The Planar Crystallography Groups as an Iconographic Analysis Tool in Islamic Art

María Antonieta Emparán

Orientalisches Seminar, Albert-Ludwigs-Universität Freiburg;
maria.antonieta.emparan@orient.uni-freiburg.de

Abstract

The mathematical discussion around the presence of the 17 crystallographic groups in the Alhambra became fertile after Edith Müller presented her doctoral thesis in 1944. However, from the history of art, only M. C. Escher has been mentioned for the use of planimetric symmetry in his study of the patterns of the Alhambra *alicatados* (mosaics tilings) for artistic purposes. The present work establishes a methodology of iconographic analysis of the Alhambra tilings from the chromatic asymmetry present in four of the examples found by mathematicians, as well as the search for a symbolic meaning in the mathematical use of work constructed by the Nasrids (1232-1492).

Introduction

In her thesis, Müller [12] claims to have found 11 of the 17 crystallographic groups in the Alhambra. Since that time the issue has been the focus of interest of some mathematicians, and the topic has not been without controversy and discussion. Over the years new crystallographic groups have been recognised in the geometric patterns of the Alhambra. According to some mathematicians, the 17 groups would be present in the Nasrid ornamental design. However, it is necessary to establish whether symmetry should be sought only in the drawing or if colouration should also be included in the analysis. Thus, considerations such as the colour, the lace and the supports of the collected examples determine the presence or not of crystallographic groups. Consequently, for some mathematicians, only 14 of these groups are present throughout the Alhambra.

This research, through iconographic analysis, argues that chromatic asymmetry is neither a mathematical error nor an oversight. The colouration would respond to a representational approach to Islamic cosmology. Therefore, chromatic asymmetry would be a symbolic tool to generate a specific aesthetic experience in the spectator. It is, therefore, necessary to face these works with the hypothesis that there is purposefulness in the arrangement of colours. Such intentionality would be responsible for chromatic asymmetry and not the randomness or ignorance of symmetrical properties in a tessellation.

Two fundamental facts emerge when approaching this mathematical problem from the history of art. The first considers the progress and artistic development within the society of Granada. It had to prosper in an environment of constant struggle for survival through frontier agreements and treaties. On the other hand, it is necessary to pay attention to the historical motivations and reasons for the palatine buildings. While the oldest palaces were built during periods of peace, the most aesthetically prominent ones were constructed during times of great armed agitation. It is in these monuments, constructed during the war, where the tilings in this paper are analysed.

Four examples of tessellations in which chromatic asymmetry exists from a mathematical point of view will be studied. Aesthetic parameters will be established around this asymmetry. Finally, a symbolic analysis using the iconographic method will be proposed to one of these examples. As a representative case, a pattern that has two versions of chromatic distribution was selected to compare. It also has one of the most disagreed upon mathematical discussions.

Alhambra, a context

The architectural complex known as the Alhambra was the palatine city of the Nasrid dynasty. After the fall of the Almohad Empire in 1269, the Nasrids became the last Islamic enclave in the Iberian Peninsula. Since 1238, when Sultan Muhammad Ibn Nasr (1195-1273; reign 1232-1273) took the city of Granada until the fall of the dynasty in 1492, the Alhambra was the palatine city of the Nasrids. This architectural ensemble consists of a variety of palaces, towers, gardens and other buildings. Each Sultan who came to the throne carried out some construction or architectural reform in the fortified city. The tiles to be analysed in this study are found in four of the majestic buildings inside the Alhambra. At the same time, these buildings were built or renovated during the period of most significant war agitation and flourishing of the Dynasty [1].

