Extensible Structures of Interlinking *SL* Strands

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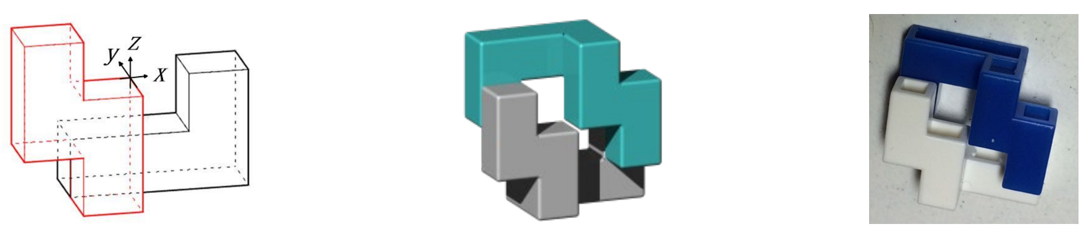
Abstract

An *SL* block is an octocube that can be systematically concatenated into large variations of interlocking structures called *SL* strands. An *SL* strand can be derived with a non-intersecting path in a directional grid that is specified according to the orientation of the initial *SL* block. This study considers using planar *SL* strands to build interlinking structures that may extend to indefinite sizes in one, two and three dimensions.

SL Block, Concatenation and Strand

An *SL* block is an octocube consisting of an *S*-shaped and an *L*-shaped tetracubes attaching to each other along sides (Figure 1a). Two *SL* blocks can be arranged into a conjugate pair (Figure 1b). Conjugate pairs can be consecutively concatenated with other pairs to form *SL* strands. Seven ways to combine *SL* conjugate pairs named as ***h, s, t, d, a, y, u*** concatenation were proposed by Shih [1] and Chou [2]. Among all types of concatenations, only ***h*** and ***a*** concatenations do not change the height of the bottom surface when inserting an additional conjugate pair. This paper discusses interlinking structures of planar *SL* strands that can extend in one, two and three dimensions by using ***h*** and ***a*** concatenations of *SL* conjugate pairs exclusively.



(a) (b) (c)

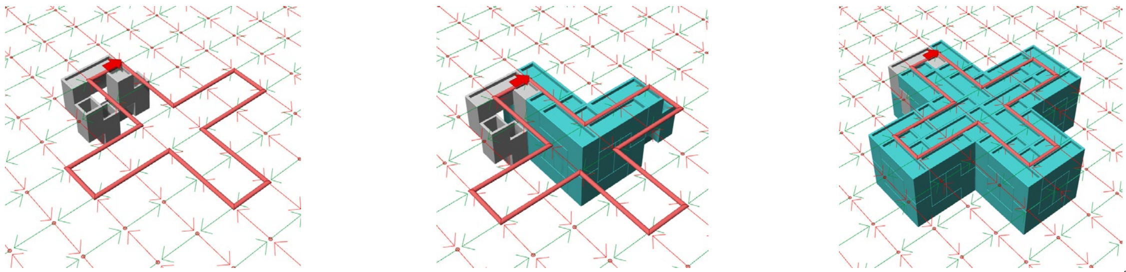
Figure 1: *(a) An SL block, (b) a conjugate pair, (c) photo of a conjugate pair*

Figure 2 shows ***h*** and ***a*** concatenations of *SL* pairs. The initial pairs are shown as half transparent on the left hand sides and the add on pairs are shown in cyan on the right ends of Figure 1a and 1b. Rotations and translations are represented as $\boldsymbol{R}_{x(angle)}$, $\boldsymbol{R}_{y(angle)}$, $\boldsymbol{R}_{z(angle)}$ and $\boldsymbol{T}_{(x\ y\ z)}$ respectively. The ***h*** concatenation makes a straight forward extension of the planar *SL* strand, while the ***a*** concatenation introduces a 90 degree right or left turn, depending on whether the preceding conjugate pair is installed with the up-right or up-side-down orientations. The concavities on top surfaces of rendered images and photos are designed for injection-molding production and can be used as visual references to the orientations of *SL* blocks.

