Escher's Polyhedral Models

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Abstract

Dutch graphic artist M.C. Escher (1898-1972) had a passion for order and symmetry and sought to express this in several of his works. He studied regular polyhedra and their many variations: duals, compounds, stellations, and more. He sought information from his half-brother, geology professor Berend, and in books, but to really understand these tantalizing bodies, made models that could be held, rotated, and viewed from many angles.

Dedication: *This paper is dedicated to the memory of George A. Escher (1926-2018), eldest son of M.C. Escher, for his friendship, generosity, and encouragement over the last 30 years as I sought to learn, understand, and communicate the astonishing work of his father.*

Order and regularity

Order and regularity were passions of the Dutch graphic artist M.C. Escher (1898-1972). In 1965, in accepting the Culture Prize of the City of Hilversum he confessed, "The laws of the phenomena around us—order, regularity, cyclical repetitions, and renewals—have assumed greater and greater importance for me. The awareness of their presence gives me peace and provides me with support. I try in my prints to testify that we live in a beautiful and orderly world, and not in a formless chaos, as it sometimes seems." [7, p.21], [12, p.102] His 1947 print *Crystal* (Figure 1) was his first testament to this thought.

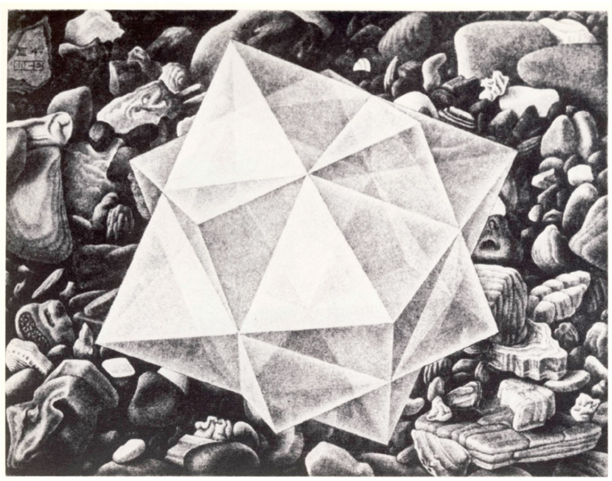

Figure 1: *M.C. Escher.* ***Crystal****. 1947, Mezzotint.*

Just after finishing the print, Escher wrote to his friends Bas and Len Kist in December 29, 1947, "I have called it *Crystal* because 'Order and Chaos' sounds too ponderous, though that title actually gives a better indication of my intention. Let me add that Chaos is present everywhere in countless ways and forms, while Order remains an unattainable ideal: the magnificent fusion of a cube and an octahedron does not exist. Nevertheless, we can hope for it." [1, p.60] Escher's beautiful crystal-like depiction of the dual cube and octahedron was the first of four times that he displayed this compound in his prints. Nineteen of his prints depict regular polyhedra and/or their variations, drawn as solids or as Leonardo-style cages.

Escher's interest in regular polyhedra, their symmetries, dualities, compounds and stellations likely began around 1935 when he and his family moved from Rome to Switzerland, leaving behind the possibility of continuing his drawing excursions in the Italian countryside. That same year his half-brother Berend (Beer), a professor of geology, produced a book on mineralogy and crystallography [4] and also introduced him to some crystallographic literature [10]. Escher was astonished at the beauty of crystals, and wrote, "Long before there were people on the earth, crystals grew in the earth's crust. Then came a day when, for the very first time, a human being perceived one of these glittering fragments of regularity, or maybe he struck against it with his stone ax, it broke away and fell at his feet, then he picked it up and gazed at it lying there in his open hand. And he marveled." [5]

Escher was aware that much of the public would not appreciate his depiction of abstract mathematical objects. Yet he defended this work, "If you asked me why I am creating such nonsense—something so objective with nothing personal in it—I can only say that I can't leave it alone. As far as I know, this problem [of drawing polyhedra] was worked on circa 1500 and 1600 by Dürer, Pacioli, Barbaro, and even Leonardo, and no one solved it in a satisfactory manner. They probably were as interested as I am—the beauty and the order of regular solids are overwhelming. Nothing can be done about these forms because they are simply there." [1, p.146], [6, p.52], [12, p.184]