The four buildings that were constructed between the sultanates of Yusuf I (1318-1354; reign 1333-1354) and Muhammad VII (1370-1408; reign 1392-1408) are: The *Pórtico Dorado* (Golden Gateway) which served as the entrance to the constructions made by Muhammad V (1338-1391; reign 1354-1359/1362-1391); The *Baño de Comares* (Bath of Comares) which is located between the *Patio de los Arrayanes* (Myrtles Courtyard) and the *Patio de los Leones* (Court of the Lions). It has several rooms such as a sauna, hot and cold water baths and a rest area known as the *Sala de las Camas* (Room of the Beds). Yusuf I reformed this bath; Yusuf I also constructed the *Patio de los Arrayanes.* It serves as the antechamber to the *Salón de Comares* (Comares Hall) where the Sultan's throne was located. This courtyard has a large pool with a large water mirror as usual in Islamic gardens. The *Torre de las Infantas* (Infants' Tower), built by Muhammad VII, serves a dual architectural purpose. In addition to being a watchtower, it is a residential building with a central reception room [6] and [10].

After the sultanate of Muhammad VII, the Nasrid dynasty never recovered the splendour of the times of Yusuf I and Muhammad V. The decline manifested itself in the lack of constructive capacity together with high political tensions. Finally, Muhammed XII (1460-1533; reign 1482-1483/1487-1492), known as Boabdil, surrendered the city to the Catholic kings on 2 January 1492 [1].

Art history and geometric art

Art history is a discipline that has multiple methodologies to approach work. Edwin Panofsky has developed one of these; this is the iconographic method [13]. Panofsky's method seeks to understand the symbolic content within the work of art. To obtain this information, it is necessary to carry out work in three phases: first, a description of the represented scene is necessary. Then, a recognition of the represented motif is made using external references such as documents, mythology, traditional folklore, etc. Finally, an analysis is carried out using different disciplines such as history, sociology, anthropology, theology, etc. with the purpose to understand the real symbolic content. This is an efficient method for mimetic or figurative art. However, what could be expected from this method when applied to an abstract or geometric work of art?

As an example, let us take Piet Mondrian's work developed in Paris between 1919-38. In the first step, the pre-iconographic, it is identified that there are quadrilateral figures in white, red, blue and yellow delimited by black stripes. It is possible in this step to determine the measures of the quadrilaterals, the tone of the colours, for example. For the second step, the iconographic description, as it is an entirely abstract and geometric work, it is possible to indicate the name of the work or some description of similar characteristics, for example, "Large Composition with Red, Blue and Yellow" (1928) [2]. In the third and final stage, iconological analysis, the artist's affiliation with Theosophy, the manifests of the De Stijl group and the history of the artistic avant-garde of the 20th century should be considered [5]. It is, therefore, possible to point out that this work not only represents but also seeks to refound society spiritually employing a new art.

However, what happens when the object of study is Islamic geometric art? Does the absence of authors, manuals, artistic treatises, contractual documents make it impossible to carry out research and symbolic interpretation of these works of art? It is due to the absence of these primary sources that the Panofsky method, added to the fact that it is designed for figurative works, is only used by art historians

with few exceptions. The history of art has approached Islamic geometric art from formal methods, that is, description and cataloguing of ornaments according to their forms, periods and geographical areas.

The challenge of a symbolic interpretation of the geometrical patterns is possible. Works such as the one done by Darío Cabanelas with the ceiling of the *Salón de Comares* of the Alhambra prove it [4]. It is thanks to his study that it is known that this dome, through its wooden tile stars, represents the seven heavens of Islamic cosmology next to the divine throne and the cosmic tree. Nevertheless, the debt of art history to geometric ornaments is still very significant. As a reflection of this is the widespread idea that there is no background in ornamental geometric designs, as well as no systematic mathematical development.

A brief definition of the symmetry of the 17 crystallographic groups

In 1891 Evgraf Fedorov and later George Polya, in 1924, pointed out that there are only 17 groups, combining isometric movements, capable of filling a plane symmetrically with the same tesserae. The isometric movements involved are the translation, rotation, reflection and glide reflection. To maintain the symmetry of the geometric pattern two necessary conditions must be respected. The first is that only rotations of order 2, 3, 4 and 6 can be present. The second condition is the existence of 5 types of lattices: parallelogram, rectangular, rhombic, square and hexagonal. Under these parameters, the same tesserae can cover a plane up to infinity symmetrically.

