A Minimal Art Object with Six Horses in a Carousel

Walt van Ballegooijen[1] and Hans Kuiper[2]

[1]Artist, Wijk en Aalburg, The Netherlands; waltvanb@xs4all.nl
[2]Artist, Bunnik, The Netherlands; hans.kuiper@net.hcc.nl

Abstract

This paper describes our next step in the development of math objects with hidden graphics. It is a follow up on our minimal art objects with the three faces of Gödel, Escher, and Bach, and the four faces of The Beatles. Again we use a low resolution optical minimal art technique. This year we made an object with the silhouettes of six horses. We describe the shape of a suitable unit cell, the shape of the object composed with these unit cells, and the shape of the grid that shows the images. Finally we explain several ways to display the object.

Starting Point

Our earlier contributions in creating minimal art objects with hidden graphics are described in [1,2,3,4,5]. We developed the GEB-object showing three images and the Beatles-object showing four images. Our next question was: Is it still possible to further increase the number of images? We answered that question by developing the minimal art object displayed in Figure 1. But we suspect that its six images are about the maximum number possible, a further increase will lead to images that are too vague to be recognized.

Our challenge was again to design a cell with the right structural and optical properties. Each cell must contribute one tile to six different images and moreover all these cells must be connected in one minimal art object. The object we created has the shape of a trapezium. It has 361 cells, all lying in a flat surface. We designed our object in Rhino [6] and had it printed by Shapeways [7].

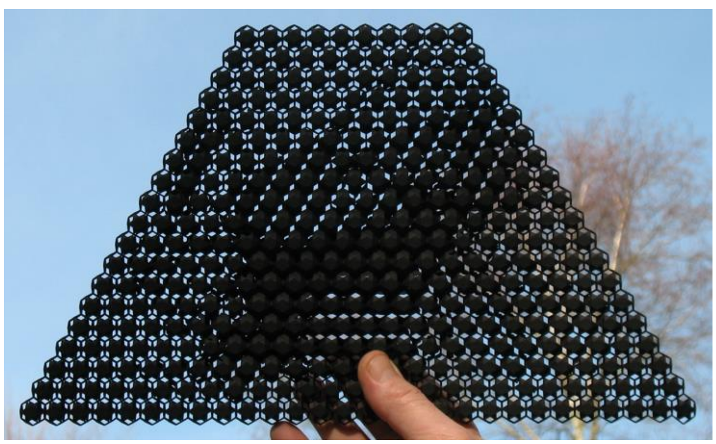


Figure 1: *Minimal art object with 6 hidden images.*

The Design Process

We searched for a suitable unit cell with 6 viewing directions. Existing Platonic or Archimedean solids did not fulfil our demands. A specially faceted hexagonal prism was designed to optimize the optical properties, by maximizing the area of the viewing holes. The cell is derived from the optical requirements. Six "tubes" with a square cross-section are designed to act as viewing holes in the six viewing directions (see Figure 2a). The area of these cross-sections is maximized, while avoiding to create intersections at the ends. All tube axes intersect each other in the centre of the cell. Each axis intersects the ground plane at an angle of $\arctan(1/\sqrt{2}) \approx 35.26°$. The convex hull of the 6 tubes leads to the final cell shape (see Figure 2b). This unit cell has 32 surfaces: 2 regular hexagons (on top and bottom), 12 squares (viewing holes), 6 hexagons (around for structural purpose) and 12 triangles.

The cells are connected by their lateral hexagons (see Figure 2c). The optical tiling, seen at the right angle, leads to a regular square grid of holes (see Figures 2d en 2e). So the unit cells are perforated perpendicular to the squares to simulate the white areas in the image. The area of these holes related to the total area is shown in Figure 2f (dotted lines indicate the outlines of the tiles that subdivide the image).