(a) ***h*** concatenation: $\boldsymbol{R}_{x(180)}\boldsymbol{T}_{(2\ 0\ 0)}$ (b) ***a*** concatenation: $\boldsymbol{R}_{z(-90)}\boldsymbol{T}_{(1\ -1\ 0)}$

Figure 2: ***h*** *and* ***a*** *concatenations of SL conjugate pairs*

Consecutive applications of ***h*** and ***a*** concatenations result in a planar *SL* strand that follows a non-intersecting path in a directional grid defined by the orientation of the initial pair. Figure 3 shows the grid and the construction process of one *SL* strand with its corresponding path in the grid. An *SL* strand is represented as its generating sequence of concatenations. Ends of strands may join to form closed loops.

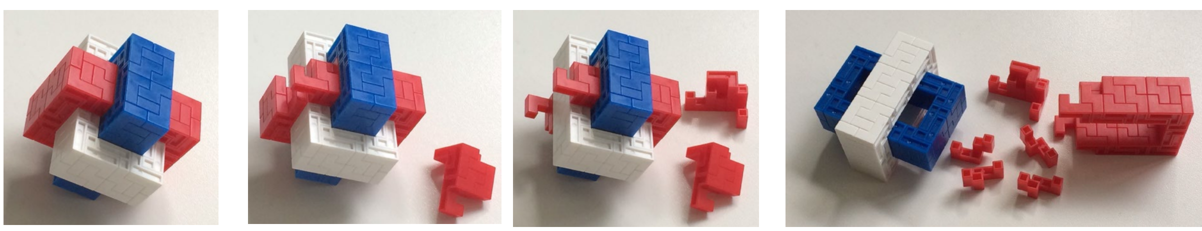


(a) The initial pair and the path (b) The *SL* strand ***ahaha*** (c) The *SL* strand ***(ahaha)***^{4}

Figure 3: *The directional grid, the traversing path and the corresponding SL strand*

Interlinking of *SL* strands

Multiple strands can be arranged into interlinking configurations. In this study, interlinking is defined as the situations that no one strand can be removed without breaking any strands. Figure 4 shows three interlinked Borromean rings built with 96 *SL* blocks and the process of disassembly. The structure is considered interlinked because none of the three rectangular strands can be removed without breaking any strands. The only way to disassemble the structure starts from taking out 2 of the 12 corners, with 4 joining *SL* blocks as one whole each time (Figure 4b, c and d).



(a) (b) (c) (d)

Figure 4: *The dissembling process of Borromean rings*

One-dimensional extensible structure

Interlinking structures with one dimensional extensibility means that with repetitive patterns, the structure may extend its size along one dimension indefinitely, and only blocks located at the ends can be removed without breaking other blocks. The example shown in Figure 5 uses multiple square loop strands and a long strand as a center core to penetrate all loops and lock at both ends. To expand the length of the structure simply add more square loop strands and increase the length of the center core. Traversing paths of *SL* loop strands are shown in red.

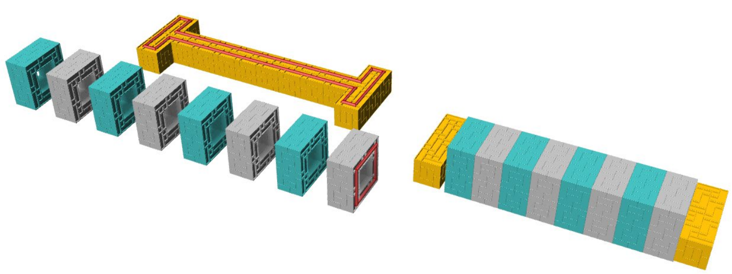
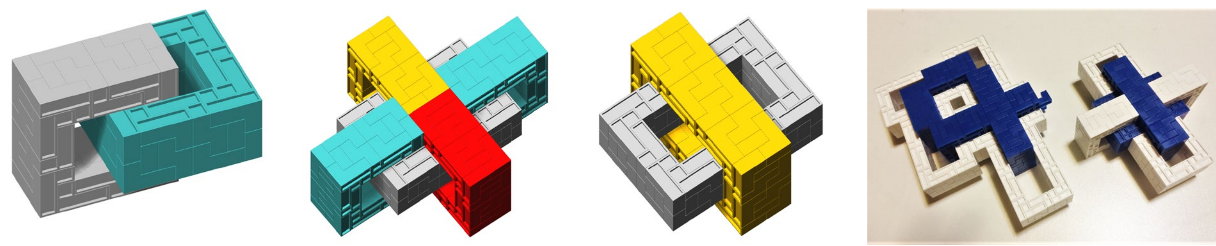