There are different systems for naming these 17 crystallographic groups. This paper uses the system established by the International Union of Crystallography. On notation, isometric movements and recognition system, information can be found in [9] and [16].

Tiling seen through mathematics

While art history continues to see abstract figures that are difficult to deal with in Islamic geometric ornaments, the world of mathematics has taken more significant steps in this regard. Proof of this are the various presentations in this space concerning Islamic tilings and multiple areas of mathematics as a methodology of analysis. The most successful discussion is the one that has developed around the existence or not of the 17 crystallographic groups in the Alhambra.

For design, architecture, visual arts and anthropology, among other disciplines, the symmetry of the plane is a great work and inspirational instrument. Despite the significant development of this discussion, for art historians the “cryptic” nature of mathematical language has resulted in little engagement in this discussion. Oleg Grabar only observed the existence of Müller's 1944 thesis about the presence of 11 groups of planimetric symmetry of the Alhambra *alicatados* in his work “The Alhambra: iconography, forms and values” [6]. Grabar highlights the importance of carrying out an interdisciplinary dialogue to gain knowledge of what mathematics says about the tilings of the Alhambra. However, while art history ignores this possibility, mathematics has developed an interesting debate on this subject.

The research of mathematicians Branko Grünbaum, Zdenka Grünbaum and Geoffrey Shephard, Rafael Pérez Gómez and José Montesinos Amilibia are reviewed in this paper. Among these, an interesting debate takes place which focuses on the criteria for establishing the search for the 17 crystallographic groups within the palatine city of the Alhambra. One of these defines what a tiling is and, therefore, determines which structures (ceramic, plaster, paint, etc.) could be included within the analysis or not. Another important criterion is to clarify if only the drawing of the design is to be considered ignoring the chromatic element. The last element is relevant since in many of the tilings do not have a chromatic symmetry. In other cases, the same drawing corresponds to different crystallographic groups depending on the chromatic organisation.

Grünbaum [8] established these criteria and alluded to Gómez and Montesinos questioning the examples they present to exemplify the presence of the groups pg, p2, pgg and p3m1. Faced with this, Gómez, in a second work, justifies his selection of study material according to the criteria of the former, but insisting, once again, on his standards [14]. The discussion was finally settled by Lynn Bodner [3]

Bodner points out that in the Alhambra only 14 groups are represented, with no examples of the groups p2, pmg and p3m1, finding patterns for the groups pg and pgg [3].

According to Grünbaum, the Nasrids were not interested in doing work that had symmetrical mathematical properties because 500 years ago this was completely irrelevant [8]. These statements lead to the question of whether the mathematical symmetry in the coloration of patterns was irrelevant for Nasrid artisans.

Planimetric crystallography as a tool for art history

As an art historian, planimetric symmetry emerges as a tool for analysis artwork. Undoubtedly, the methodology of work from anthropology developed by Dorothy Washburn [16], [17] and [18], allows us to link mathematics and art history. Washburn's methodology indicates that symmetry in ornamental designs is a tool of symbolic expression. Only those who belong to the culture that produces these ornaments can access the metaphorical content.

It is entirely relevant for performing the symmetry classification considering the colour as well as the drawing. In this unique way, it is possible to observe the anomalies that allow analysis and not only a classification. Under the criterion of chromatic asymmetry, there are four tiles inside the Alhambra whose drawing is part of a crystallographic group but whose colouration is asymmetric or belongs to another group.

The Clavos (Nails) of the Pórtico Dorado

This tiling corresponds to those found in the columns of the *Portico del Cuarto Dorado* (**Figure 1**), whose pieces are known as *Clavo* (nail) or el *Avion* (Airplane). The design of this pattern, considering the chromatic element, corresponds to the group pm (**Figure 3**). While only the drawing is part of the group p4g (**Figure 4**) with a rotation of order 4, according to the classification made by Grünbaum, Grünbaum and Shephard [7]. The classification is made considering each of the *Clavo* as a repetition unit. However, they overlook the fact that the vertical *Clavo* are thicker than the horizontal ones. Therefore, there would be an error of classification. On the other hand, both Pérez Gómez [14] and Montesinos [11] analyse a piece from the Alhambra museum and classify it within the cm group (**Figure 5**) by taking a triangular section that involves part of five *Clavos* (**Figure 2**). According to Grünbaum, this is one of the examples that Müller forgot in her thesis to complete a total of 13 crystallographic groups represented in the Alhambra. The chromatic asymmetry does not allow this tile to be classified into p4g and cm groups. However, in visual and non-mathematical terms it maintains a great symmetrical equilibrium.