Escher's models

Escher was a keen observer and could draw the most intricate details found in nature and architecture, but lamented his inability to just imagine an object in order to draw it. "Yes, I am quite incapable of drawing, even the more abstract things such as knots and Möbius rings, so I make paper models of them first and then copy them as accurately as I can." [2, p.68]. His models of polyhedral forms were essential for making prints that depicted these forms and for producing carved wooden sculptures with precise polyhedral symmetries. For his print *Flatworms*, he made marble-sized plasticine tetrahedra and octahedra to stick together to form the figures seen there [2, p.97]. He also designed a flat net for an icosahedral metal candy tin, decorated with shells and starfish [1, p.151], [2, p.60].

Pictures of four of Escher's paper "star" models are in [1, p.146] — a stella octangula, a stellated dodecahedron, a compound of five tetrahedra, and a weaving of starfish-like pentagrams. Other pictures of his models are in [1, p.72], [2, p.69, 94], [3], [10, pp. 246-47], [11, p.188]. Two, and likely several more geometric models were inspired by photos in Max Brückner's classic work, which Escher cites on his color drawings of the compounds of three cubes and of five tetrahedra [8], [12, p.99], [10, p.247]. Escher carved a beautiful wooden sculpture of twelve flowers derived from his model of the compound of five tetrahedra (Figures 2, 3). On that model, (Figure 2, left), you can see his penciled curves of leaves on five faces that, in the carved form, would become a calyx, replacing the sharp edges of the model.

Figure 2: *(Left and far right) Escher's cardboard model of the compound of five tetrahedra. (Center) Escher's **Polyhedron With Flowers**. 1958, Maple, diam. 6 5/16 in. Collection Dr. Stephen R. Turner.*

Figure 3: *(Left) Escher explains his model of five intersecting tetrahedra to a student. (Center) Escher holds his related sculpture; his stick model hangs above. Photos by Bruno Ernst. [3] (Right)* ***Polyhedron with Flowers*** *carving, showing its rotational dodecahedral symmetry. Collection Dr. Stephen R. Turner.*

In a slide lecture, Escher described the model and sculpture seen in Figures 2 and 3: "…[the model] presents twelve stellated concavities, each made up of five faces, which remind one of the leaves of a flower. It was this association that suggested to me the idea of making a wood sculpture. It presents twelve calyxes, which are like the twelve pentagons of a regular dodecahedron. In the center of each calyx is a pistil with stamens. These twelve pistils are the points of a regular icosahedron." [7, p.69]

An Enigmatic Print

Early in 1961, Escher received a package of *Geo D-Stix*, colored sticks with plastic connectors, recommended to him by Arthur Loeb, an admirer from the Electrical Engineering Department at M.I.T. [9]. Using these, Escher built a nest of the Platonic solids and hung it from the ceiling of his studio. Outermost was a red icosahedron with a yellow dodecahedron in dual position, then a blue cube with green octahedron in dual position, and finally, a black tetrahedron inscribed in the cube (Figure 3, center; Figure 4, left). In May of that year, he completed the print *Four Regular Solids* (Figure 4, right).