Figure 5: *An interlocking structure with one-dimensional extensibility*

Two-dimensional extensible structure

Two types of structures with two-dimensional extensibility are proposed. The first type is called "chain mail". It consists of a network of interlinking loops formed by rectangular *SL* loop strands. In the simplest case, two neighboring loop strands are properly interlinked by passing one edge through the hole of the other loop strand (Figure 6a), or four other loops may interlink with a suitably elongated loop strand (Figure 6b). Alternatively, two stretched loops may be entangled as in a Borromean configuration (Figure 6c). The configuration 6b leads to the assemblies shown in Figure 7. Those two assemblies have a different pattern of the horizontal gray loop strands.



(a) Interlinking by one edge crossing (b) Four interlinking loop strands (c) Borromean configuration (d) Structures under construction

Figure 6: *Interlinking loops as the basic configurative pattern of chain mail structures*

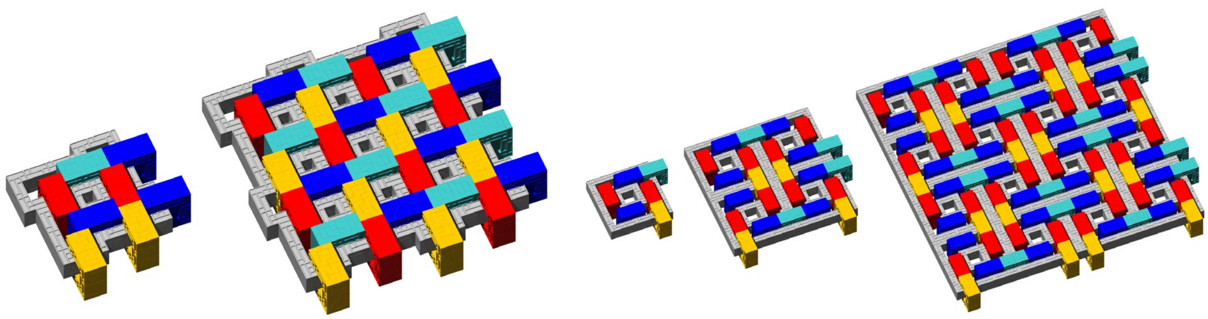
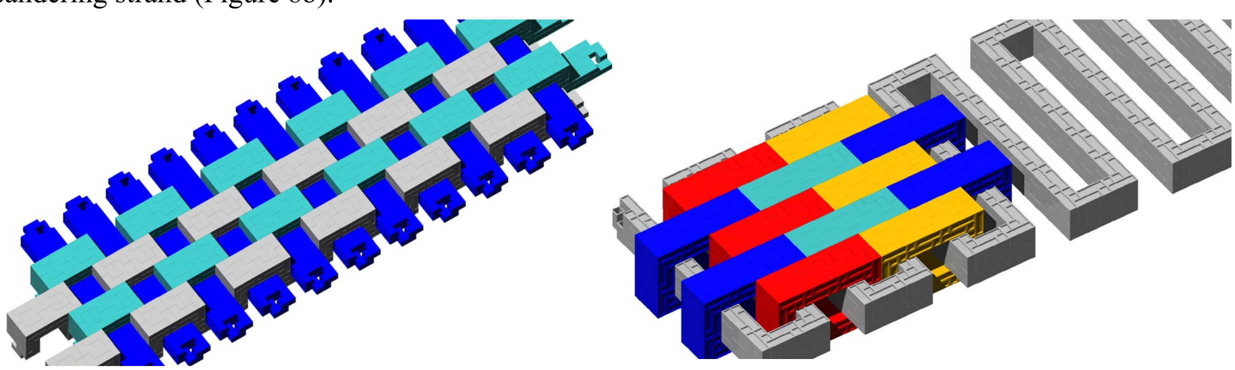