Figure 1: *Clavo tiling*

Figure 2: *Clavo tiling*

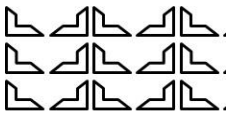

Figure 3: *A pm pattern*

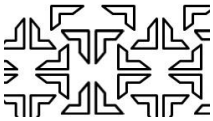

Figure 4: *A p4g pattern*

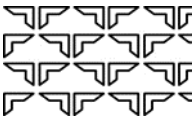

Figure 5: *A cm pattern*

Pajarita Nazarí (Nasrid Birdie) from the Baño de Comares

In the *Baño de Comares*, there are two tilings composed by the pieces denominated *Pajarita Nazarí*. One of these is in the *Sala de las Camas* (**Figure 6**) while the other is in the *Cámara Fría* (Cold Chamber)

(**Figure 7**). The pattern in the *Sala de las Camas* has a chromatic arrangement with an evident organised intention. However, the tiling in the *Cámara Fría* has a random chromatic order, as Montesinos points out [11]. This author, conveniently, as Grünbaum [8] observes, states that only two colours should be considered in the second tiling. It is clear to Montesinos that the Nasrids knew the combination of black and white, so the author classifies these patterns in the p3 group (**Figure 8**). Montesinos argues that if one considers the chromatic element of the tiling in the *Sala de las Camas*, the symmetry would be fundamental. Moreover, if only a fragment of this *alicatado* survived, it could be suggested that the chromatic distribution corresponds to group p3.

Furthermore, Pérez Gómez eliminates the chromatic element for the analysis of this pattern and classifies it as p6 (**Figure 9**) when he considers an isosceles triangle as a primary piece [14]. The proposal, as well as Montesinos', has the solution of eliminating or changing the chromatic aspect. However, it is necessary to observe in detail the difference in harmony in the composition between the two tilings. While one has a visual symmetry, the other is entirely chaotic chromatically. Since the Nasrids understand the symmetry of the p3 group, randomness must be excluded to explain the chromatic arrangement of the tiling in the *Cámara Fría*. The chromatic asymmetry of the *Cámara Fria* could be intentional.

Figure 6: *Sala de las Camas tiling*

Figure 7: *Cámara Fría tiling*

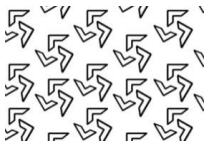

Figure 8: *A p3 pattern*

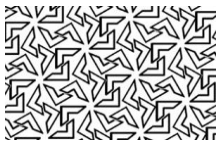

Figure 9: *A p6 pattern*

Side Alcoves of the Patio de los Arrayanes

Similar to the tiling of the *Baño de Comares*, in the side alcoves of the portal in the *Patio de los Arrayanes*, there are two tilings also built with *Pajarita Nazarí* (**Figure 10**). Another common element is chromatic; while one was constructed with a voluntary organisation, in the other it is difficult to detect a chromatic pattern. Conscious of this situation, Grünbaum and company decide to classify this pattern in two groups; one considering the chromatic aspect of the left iwan tiling only and the other considering just the drawing. The first is classified as p1 (**Figure 11**) and the second as p3 (**Figure 8**) [7]. Montesinos does the same classification for both versions of the tilings [11] meanwhile Pérez Gómez only classifies the pattern of the left side alcove as p1 [14].

As with the *alicatados* in the *Baño de Comares*, it is necessary to pay attention to the chromatic factor. Why did the artisans symmetrically arrange the colours in one tiling, while in the other, in a mathematically and visually random way?