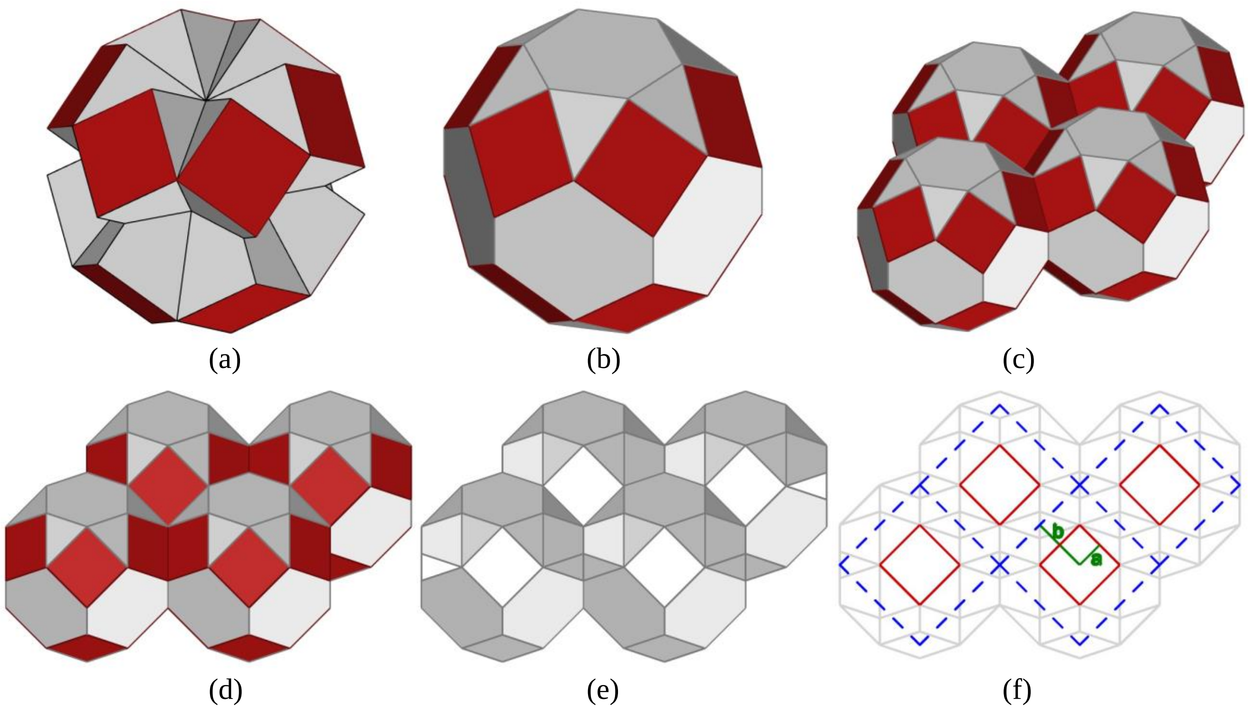


Figure 2: *The design process in steps: (a) six intersecting "tubes", (b) convex hull gives unit cell, (c) four connected unit cells, (d) view from one of the six viewing directions, (e) with maximal open holes, (f) detail showing the overlaps; a:b=1:2 so* $a^2{:}b^2=1{:}4$.

We consider our object as minimal if and only if every hole in the image is created by only one unit cell in the object. That means that in the viewing directions every single unit cell is visible, and no unit cell is hidden behind another.

It was not possible to connect the cells beyond the flat surface without losing the minimal art property. So the structure we made is a straight forward flat slab of unit cells in a hexagonal grid (see Figure 3). We could have decided to create an object with freely chosen borders, but we preferred to display the object in such a way that from one viewing point one can see all the 6 hidden images (1 direct view and its 5 reflections) with the help of two mirrors, meeting in a 60° angle. An equilateral triangle was the logical shape for that, but we chose the shape of a trapezium because of the maximal dimensions of the bounding box of the 3D-printing machine. The trapezium is taken from this equilateral triangle. That provides a seamless fit with the mirrors.