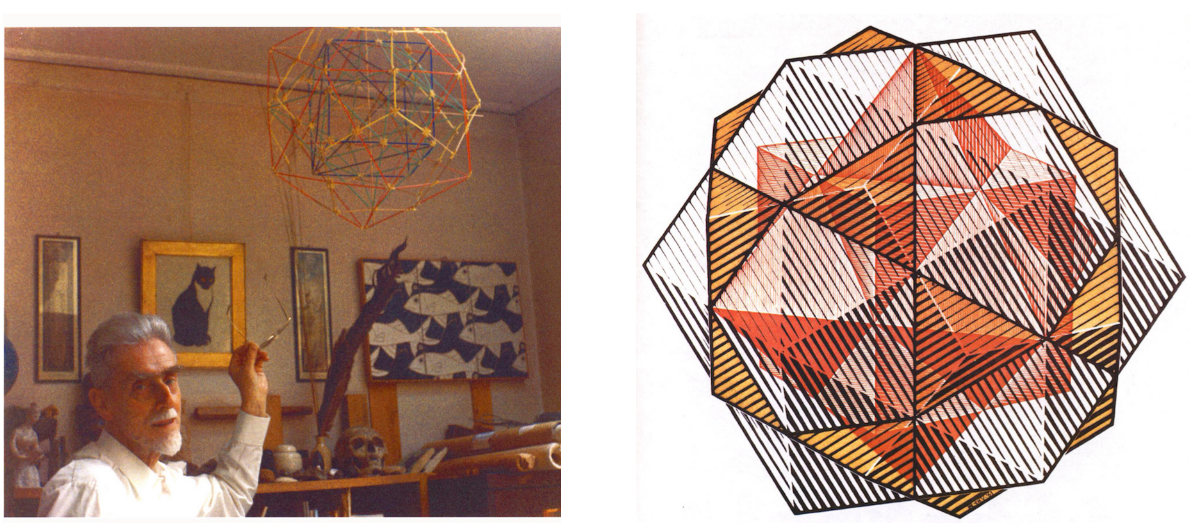

Figure 4: *(Left) Escher in his studio with his stick model of nested Platonic Solids. Photo by Bruno Ernst. [3] (Right) M.C. Escher.* ***Four Regular Solids.*** *May 1961, Woodcut.*

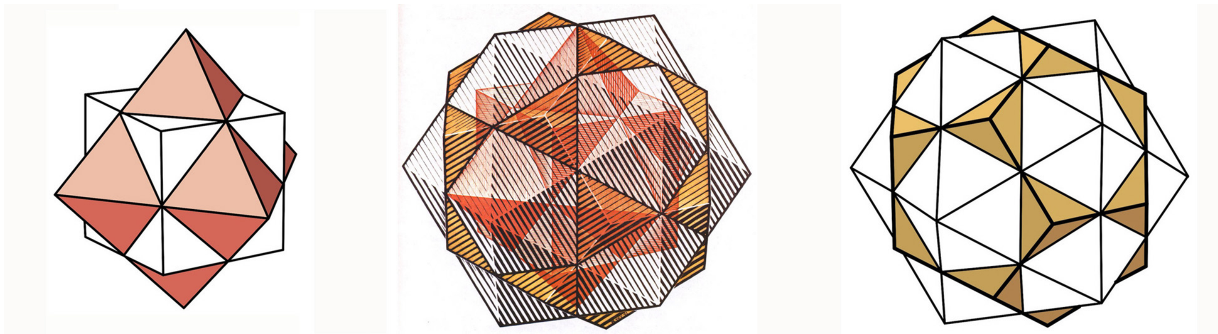

Figure 5: *Escher's print parsed to show the two separate compounds of dual solids.*

He wrote, "I worked my fingers to the bone for more than a month to construct, in one print, four regular solids with 6, 8, 12, and 20 faces. … Of course, I wanted to include number five, the tetrahedron, but that was too much —not for me but for the 'observer.'…This is definitely a print that the public will have trouble understanding and therefore won't generate many sales. But I am pleased with it." [6, p.52]

In the print, Escher clothed his stick models to become solids. To an uninitiated viewer, the print is an enigma; only its title hints what it depicts. To parse it, focus on just one color. Except for the compound's outline and where the icosahedron pierces the dodecahedron, Escher does not outline edges of the solids in black. Most edges of the dodecahedron and the icosahedron are thin white lines, while most edges of the fused cube and octahedron are defined by the shading on their faces. Figure 5 parses the figure into its two separate compounds, with Escher's color on only one of a compound's two solids. Escher invites the viewer to look closely, more than once, to discern his print's often hidden surprises.

Acknowledgements

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