Figure 10: *Side alcoves of the Patio de los Arrayanes tiling*

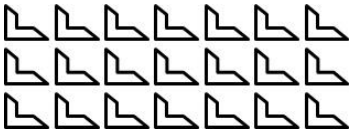

Figure 11: *A p1 pattern*

Side arches in Torre de las Infantas

Montesinos and Pérez Gómez analyse this tile (**Figure 12**). Both mathematicians classify this pattern into the group p4g (**Figure 13**) [11] and [14]. To perform this analysis, it was necessary to consider only the design of the tiling drawing was leaving aside the chromatic aspect since it is mathematically asymmetrical. The pattern, however, has the property that the colours are correctly ordered maintaining a balance and visual symmetry by organising the colours in concentric rectangles.

Figure 12: *Side arches in Torre de las Infantas tiling*

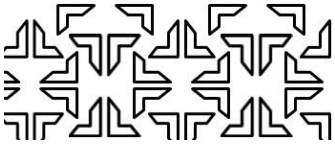

Figure 13: *A p4g pattern*

Metaphorical Symbology

Two critical elements that have been replicated in this exposition must be remembered. When Grünbaum speaks of the tiling of the *Clavo* of the *Pórtico Dorado*, he points out that chromatic symmetry was not something of interest to the people of 500 years ago. Moreover, it is not possible that they were aware of the 17 crystallographic groups [8]. Furthermore, Montesinos points out that the Nasrids knew very well the symmetrical properties of the p3 group when he refers to the bow ties in the *Baño de Comares* [11].

From the history of art, following the anthropological thesis of Dorothy Washburn [16], [17] and [18], this chromatic asymmetry is analysed iconographically. Unlike the anthropological work performed by Washburn, with the symmetry performed by the repetition of an abstracted motif, the iconographic method, in this case, seeks in geometric abstraction a metaphorical content. While the first seeks a symbol in the symmetrical repetition of a figurative motif, the second makes a symbolic interpretation of the geometric design. Considering the drawing of the design, Nasrid artisans knew the property of symmetry in the plane. A formal observation can only confirm this. At the same time consideration must be given to the list of colours in the wooden *zafate* (a small decorative piece of polygonal wood) of the ceiling of the *Torre de Comares* (Comares' Tower) [4]. This list demonstrates the intentionality in the arrangement of colours with a symbolic purpose. This geometrical pattern represents the seven heavens of the Islamic worldview, together with the divine throne and the cosmic tree.

Since there are no primary sources are indicating the reasons for the construction of the ornament in question, it will be necessary to review the historical context of the moment of the building of the monument in which it is located. Consideration of the function of the building in question is also required. Finally, it is necessary to read the inscriptions that accompany the tilings and to obtain a greater understanding of the context of the entire space in which the geometrical pattern is located.

In this way, it will be possible to know if the construction of the building is due to commemorative, functional, political or other reasons. The epigraphic inscriptions perform the role of informing dates and goals for the construction. By obtaining information on the historical, political, architectural and functional context, symbolic relationships can be established for chromatic asymmetry.

Water in the *Baño de Comares*

One of the most abundant components of the *Baño de Comares* is undoubtedly water. However, this is not the only element required to make the bathroom functional; the other is fire. Also, it is precisely about these two elements that the poems were written by Ibn al-Yayyab (1274-1349) speak in symbolic terms. Regardless of the metaphor used by Ibn al-Yayyab to refer to the military campaigns led by Sultan Yusuf

I and his qualities as a political and military strategist, the focus must be on the interplay between fire and water.

Let us pay attention to the fragments of the poems in question:

"In it, the most diverse desires are harmonised, / filled with the two opposites: water and fire. "

"Both share the two qualities of their nobility:/ ardent value and universal generosity. / And it is that one spills cold water, while its opposite hot water pours. " [15]

Attending again to the tiling pattern (**Figures 6 and 7**), it can be seen that the undulating contour of the bow tie simulates the movement of water; therefore, this *alicatado*, inside the *Baño de Comares*, could represent the water. However, it can be seen that there are two tilings whose chromatic dispositions differ. While the tiling in the *Sala de las Camas* has a pattern in the order of its colours that can even be classified in group p1, that of the *Cámara Fría* is entirely chaotic. Thus, in the *Sala de las Camas* there is a balance between water and fire due to the temperate environment. Moreover, like Ibn al-Yayyab's poem said, "diverse desires are harmonised". The harmony could be represented visually through the chromatic order.