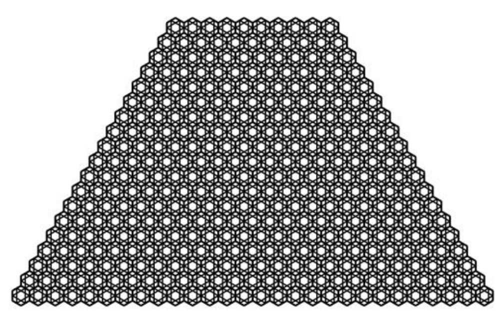
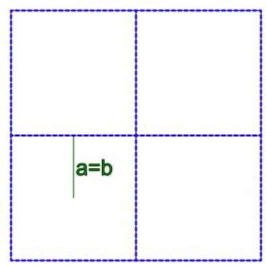
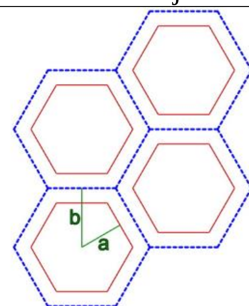
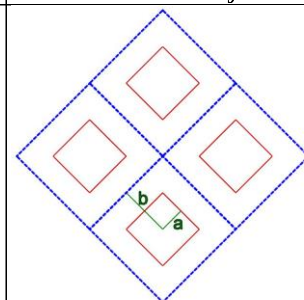


Figure 3: *Trapezium of unit cells, seen from above.*

Optical Minimal Art Graphics

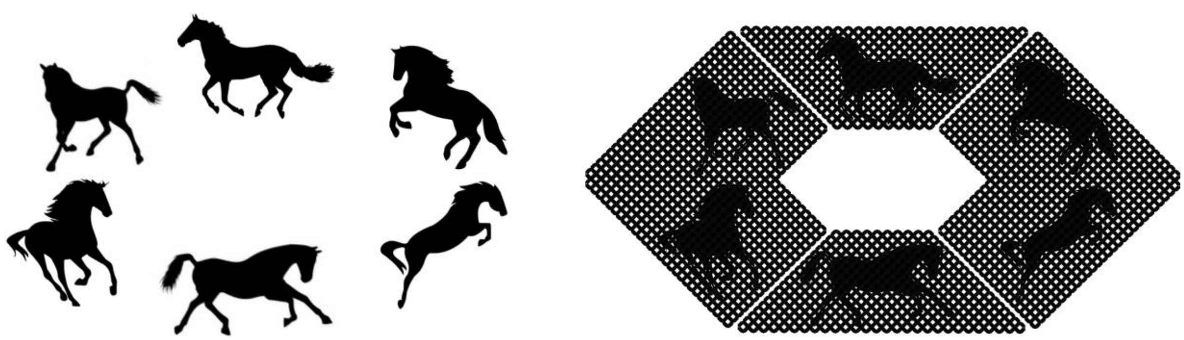
In our "GEB-object'' we displayed 3 images (in 3 dimensions) and we could use 100% of the available area for optical minimal art imaging. We define Transparency as the ratio (Available Area/Total Area). In our "Beatles-object" we displayed 4 images and the Transparency was only 56.25%. In our "Six Horses-object" with 6 images the Transparency goes down to 25%. These three observations lead to Walt's conjecture: Transparency = (Dimensions/Images)^2, or shorter for 3 dimensions: T = $(3/n)^2$, see Table 1.

Table 1: *Walt's conjecture about maximal available Transparency.*

	GEB-object	Beatles-object	Six Horses-object
a = available b = total			
n = number of images	3	4	6
T = Transparency	$(3/3)^2$ = 100%	$(3/4)^2$ = 56.25%	$(3/6)^2$ = 25%

For this piece of art we decided to use black figures on a white background. Images with different shades of grey would not be recognizable enough in this low resolution, due to the low Transparency of 25%. We have chosen to display 6 silhouettes of horses [8], because these horses can interact together as if they are galloping in a carousel, which improves the recognizability (see Figure 4a).

