Setting a Creative Math Task with SET 3D: Modeling Physical Pieces through Digital Resources

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Abstract

This paper presents and discusses teaching experiences in which mathematical and technological competencies of students are developed through games and puzzles. In-service teachers were encouraged to play some games in its physical form and then challenged to digitally recreate them. Observations from these experiences suggest that such activities could be successfully integrated into the curriculum to promote creative and critical thinking.

Introduction

SET© is a logic card game that requires players to concentrate intensely while playing with a deck of 81 cards that varies in four features across three possibilities: shape (diamond, squiggle, oval), quantity (one, two, or three), shading (full, striped or empty) and color (red, green, or purple). There is only one card of each type in the deck, e.g. a card with three purple striped ovals. A "set" is formed when three cards form a set that includes either all cards are the same or all different by the figures. For example, the following cards form a set: two green open squiggles, two violet filled squiggles, and two red striped squiggles, because all of them have the quantity of two squiggles and each card differs in shading and color. The basic idea is for the dealer to lay out twelve cards on the table and wait until someone identifies and claims to have found a set of three cards by saying out loud "Set!". If it is a proper set, then the player who announced the "Set" collects the cards and points, then the dealer adds three new cards to the table. Despite the game seemingly being simple it does require intense attention of the players in analyzing a range of possibilities through the twelve cards and the use of this game has proven to be successful among those who appreciate mathematics. Such affection for the game was observed spontaneously through practices performed in a program of preparation for the National Mathematical Olympiads by one of the authors, in Brazil. In Iceland and Denmark, some lower secondary school teachers reported that they sometimes use a set web-applet to collect students' attention at the beginning of their mathematics classes. The game can also raise many mathematical questions (e.g. about combinatorics) and in this paper we intend to connect it to recent advances in the development of 3D printing technologies. We consider developing a teacher training course that would combine and connect geometric modeling and combinatorial thinking, and pay special attention to spatial reasoning. We focus our task design on the use of different representations and observe how they can be used in teaching and learning of specific concepts and how they can foster mathematical learning as well as creative and critical thinking.

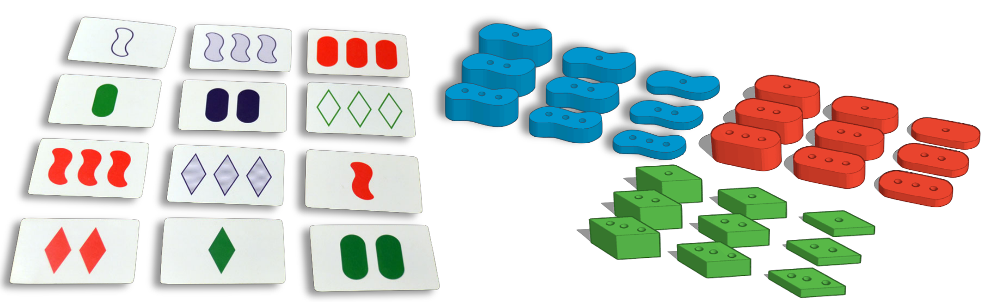


Figure 1: *The game Set in the original 2D card game form on the left and 3D-printed pieces on the right.*

In our preparation for the first teacher workshop in Israel, we decided to map the quantity in the 2D version to be the height in the 3D version and to map the shading in the 2D version to be the number of holes in the 3D version as can be seen in figure 1. Nevertheless, we realized that it would be an interesting task to involve the audience in the game design and ask them to come up with their own mappings. This requires them to map the game from 2D to 3D by themselves and at same time to design and develop a functional 3D game on their own, which inevitably would connect them to the product. This is why at the teacher training workshops in Denmark and Iceland, participants were required to think for themselves before being presented with our own ideas. Some geometrical and combinatorical questions would be raised during the modeling process. However, some non-mathematical skills, such as creative and critical thinking, collaboration and instrumentalization of technology were also observed in the first approaches with teachers, and we will report on those as well in the following text.