On the other hand, in the *Cámara Fría*, the fire element is absent, thus an imbalance and loss of harmony, as the water is without its complement. The disharmony and imbalance would be represented visually through a random arrangement of colours that would intensify the effect of muscle tension. The aesthetic experience of contemplating the tessellation together with the exposure to cold water would recall the courage of the Sultan on the battlefield. On the contrary, the harmony of the elements water and fire produce relaxation and rest that reminds of the "universal generosity" developed by the Sultan in peacetime.

Sultan's policy: three symbolic proposals

Diplomacy in the Pórtico Dorado

After the triumph in the Battle of Algeciras, Muhammad V, in his second sultanate, built the *Portico Dorado* in commemoration [10]. The Nazarí Kingdom lived a period of peace that depended on a balance in the relations with the neighbouring kingdoms and emirates [1]. This balance could be seen reflected in the proportion of colours present in the tiling of the *clavos*: ½ of white pieces, ¼ black, 1/12 blue, 1/12 green and 1/12 brown [8]. We could speculatively point out that each colour would represent the diplomatic relations in order of importance that Granada maintained. Each colour would represent: Black - Castile; Blue - Aragon; Green - Fez; Brown - Tremecén; And, although the Nasrid colour was red, due to the more significant presence of white, in this composition it would be represented by white.

Peace and chaos in the Patio de los Arrayanes

In the *Patio de los Arrayanes* there is the *Pajarita Nazarí*. As in the *Baño de Comares*, this one represents the water present in the mirror of water of the patio. This space, also built after the Battle of Algeciras, precedes the throne room. Both the supplicants and the official and diplomatic visits saw these tiles when they entered and left their audience with the sultan [10]. The epigraphic texts focus on pointing out the faithful or followers of Muhammad V and infidels or traitors of him [15]. Therefore, the tile with chromatic symmetry would represent the return to the throne of Muhammed V and his victory in the Battle of Algeciras. While the tiling with chromatic asymmetry would represent the usurpation of power by Ismail II (1339-1360; kingdom 1359-1360) and Muhammad VI (1332-1362; kingdom 1360-1362) and their conspiracies with tragic results. Thus, these tiles would recall the consequences of both ways of acting concerning the sultan: order and peace on the one hand and chaos on the other.

Reconcentration of power in the Torre de las Infantas

After the death of Muhammad V, the Nazarí Kingdom began to fall into constant decadence. His grandson, Muhammad VII, tried to re-establish some of the former glories and state of peace. He tried unsuccessfully to follow in the footsteps of his grandfather. After the death of his father, he usurped the

throne from his brother Yusuf III (1376-1417; reign 1408-1417) whom he confined [1]. For these reasons, he focused all his interest in his figure and protected himself from possible attacks and even envy, as attested by the epigraphic poems in the palace he built [15]. The tiling flanking the entrance to the throne room of the *Torre de las Infantas*, despite having a mathematical chromatic asymmetry, has a great visual balance. The colours arranged in concentric frames could reflect the need to focus attention on the sultan, as well as to keep protected and surrounded by those close to him.

Conclusion

For the history of art, planimetric symmetry and mathematical discussion concerning its presence in the Alhambra is a crucial instrument to develop an analysis of the work around geometric ornamental art. Thanks to this tool, it is possible to determine that the absence of chromatic symmetry in some of the tilings of the Alhambra is not due to ignorance on the part of the Nasrid craftsmen of the existence of symmetry in the plane. The chromatic asymmetry is also not due to randomness in the distribution of colours. On the contrary, it can be seen that Nasrid artisans arrange colours asymmetrically with a metaphoric purpose to represent historical and political qualities and situations.

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