Figure 7: *Two examples of extensible chain mail structures*

The second type of structure with two-dimensional extensibility is a type of weaving. The simplest weaving takes a group of straight strands as warp and a second group of strands, meandering up and down, as weft (Figure 8a). A second weaving example uses a very long meandering strand that travels back and forth across the weaving area. Elongated rectangular loop strands are used to link neighboring tracks of the meandering strand (Figure 8b).



(a) blue strands: $\boldsymbol{h^{12}}$, cyan and grey strands: $\boldsymbol{(ah^2ahah^2ah)^6}$

(b) grey strands: $(\mathbf{ah}^2\mathbf{ah}^{11})^{22}$, blue/cyan/red/yellow strands: $(\mathbf{ah}^2\mathbf{ah}^4)^2$

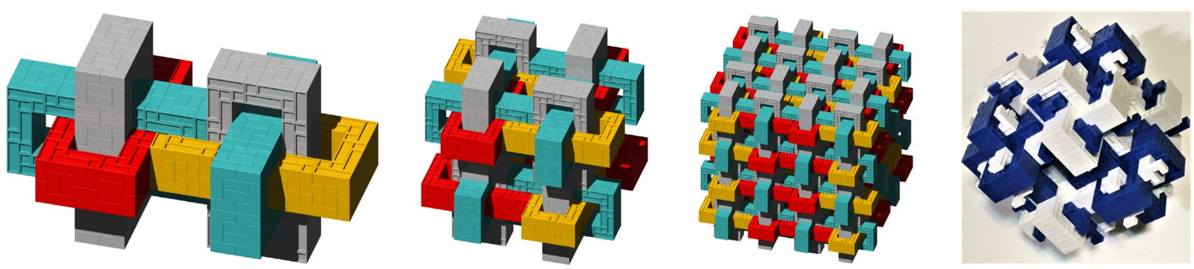
Figure 8: *Two examples of weaving structure*

Mosaic art can be created by properly arranging colored *SL* blocks in a weaving configuration. Figure 9 shows mosaic portraits of Audrey Hepburn and Gregory Peck, who acted as leading roles in the 1953 film "Roman Holiday". 14584 interlinking *SL* blocks were assembled into 304 interlinking loop strands to make the panel.

Figure 9: *Mosaic portraits built with interwoven loop strands*

Three-dimensional extensible structures

Coupled structures with three-dimensional extensibility may use elongated interlinked loop strands pointing in all three dimensions. A good starting point is the Borromean arrangement of three rings. All rectangular loop strands are stretched some more, so that neighboring Borromean clusters can interlink with one another in all three dimensions (Fig.10).



(a) Two sets of linking Borromean rings with ring strand $(ah^2ah^6)^2$ (b) 8 sets of linking Borromean rings (c) 64 sets of linking Borromean rings (d) Borromean rings array in construction

Figure 10: *Three-dimensional arrays of Borromean rings sets*

Summary and Conclusions

Examples shown in this study use one single element to build integrated structures that in principle can be extended into indefinite sizes in one, two and three dimensions. These structures derive their beauty through the integrity of geometrical forms and the intricate but simple relationships among the constructing elements that make the whole more than the sum of parts. Two types of concatenations are used in this study. Studies of the other five types of concatenations may further explore the potential of building extensible structures with non-planar *SL* strands.

References

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[2] L.W. Chou. " The Study on SL-Blocks." Master Thesis, Department of Applied Mathematics, Chiayi University, Chiayi, Taiwan.