Three-Dimensional Symmetries in Dance and Other Movement Arts

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Abstract

Choreographers characteristically use both symmetry and asymmetry as part of the palette with which they compose movement sequences, and dance floor patterns and formations often display a variety of planar symmetries. Three-dimensional symmetries also appear, especially in acrobatic compositions, but also in movement arts that overlap with athletic events, such as formation skydiving. We will examine some of these three-dimensional symmetries and the ways that transitions may be designed to move from one such symmetric formation to another, looking in detail at (a) symmetries in one body, (b) three-dimensional symmetries between dancers, and (c) three-dimensional symmetry switching.

Introduction

There are many of types of symmetry in dance and other movement arts. These include: (a) symmetry within one body, for example, how the shape of one limb might be rotated or reflected to give the shape of another limb; (b) symmetries relating one dancer's body to another's or to those in a group, all considered in stationary position; (c) symmetries in time, that is, how one configuration of dancers moves to form another shape, symmetric to the first, in time; (d) how a body in space relates to a larger structure such as a rectangular stage or stage set; (e) how one set of body symmetries morphs into another in time. Additionally, there are many points of view from which one might consider these symmetries. We might examine: (i) how one might create any given symmetry in a dance; (ii) how particular dance or movement forms tend to use certain symmetries more than others; (iii) connections between choreographic symmetries and other elements of production such as lighting, set design, costume, music, or multimedia; (iv) connections between aspects of a culture and the symmetries employed by that culture's dance forms. Such appearances of symmetries often arise simultaneously or sequentially in performance. In this paper we will take a preliminary look at three-dimensional symmetries in relation to several of these points of view in dance and related movement forms. We will also look at ways one set of simple symmetries might change into another without breaking symmetry, something akin to parquet deformations in visual arts [5]. The point of view here is to go beyond how the body may simply be used to form various angles and shapes in order to include how dancers and choreographers are adept at using such shaping to accentuate body symmetries.

Types of Dance Symmetry

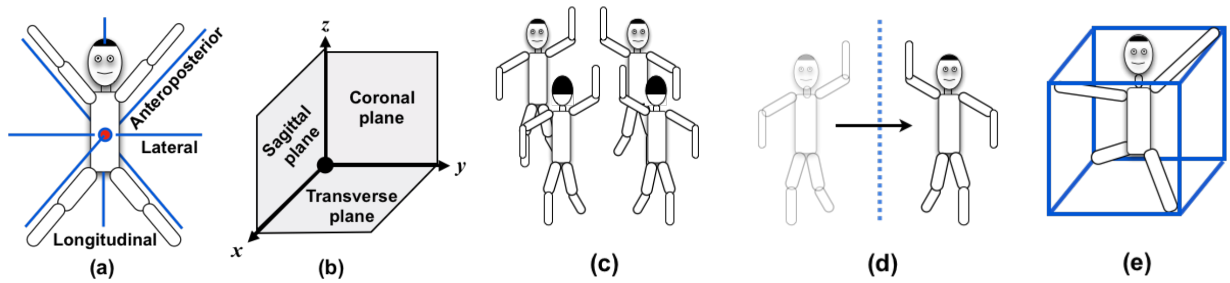


Figure 1: *Types of dance symmetries: (a) symmetries within one body, (b) symmetry planes (c) symmetries between dancers, (d) symmetry in time and space, (e) relation to external symmetries,.*

Figure 1(a) includes three mutually perpendicular lines through the center of the body along with their medical names: the front-back anteroposterior or x-axis line, the side-to-side lateral or y-axis line, and the top-to-bottom longitudinal or z-axis line. If we assume that the dancer shown is standing (or perhaps lying on the floor!) with both legs and arms directly to the side, then the lines shown represent planes of reflection symmetry. In this paper we refer to the medical vocabulary for the associated planes in Figure 1(b): the transverse or x-y plane which contains the lateral and anteroposterior lines, and similarly the sagittal or x-z plane, and the coronal or y-z plane.