The process of creating the shapes of the holes in the cells was similar to the method we described in [5]. The black areas were created by the remaining non-perforated areas of the hexagons and the (overlapping) projections of the remaining polygons. The resulting design is shown in Figure 4b.



(a) (b)

Figure 4: *Six horses in a carousel: (a) original pictures, (b) theoretical design with five reflections.*

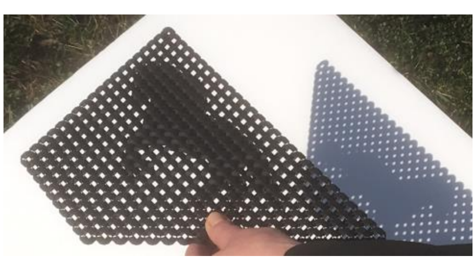


Figure 5: *One of the six direct views.*

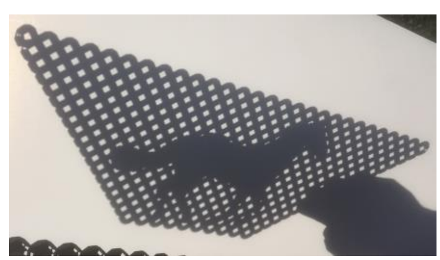
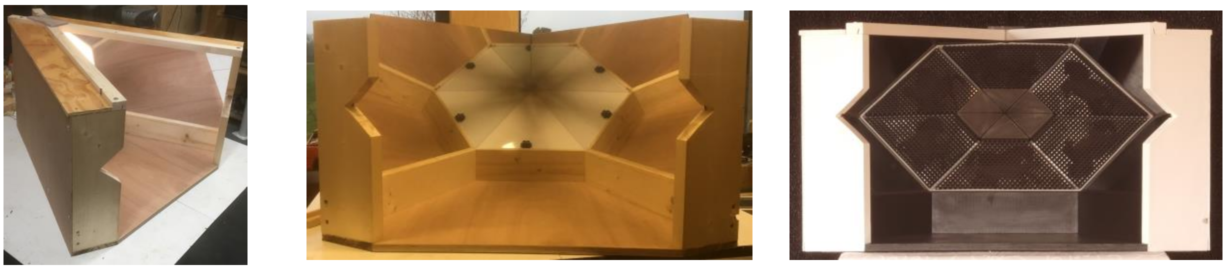


Figure 6: *One of the six shadows.*

Different Ways to Display the Object

One can view the black object against a white background from six different viewing points (see Figure 5). Another way is to look at its shadow in six different orientations of the object (see Figure 6). But the most spectacular way is to see the black object in one view with its five reflections in a display that we constructed with two mirrors and a white lighted background (see Figures 7 and 8).



(a) (b) (c)

Figure 7: *The constructed display with one lighted background and two mirrors: (a) and (b) halfway construction, (c) final construction with the lighted background switched off.*

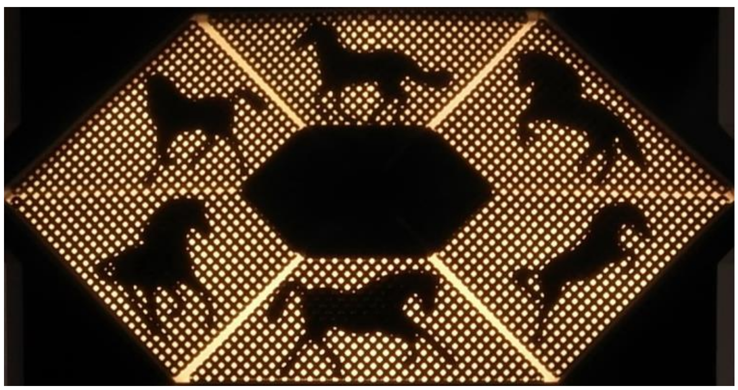


Figure 8: *The final artwork, with the lighted background switched on.*

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