Polyhedral Music

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Abstract

When a point moves in 3D space, its path can be described by lists of spatial coordinates. We have tried to hear the music of some regular solids by moving one or more points along their edges and converting the spatial coordinates into auditory ones.

Introduction

Is it possible to hear a 3D shape? If the shape of a regular solid is converted into sound, can we hear similar regularities in the sound as we see in the shape? If we create a musical interpretation of a beautiful polyhedron, is the music produced also beautiful?

A polyhedron can be specified, for example, by drawing its edges. This kind of drawing can be done by moving a point in three spatial dimensions. The movement also happens in the fourth dimension, time.

Specifying a sound also requires three dimensions. However, these dimensions are not spatial, but auditory ones: frequency, volume and timbre.

We introduce here some ways of drawing solids, in other words, different ways in which points can travel along the edges of shapes. We also demonstrate some ways to translate spatial movement into sound. These methods might be useful tools when composing polyhedral music.

The Drawing

A drawing can be made by using one or more points. For instance, octa-, cubocta- or icosidodecahedral edges can be drawn neatly along an Eulerian path through all the edges. In case the polyhedral vertices have odd degree connections, the single moving point must either move through some of the edges more than once, or “jump” to draw all edges. A single drawing point would, naturally, only produce monophonic audio.

A polyphonic result can be achieved by several methods. Let us select, for example, a cube and a plane outside the cube, perpendicular to one of the cubic diagonals. When the plane moves through the cube along the diagonal, the plane at first touches the nearest vertex. This one intersection point immediately splits into three and later into six points. The six points then merge into three points again, and finally the movement stops at the opposite vertex.

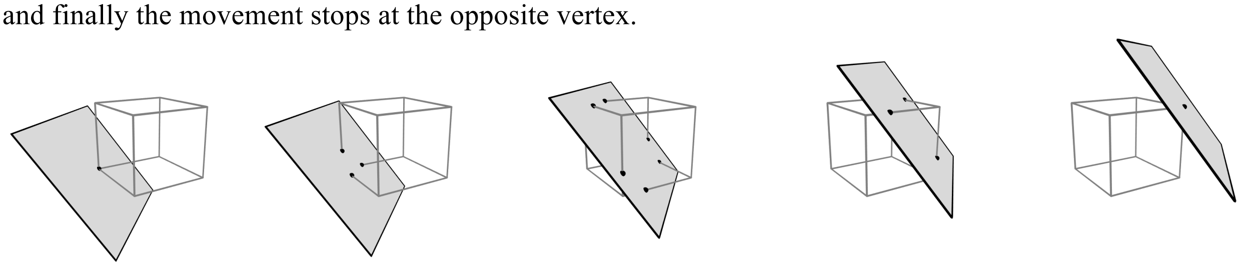

Figure 1: *A cube and an intersecting plane in five positions*

The edges of some polyhedra can be understood as arrangements of "great polygons" or, if the polyhedra are spherical, great circles. The most regular ones are the octahedron, which consists of three squares, the cuboctahedron, which consists of four hexagons, and the icosidodecahedron, which consists of six decagons. A point can be sent to travel around one of the polygons, and as soon as it intersects with another polygon, a new point can be sent to travel around that polygon. This movement of 3, 4 or 6 points can be continuous, and if the speeds of the different points are not the same, but vary irrationally, the same combination of points (the same polyphonic sound) never sounds twice.