A number of these possible symmetries were outlined by Robert Wechsler [9,10] in relation to ways of “reversing” movement patterns, especially in the modern dance technique developed by Merce Cunningham. Such symmetries in actual bodies are always inexact. However, the often fleeting yet still apparent reflections in the planes shown can elicit the perception of symmetry in audience members' imaginations. Less exact then a sagittal plane reflection is that in the transverse plane, indicating the similarity between the extended arms and extended legs (and ignoring the head!). The coronal plane illustrates how the dancer seen in silhouette may be difficult to distinguish as to whether the body is facing toward or away from the observer. Even less exact are the two diagonal planes indicated in Figure 1(a) which seem to ignore the torso and reflect an arm to the opposite leg. The *x*-, *y*-, and *z*- axis lines may each also be considered as the central lines for 180° rotational symmetries. If the arms and legs are approximately at 45°, directed respectively upwards or downwards, then the *x*-axis is also the center for 90° and 270° rotations, as might be glimpsed in a cartwheel.

Figure 1(c) shows symmetry between or among dancers in static positions. Note that the graphic shows a formation with two vertical and perpendicular planes of reflection symmetry, even though each dancer has only asymmetry in her or his body. Figure 1(d) depicts a reflection in time and space, again involving a dancer showing only asymmetry at the start and end of the movement sequence. Here we suppose the dancer has moved from our left to right, switching from left arm high to right arm high. Figure 1(e) shows a dancer in relation to a larger box, with limbs extended toward four non-adjacent corners of the box, forming a tetrahedral body shape, as those are the vertices of an inscribed tetrahedron.

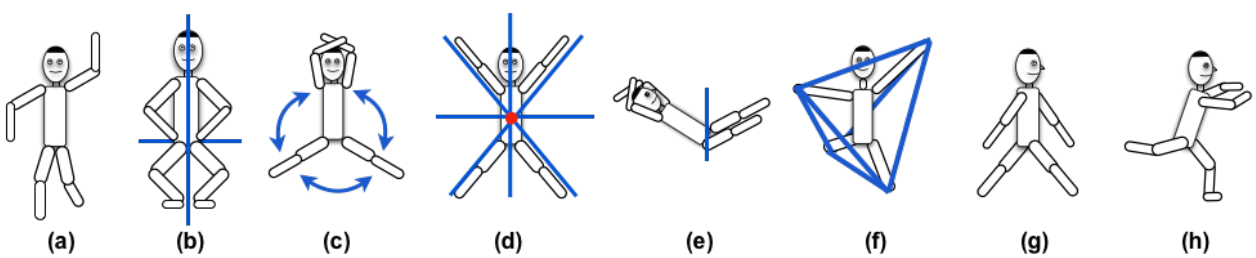


Figure 2: *Dance symmetries in one body: (a) asymmetry, (b) bilateral and horizontal reflection planes, (c) three right angles with 3-fold rotation, (d) multiple reflection planes and rotations, (e) symmetry of a vee, (f) tetrahedral symmetry, (g) rotary translation (helical), (h) parallel limbs*

Symmetries in One Body

Figure 2 shows a number of possible symmetries within one dancer's body. Figure 2(b) shows a dancer with a bilateral reflection through the sagittal plane and a reflection between arms and legs through the transverse plane. This dancer's position (common, for example, in the classical Indian dance form Bharatanatyam) may also have a coronal reflection plane, depending on the positions of the arms and legs. Figure 2(c) shows a position with three right angles, and an approximate three-fold rotation around the pelvis. Figure 2(d) has the same symmetries as Figure 1(a). Figure 2(e) has a reflection plane showing the upper and lower bodies as approximate mirror images. Figure 2(f) shows a dancer reaching with arms and legs toward the vertices of a tetrahedron, and is identical to the dancer inscribed in a cube in Figure 1(e). Figure 2(g) shows the common helical symmetry of the oppositional shape in limbs when walking, in which the position of the legs rotated a little less than 180° and translated upward becomes the position of the arms. Figure 2(h) shows a position known as back attitude; here the right leg held behind the dancer is parallel to the arms and when rotated and translated upward becomes the shape of the left arm.

