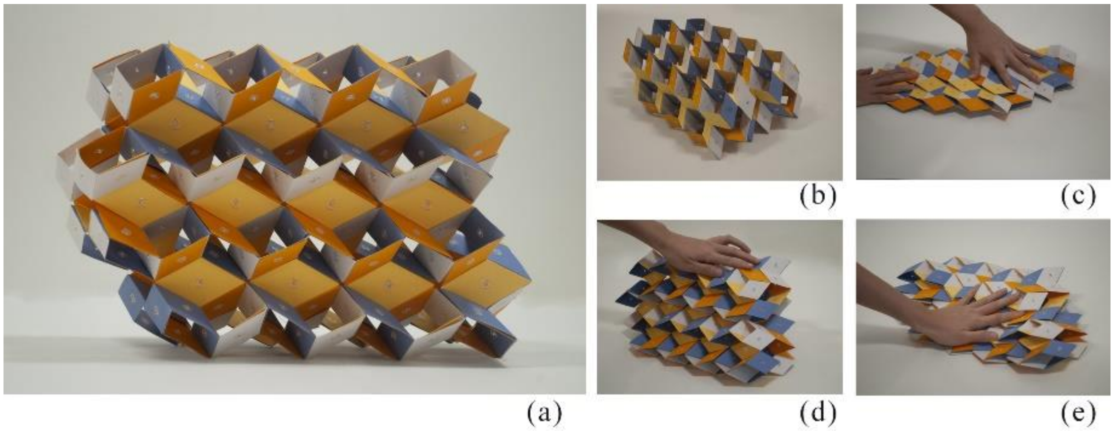

Figure 8: *Weaving of Butterfly surface using Mi Teintes paper: (a) front view, (b)&(c) in deployed and folded stages in the first direction, (d)&(e) in deployed and folded stages in the second direction.*

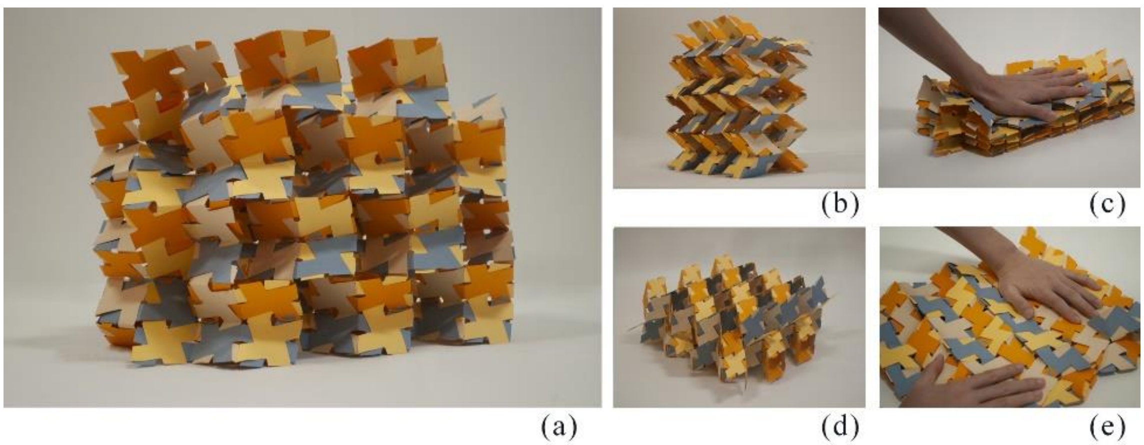

Figure 9: *Weaving of Dos Equis surface using Mi Teintes paper: (a) front view, (b)&(c) in deployed and folded stages in the first direction, (d)&(e) in deployed and folded stages in the second direction.*

Conclusion

Above we present various examples of infinite bi-foldable polyhedral complexes woven with paper strips. We believe that such structures, and the way of making such structure, can go beyond its pure artistic explorations to have implications for the building of smart metamaterials, air or hydraulic filtration systems, robots, large-scale inflatable structures, breathable architectural skins, and many more.

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