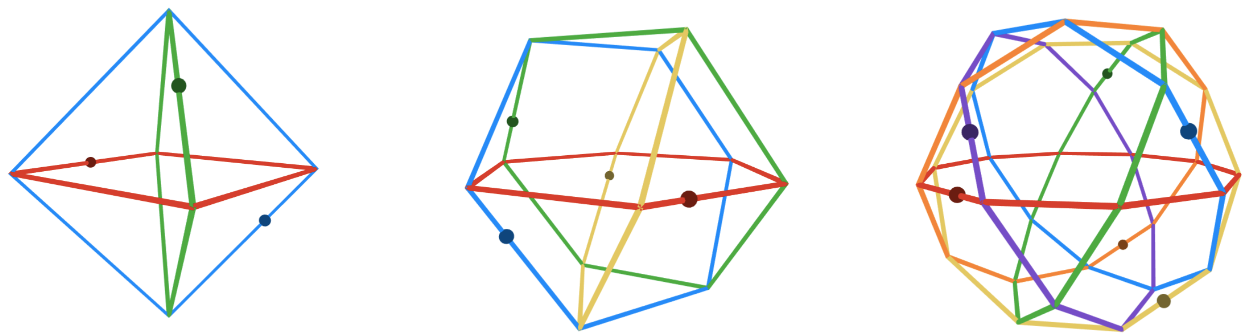

Figure 2: *The flat-faced octahedron, cuboctahedron and icosidodecahedron*

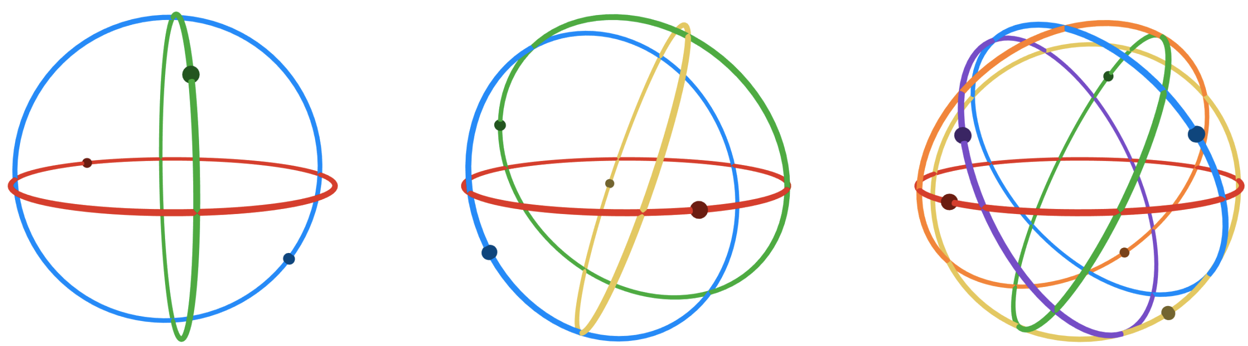

Figure 3: *The spherical octahedron, cuboctahedron and icosidodecahedron*

The described methods of drawing – the Eulerian path, the planar intersections and the great polygon or great circle paths – are only examples. There are an unlimited number of ways to move one or more points along the edges of a polyhedron. The speeds of the moving points may also vary, as may the routes of the points, etc.

The Sound

The 3D movement of the drawing points may be aurally interpreted, as follows:
– The volume of the sound gradually increases when the value of z increases.
– The pitch gradually increases when the value of y increases.
– The timbre varies when the value of x varies, in a way described below.

Unlike pitch and loudness, timbre is multidimensional and not linear. In order to adapt timbre to this demonstration, we selected a specific timbre line in advance. In our video, the timbre line is a sequence of vowels as pronounced e.g. in German. It can be described by the letters a, e, i, o, u, ü, ä, and ö.

The possibilities of different timbre lines are, of course, unlimited. Promising alternatives include a set of orchestral instruments, birdsong or simply synthetic sounds.

When drawing (i.e. moving the points), the movement is likely to be continuous. It is also natural to have a continuous change of volume while the timbre may vary either continuously or step by step. The pitch can vary discretely, as usual in music, for example, in 12 steps per octave or following the major or minor scale.

Examples

The reader may open vimeo.com/polyhedralmusic to see the movements and hear the sounds of the examples described below. This, of course, makes the following written descriptions easier to understand.

At first, let us observe how the sound changes when the point moves.
– When the point moves closer and further away, the volume of the sound respectively increases or decreases.
– When the point moves up and down, the frequency of the sound respectively increases and decreases.
– When the point moves from left to right, the timbre respectively moves from *a* to *ö*.
To summarize, when the point moves along the edges of an octahedron, the sound simultaneously changes in volume, pitch and timbre.