Three-Dimensional Symmetries between Dancers

Figure 3 shows a number of two- and three-dimensional symmetries between dancers, and also between pairs of hands; the hand figures are included as they demonstrate experimentation with upside down formations without performers having to actually invert their bodies! (See [2, pp 312-313] for similar diagrams of wallpaper groups.) Such symmetries are known as isometries, transformations of Euclidean space which preserve distances. Translation, rotation, reflection, and glide reflection have been considered as symmetries between dancers in the plane in [3,5,7]. Two additional types of three-dimensional symmetries are rotary (or "rotatory" or helical) translation, in which one dancer's shape is rotated and then translated along the rotation axis; and rotary (or "rotatory") reflection, in which an initial rotation by one dancer's shape is followed by a reflection perpendicular to the axis line of rotation. Rotary translation was also seen in Figures 2(g) and (h) in the opposition of arms and legs when walking and the positions of arms and legs in ballet positions like arabesque and attitude. The rightmost graphic in Figure 3 shows a dancer formation with all six rotation and reflection symmetries of the dihedral group D_3.

In [3,5] we detailed how to create deformations in time which cycle through all the one-dimensional frieze symmetry patterns without breaking symmetry. In [5] we looked at similar ways for groups of dancers to cycle through all 17 wallpaper group symmetries in the plane. We note that for some applications involving two-dimensions it is helpful to consider a position showing reflection between dancers as a glide that incorporates translation by distance zero, and a position showing translation as a rotation centered at a point infinitely distant. Similarly, one may regard a rotation in three-dimensions as a rotary translation with zero translation, and a reflection as a rotary reflection with zero rotation. This perspective allows us to consider Euclidean isometries as essentially being of two types: proper or pure rotations which are orientation preserving, and improper rotations which switch orientations of figures.

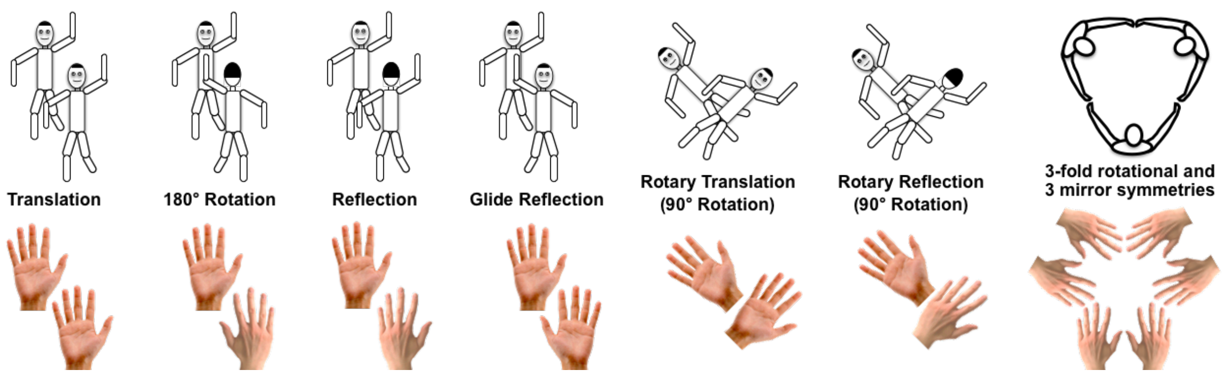


Figure 3: *Two and three dimensional dance symmetries.* .