Teacher Training with SET

In July of 2018 we offered a workshop in Shlomi, Israel, for a group of approximately 90 teachers, who were also teacher trainers and responsible for disseminating new approaches for mathematics teaching, especially regarding the use of technologies. The whole group was split into three approximately equally sized groups that each took part in a 90 minutes workshop. During the first 25-30 minutes, they played in four smaller groups with the original card version of Set and the pre-prepared 3D-printed version (Figure 2 (a) and (b)). The remaining time was reserved for the modeling process. During one of the 3D version game sessions after some head movements to better distinguish the heights among the pieces, one participant said: "*comparing to the other, this version requests some spatial comprehension in order to identify the different sizes from the same perspective.*" The rules of the game did not change, but the 3D version of the game seemed to gave participants a slightly greater level of challenges. Despite the greater challenge, teachers seemed more eager to collect the pieces in the game. In the modeling process, they started to ask themselves how many pieces belonged in the game and realized by themselves, after some discussions, that it would not be necessary to model all of them since the coloring fact was an output attribute that relied on the selection of the 3D printer's filaments. This discovery allowed them to simplify the modeling process and they decided to use other features than color when splitting the set of 3D pieces into subsets. A relevant aspect to outline as well is one teachers' excitement observed when was asked to volunteer to show their models and share some software hints with her colleagues. Beside the collaborative spirit, it boosted her self-confidence.

(a) (b) (c)

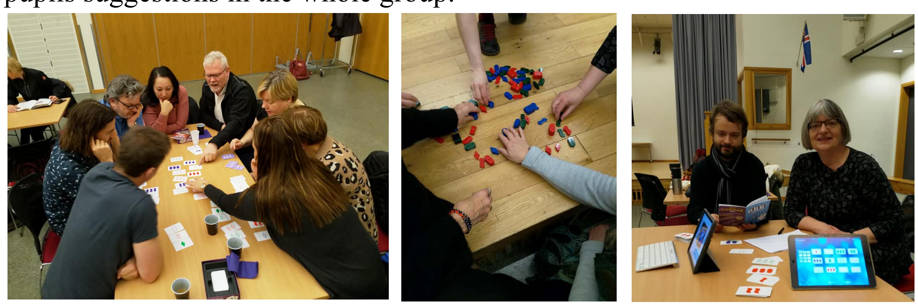
Figure 2: *In-service teachers playing and adapting the game in Israel*

Another workshop for in-service teachers was piloted at the 9th Nordic and Baltic GeoGebra Conference in Copenhagen, Denmark in September 2018 and developed further at a local mathematics teaching association event in Reykjavík, Iceland in November 2018 and Akureyri, Iceland in March 2019. First, the game was introduced by dividing participants into groups of 8-10 people with at least one „expert" (someone who knew the rules of the game, we can expect to find experts in almost any group) per group, explaining the game through playing it for 15-20 minutes.

Because many teachers did not know the game beforehand, an emphasis was made on giving clear and thorough explanations for each set and to analyze verbally why some sets indeed were not sets. Next, participants were prompted to think for themselves for two minutes about isomorphisms for the game from the presented 2D version to ideas of a 3D version. After thinking each for oneself they were asked to discuss their ideas in pairs for 10 minutes and then share within their group. At the end, all the ideas were gathered and posted on a white board.

Teachers came up with many new ideas including some thoughts regarding blind students like surface texture and material weight. After discussing the results shortly, some prior example 3D-printed ideas were presented and everybody gathered around the pieces to try out the game in 3D. Teachers were pleased to see that the presented 3D version included features that had been among their own ideas. Also, they were excited to see if they could print other ideas that they had come up with at local FabLabs.

At the workshop in March 2019, teachers noted that when working on the lower secondary school level, they would rarely use the group size of 8-10 pupils but rather ask pupils to work on their designs in groups of 2-3. However, they might introduce the game by asking them to make teams of four students each competing against each other, thus not requiring too many decks of Set. One teacher who knew the Set web applet and was familiar with connected classroom technology said that after introducing the game she might revisit it by projecting the applet to the wall and ask students to make a sign as soon as they had found a set. Then she would gather the suggested sets with connected classroom technology and discuss her pupils suggestions in the whole group.