Secondly, let us observe a polyphonic cubic melody according to Figure 1, where a plane moves along one of the diagonals of a cube. In the video, the intersection points are bright. The sound is at first triphonic, then hexaphonic, and finally triphonic again.

Thirdly, let us observe a continuous triphonic octahedral composition, using an octahedron where the edges are considered as three squares.
1: The point starts off on the "north pole". After 90° rotation, when it intersects another square, another point starts to move along that "equator square". When the first point reaches the "south pole" a third point appears and starts to move along the third square. A chromatic scale is played as the points move.
2: As 1, but using a spherical octahedron.
3: As 2, but with the speeds of the points in the ratio 1:√2:√3 and with several rounds to demonstrate how the triphonic composition never repeats itself. Glissando pitch.

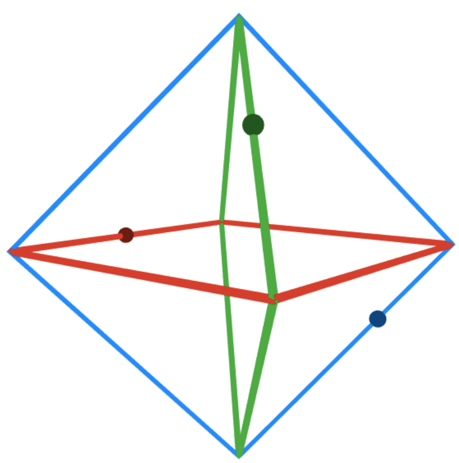

Figure 4: *An octahedron*

Finally, let us finish off with a quadrophonic cuboctahedral composition. It is similar to the previous example, but the points move along the cuboctahedral edges: at first along the hexagons, then along the great circles. The composition is played twice, in C major scale and in C minor pentatonic scale.

And after the end, as an encore, let us hear a hexaphonic icosidodecahedral Grande Finale. It is similar to the previous cuboctahedral demonstration, but is based on a spherical icosidodecahedron, with the sound played glissando, and with the six points moving along the different great circles at different speeds: 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$ and $\sqrt{7}$.

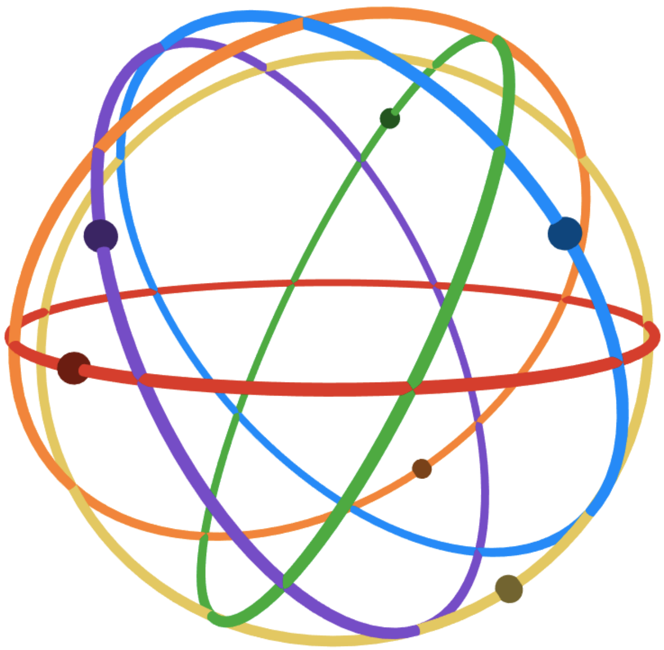

Figure 5: *A spherical icosidodecahedron*

Summary and Conclusions

This paper is just a beginning, and we offer it as a provocation for anyone interested in finding more mathematical relations and decoding methods between geometry and music.

Two questions naturally suggest themselves:
Would it be possible to compose enjoyable music with the method presented above? Or, more precisely, what different kinds of music could be produced by using different shapes?
Can the method be used to study whether any interesting geometric shapes could be found in classical or popular music? For example, what kinds of shapes could be found in the national anthems of different countries?