Symmetry Patterns from Multiple Identically Patterned Cubes

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Abstract

The six faces of a cube can be patterned with copies of a simple motif and its mirror image so that all seven frieze patterns and twelve of the seventeen wallpaper patterns can be produced using a set of patterned cubes. This is illustrated using a twenty copies of a cube patterned with a footprint motif. Compared with a set of tiles of multiple types, a set of such identical cubes simplifies the construction process since there is only one type of manipulative.

Introduction

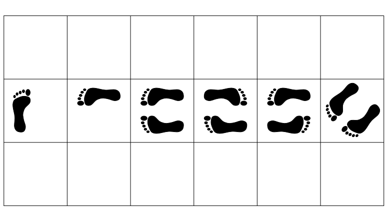
Creating symmetry patterns using a constrained system can be a fun challenge as illustrated in a recent paper about symmetry patterns in the fiber arts [2]. There are seven distinct types of repeating decorative designs on an infinite strip called frieze patterns [1]. Each of the frieze patterns can be created using a square tile decorated with an asymmetric motif and its mirror image tile. With these two types of tiles, one needs a strip two tiles high to create some of the frieze patterns. However, patterning additional tiles with two copies of the motif will allow all frieze patterns to be created with a strip that is just one tile high.

Similarly, there are seventeen distinct types of decorative designs that repeat in two directions on the plane. These are called wallpaper patterns [1]. Twelve of these wallpaper patterns can be created using a square tile decorated with an asymmetric motif and its mirror image tile plus a blank tile. As with the frieze patterns, the addition of more tiles with two motifs can reduce the number of tile types needed to construct a particular symmetry pattern. The remaining five wallpaper patterns require a triangular tile and are not considered in this paper.

This paper explores using a set of multiple identical copies of a specially patterned cube that allows one to create all the frieze patterns and twelve of the wallpaper patterns.

A Patterned Cube

One can decorate the faces of a cube with motifs having the same symmetries as the patterns shown in Figure 1 to construct the seven frieze patterns and the twelve wallpaper patterns obtainable from square tiles.



(a) *A template for creating a patterned cube using a modular method.*

(b) *The template in (a) cut into six strips.*

(c) *Two assembled patterned cubes using the strips in (b).*

Figure 1: *A patterning of the six faces of a cube allowing the construction of all seven frieze patterns and twelve of the seventeen wallpaper patterns as shown in Figure 2. The rotated single foot in the template (a) results in the mirror symmetry seen in the assembled cube (c).*

(a) ∗442 *(p4m)*

(b) ∗2222 *(pmm)*

(c) ∗∗ *(pm)*

(d) 2∗22 *(cmm)*

(e) 4∗2 *(p4g)*

(f) 22∗ *(pmg)*

(g) ∗× *(cm)*

(h) ×× *(pg)*

(i) 22× *(pgg)*

(j) 442 *(p4)*

(k) 2222 *(p2)*

(l) ◯ *(p1)*

Figure 2: *The symmetry patterns constructable from a cube patterned as in Figure 1.*

These wallpaper patterns are shown in Figure 2. Each frieze pattern is either a row or column of one of the wallpaper patterns. The frieze and wallpaper pattern construction is not always unique, which adds fun.

The cube construction from strips was chosen for convenience. When Figure 1(a) is printed in standard letter-sized paper, the resulting cube is 4.4 cm per side. Each strip has two folds which allow the cube to hold together without fasteners. The faces are single units, which allows easy patterning. However, any cube that can be patterned can be used. Likewise, the footprint motif (after Conway et al. [1]) can be replaced with any similar asymmetric motif. Potential applications include decorating toy building blocks, boxes, and other manipulatives. The advantage with a single block containing patterns is that the challenge of finding a particular tile type in a large collection is essentially eliminated; one only needs to rotate a block to find the face pattern needed. Large-scale blocks could also be patterned for large group activities, similar to the pattern blocks used by the National Museum of Mathematics [3].

References

[1] J. Conway, H. Burgiel, and C. Goodman-Strauss. *The Symmetries of Things*. AK Peters Wellesley, MA, 2008.

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[3] C. Lawrence. "Play Truchet: Using the Truchet Tiling to Engage the Public with Mathematics." *Proceedings of Bridges 2018*. pp. 359–362. Available online at http://archive.bridgesmathart.org/2018/bridges2018-359.pdf.