Sombrero Vueltiao – Weaving Mathematics

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Abstract

This paper describes the weaving technique of a traditional Colombian hat called *sombrero vueltiao* and some of its patterns. The making of the hat–and weaving in general–involves interesting mathematical thoughts and concepts and offers many learning opportunities, for all of which we provide examples in this paper.

Introduction

The *sombrero vueltiao* (shown in Figure 1a) is a traditional hat from the Colombian regions of Córdoba, Sucre and Bolívar. It is originally made out of *caña flecha* (a type of cane of the region) by the Zenú, an indigenous tribe from the valleys of the Sinu and San Jorge rivers in the north west of Colombia. The hat is traditionally worn by *cumbia* and *vallenato* artists, two popular Colombian folk music genres. It is one of the main handicraft products of Colombia (see [7]) and since 2004 an official national cultural symbol [1].

(a) (b) (c)

Figure 1: *(a) a sombrero vueltiao made out of 19 pairs of strands, (b) inside of a sombrero, (c) pair of strands illustrated with paper*

The quality of the hat depends on the quality of fibres that are being used and on the number of pairs of fibres/strands woven together where one pair usually consists of one black and one white strand put on top of each other (see Figure 1c). In general, the Zenú use a minimum of 15 and a maximum of 27 pairs of strands. The most common qualities are the Quinceano (15 pairs of strands, takes about 3 days to make, and is the cheapest one), the Diecinueve (19 pairs of strands, finer, takes about one week to make), and the Veintiuno (21 pairs of strands, 10-15 days, costs between 100 and 400 US Dollars). A sombrero vueltiao is made by first weaving a long braid which is then sewed together in circles. It's name sombrero *vueltiao* originates in the Spanish word *vuelta* which means *turn* and describes exactly the way the hat is made.

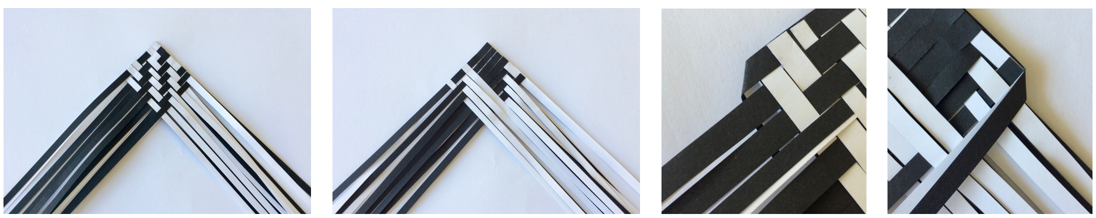
Weaving and Mathematics

Weaving arts can be found in many different cultures. Paulus Gerdes, a Dutch mathematician and one of the founders of ethnomathematics research, wrote about *interweaving arts and mathematics in Mozambique*

[2]. The art of weaving involves concepts from different mathematical branches, such as: geometric shapes, angles, patterns, tilings, symmetries (see [3]), combinatorial thoughts, such as counting patterns (we will give examples in the next section), and modelling and representations. After deciding on a pattern, the craftsperson has to find out how to weave it. Representations for the patterns have to be developed and restrictions for the weaving have to be considered. Through trial and error, the weaver creates the desired pattern–a modelling process takes place. Because of those diverse mathematical topics and its tangibility, weaving is used as an activity in mathematics education (see [4]).

Weaving Techniques and Patterns of the Zenú People

The technique the Zenú use to make the braids for the sombrero vueltiao can be seen as a mixture between weaving and braiding. There are always two groups of strands that are perpendicular to each other–like in weaving–but the strands cannot be divided into two fixed groups (in weaving called warp and weft) that always stay perpendicular to each other. You start with two groups of perpendicular pairs of strands (see for example Figure 2a) but during the weaving process the strands change direction several times (like in braiding) and the path of a strand bends at a right angle. But unlike in braiding, the strands are not flexible. The Zenú call the long woven part *trenza* which means braid.



(a) **(b)** **(c)** **(d)**

Figure 2: *(a) possible starting configuration with 15 pairs of strands, (b) possible back side of starting configuration shown in (a), (c) folding away at front side, (d) corresponding back side*

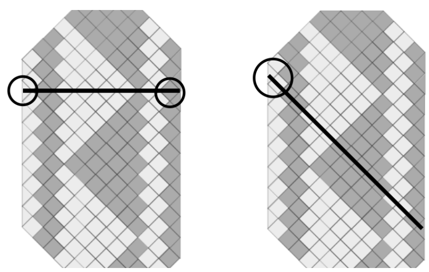
An interesting mathematical question is how many *starting configurations* and patterns are possible? Let's say the number of pairs of strands is n, where n is odd. The groups consist of $\frac{n-1}{2}$ and $\frac{n+1}{2}$ pairs of strands respectively. We assume that in one group the black side is at the top and in the other group the white side, since this is what you find in the patterns of the Zenú. One then has to decide which strand to put at the top for every two perpendicular (pairs of) strands. For the starting configuration, we have to put all n pairs of strands into place, which leads to an area of $\frac{n-1}{2} \cdot \frac{n+1}{2}$ squares, where a square is the overlapping of two perpendicular strands. That gives us $2^{\frac{n-1}{2} \cdot \frac{n+1}{2}} = 2^{\frac{n^2-1}{4}}$ possible starting configurations. These are only the possibilities for the front side. Since we are weaving with pairs of strands, you can get different patterns at the back side. For example, for the starting configuration shown in Figure 2a one can get the pattern shown in Figure 2b. For every starting configuration on the front side, we can get all $2^{\frac{n^2-1}{4}}$ patterns on the back side of the braid as well. In total that gives us $2^{\frac{n^2-1}{2}}$ possible starting configurations of which some are not interesting, or not even woven. Restrictions for the patterns will be discussed later on.