(a) (b) (c)

Figure 3: *Teachers playing the game SET in different versions in Iceland*

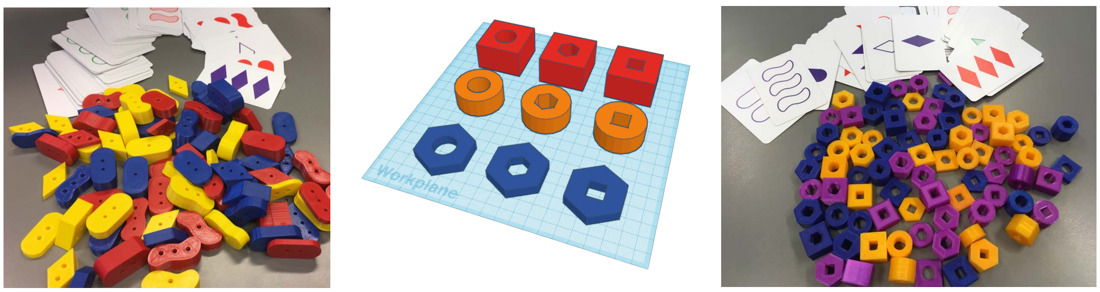
Bearing in mind Lesh, Post, and Behr (1987) suggested a classification of mathematical representations as concrete (manipulatives), language, symbolism (notation), semi-concrete (pictorial), and contextual (real-world situations) [5], we seek to observe which of these different aspects can be identified with this approach. Each representation involves different (levels of) cognitive abilities that we would wish to nurture in our students [5]. Visual forms of representation have been a widely researched topic among different representations in recent years, in part because they are more readily available but also because researchers have found that they play an important role in determining and nurturing problem-solving ability. As Arcavi (2003) states, “Visualization is being recognized as a key component of reasoning, problem solving and even proving” (p. 21) [5]. With this game design task, while the 3D printing favors the hands-on abilities, the use of software, computer-aided design (CAD) as well as Dynamic Geometry System (DGS), have broadened the technological repertoire for designing lessons. Emerging technologies provide new opportunities and call for new approaches; with the use of 3D printers it is possible to create mathematical collections in schools and they also allow teachers to create custom-built manipulatives (Bartholdi et al., 2011, in [3]).

In the sections above, we could observe that the use of digital technologies has the potential to make complex modeling problems more accessible to students and the successful implementation of technology “active” modeling tasks. In addition, such “active” modeling could enable classrooms to utilize enquiry-based approaches enabling its five phases: engagement, exploration, explanation, elaboration and evaluation [2]; together with developing cooperative learning environments (Leiss, 2007 in [4]).

However, to successfully create such classroom learning it is important to develop teachers' expertise, confidence and beliefs about the nature of mathematics learning [1] as well as the integrated technologies and innovative teaching methods. Therefore, our further research will concentrate on the preparation of teachers for these challenges.

Results and Discussions

The game design approach developed in these tasks aimed to involve the participants in a collaborative activity where they could connect with mathematical issues as well as with the challenge of designing digital resources to develop their own physical models. Based on the level of discussions and the multiple solutions that came up, we consider that the experiences tried out were positive in the sense of opening the participants' minds for new possibilities and directions in mathematics teaching.



(a) (b) (c)

Figure 4: *Two different possibilities for mapping the game Set from 2D to 3D. (a) pieces similar to the original shapes, (b) drawing in TinkerCad introducing new shapes, and (c) the corresponding pieces to the sketch in (b)*

From these experiences, we also found that the use of games as a teaching tool can be adapted as much as desired for learning/teaching differentiation. This shows another great value in using games in education. Currently we are considering extending the presented teaching experiments for them to be easier adaptable to different learning environments and classroom settings. One idea is to use more accessible technologies such as smartphones and tablets to complement the activities. Through this, we believe that a greater number of students could be reached and engaged to the learning process described in this paper.

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