After all n pairs of strands have been put into position, the pair of strands on the far left is folded such that its path bends at a right angle (see Figure 2c). The white strand, which is now at the top, will then interweave with the black strand from the unbent pairs, and the black strand will interweave with the white strand from the unbent pairs. One then takes the pair of strands furthest to the right and continues this process, alternating left and right sides. You can decide in which direction to fold the pairs of strands. If you fold away from yourself, the color of the triangle at the border (half of a square) will be the one which was on top (see Figure 2c), and when folding in the other direction the triangle is the color which was on the other side. For the

(a) **(b)** **(c)** **(d)**

Figure 3: *Typical patterns one can find on (the brim of) a sombrero vueltiao: (a) made out of paper, (b) back side of (a), (c) and (d) same patterns appearing in a sombrero vueltiao*

corresponding triangle on the back side, you always get the same color as on the front side (see Figure 2d). Following the starting configurations shown in Figure 2 one gets the patterns shown in Figure 3. It is not possible to create patterns, where all white strands travel diagonally from left to right and all black strands from right to left, when using a usual weaving technique. It only works with this double weaving method where one black and one white strand are put on top of each other and the pairs are then grouped by color.



(a) **(b)**

Figure 4: *(a) width of a braid with a triangle on the left and a square on the right, (b) one and a half squares at the beginning of the diagonal*

Of course, not every possible pattern can be found in the artworks of the Zenú. One can observe, that they usually fold in the same direction (from front to back). The border of a braid consists not only of triangles, but of a triangle on one side and a square on the other side, which is $1\frac{1}{2}$ squares in total (see Figure 4a). To get a regular pattern at the border, the Zenú always weave over (or under, depending on the side and folding direction) exactly one strand after folding to get $1\frac{1}{2}$ squares of the same color in the beginning of a diagonal (see Figure 4b). This is the case for both sides, on the left of the back side you get a white border and on the right a black border (at the front side it is the other way around). To assure, that the braid is actually woven and not completely loose, the color of the next square in the diagonal direction should be different from the first $1\frac{1}{2}$ squares. That means, that $1\frac{1}{2}$ squares on one side and 1 square on the other side have a fixed color. In many patterns of the Zenú, you actually find that the color of the next two squares is different from the first $1\frac{1}{2}$. To count the number of possible patterns for a given braid length, we consider the width of a braid (the number of squares from one side to the other), which is always $\frac{n}{2}$ (because you get one square for every two intersecting strands plus half a square at one border, which is $\frac{n-1}{2} + \frac{1}{2}$). Since we assumed that $2\frac{1}{2}$ squares have a fixed color, we have $\frac{n-5}{2}$ squares in each row for which we can choose the color. For a given braid length of l rows, we therefore get $2^{\frac{(n-5)\cdot l}{2}}$ possible patterns. Usually, one finds 6 different patterns on a sombrero vueltiao: the two patterns I present in this paper and 4 out of over 50 different patterns which the Zenú call *pintas* (symbols). The pattern shown in Figure 3d is a two-color frieze pattern of type *mg* (vertical reflection symmetry and two-color glide reflection, see [5]). The translation unit is the parallelogram consisting of a white and a black triangle. It repeats after $2 \cdot (n - 8)$ rows (the length of the short side of a triangle is $n - 8$, since a diagonal consists of n squares, of which 7 belong to the border and one square belongs to the next triangle of different color). Since the generating region (half a triangle) consists of $(\frac{n-7}{2})^2$ squares, we get $2^{(\frac{n-7}{2})^2}$ possible patterns with this type of symmetry and same length of repeat. This pattern appears at the top of the hat and on the inside of the brim. When weaving this pattern, the Zenú always use the pattern shown in Figure 3c on the other side of the braid. It contains a two-color plane symmetry of type *pgg* (two-fold rotation and two-color glide reflections, see [5]). It is usually also used at the inside of the crown of the hat (see Figure 1b), where at the outside one can find the *pintas*.

When we subtract the $1\frac{1}{2}$ squares from the border, we get that $n = 2N + 3$, a formula stated by Benjamín Puche Villadiego [6], where N is the number of squares in the middle part of the braid[1]. This formula only works when n is odd. N is then always the same integer for every row. If n is even, the number of squares at the border, and therefore the number of squares in the middle part, change: In every second row you get two squares and therefore $\frac{n-4}{2}$ squares in the middle. In the other rows you get only two triangles at the border and $\frac{n-2}{2}$ squares in the middle part. This explains why the number of pairs being used is always odd (to get those regular patterns). Furthermore, if $n = 4k - 1$, with $k \in \mathbb{N}$, which is the case for most of the numbers of pairs being used (the Veintiuno is the only exception), N is always even. Benjamín Puche Villadiego stated the formula to find out the quality of a hat (which is given by n), since it is easier to count N than the actual width (because the braids are sewed together and generally $1\frac{1}{2}$ squares are not visible).

Conclusions and Outlook

The sombrero vueltiao provides a lot of interesting and beautiful mathematical activities of which I could give only a few examples in this paper. Engaging with weaving leads to mathematical thinking and to beautiful artwork. The patterns and technique of the Zenú are not at all random, but are due to mathematical properties. During the presentation further patterns of the Zenú and mathematical questions/concepts they involve will be presented.

Acknowledgements

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[1] Villadiego actually gave the formula n=2(N+1)+1, which is of course the same.