Figure 4 depicts positions from dances choreographed by and photos directed by the author that demonstrate the symmetries mentioned above, as well as the general complexity of symmetry in dance. Figure 4(a) shows a dancer in the air in the x-position, and for which the transverse, sagittal, and coronal planes are planes of reflection symmetry. Figure 4(b) shows a leaping dancer exhibiting the symmetries of the tetrahedron. She also exhibits rotary reflection: if her leg positions are rotated 90 degrees around the vertical axis they become the mirror image of her arms. Figure 4(c) shows a dancer each of whose arms are a reflection of her left leg. Additionally, the line of her left leg extends in the same direction as her right thigh, and the extreme backward curve of her torso bring the linear shapes together. These geometric and symmetry properties of her body shape contribute to the beauty of the photographed position. This dance deals with the loss of a child in warfare, and the extreme stretch in the dancer's body is designed to help convey a terrible sense of loss. Figure 4(d) shows a dancer outlined in a door frame, and in a clear reflection of her shadow on the ground. Her right arm is a reflection of her right leg as well as a tilted reflection of her left arm, and again these elements add to the strength of the position. Figure 4(e) shows two dancers in glide symmetry. Each dancer's right-angle shaped forward arm also closely mirrors her rear leg position. Figure 4(f) simultaneously shows mirror image and translation symmetries. The arms contribute to the overall position, as all appear translated in a line, held in apparent tension. Figure 4(g) shows a dancer centrally placed inside a hoop; like Figure 4(d) this demonstrates a connection between the dancer's body shape and an external form. Figure 4(h) shows a cubic formation by four people, each in the position of Figure 2(c). Although the photo was taken at Bridges BANF 2009, a similar cubic shape by four dancers was used in a piece choreographed by the author for the Children's Dance Foundation of Birmingham, Alabama in 1998. Such a formation may be considered as showing all 48 reflection and rotation symmetries of the cube, or if we want to consider only isometries in which a pelvic vertex is transformed to another pelvis, then either the 12 rotations of the tetrahedron or the full 24 tetrahedral symmetries which include reflections. The author developed a plan for six dancers forming all five Platonic solids in 1998 [4].

Figure 4: *(a) Jane Real in The Daughters of Hypatia (2015), (b) Saki in Through the Loop, In Search of the Perfect Square (2009), (c) Maria Basile in Mosaic (2014), (d) Laurel Shastri, (Djerassi Residency, 2015), (e) Jenna Purcell and Kaylie Caires in Mosaic (2011), (f) Saki, Laurel Shastri, Lila Salhov, and Jane Real in Hypatia, (2014), (g) Saki in Hypatia (2014), (h) CW: Mike Naylor, Vi Hart, Erik Demaine, and Karl Schaffer in human cube (Bridges BANF 2009).*

Other examples of dancers moving in relation to a larger external frame include the system known as Space Harmony developed by Rudolf Laban (1879-1958) [11]. Laban designed movement exercises in which dancers are placed inside an imagined or actual icosahedron, cube or octahedron, and reach toward a sequence of their vertices in what he called choreutic scales. These 12 + 6 + 8 = 26 vertices also approximately correspond to the 12, 6, and 8 edges, faces, and vertices of a large cube, or box-like stage or studio. The choreographers Trisha Brown (in *Locus*, 1975) and Gerald Casel have created works using these 26 directions. About his work *Visiter* (2014), Casel stated,

> "I've lived in five places in five years, and I've felt a bit dislocated. Instead of telling my own story, I want to create a process for the dancers to emulate that experience. So, I've used a random number generator to direct the choreography... We made numbers correspond to places in space on a grid — 26 different directions the body

> could move. The dancers end up with instructions like 'elbow to the right diagonal on count one, head to lower left diagonal on count two.' It's the third piece I've made using this system." [1]

A number of artistic and athletic movement practices other than dance incorporate two- and three-dimensional symmetry patterns. For example, formation skydiving brings skydivers in free fall together to form patterns in the air [12]. "Big-way" involves large numbers of skydivers forming a pattern with central radial symmetry. The world record is currently 400 skydivers forming a pattern with 10-fold rotational symmetry. Competitive formation skydiving involves teams of 4, 8, 10, or 16 quickly forming one prescribed pattern after another, with an additional team member creating video documentation for use in scoring. For example, Figure 5 shows a "block" pattern in which the 4 team members create the formation known as Sidebody Donut, followed by an intermediate move in which the three skydivers forming the "donut" swivel a complete turn, and then form the pattern known as Side Flake Donut. Large indoor wind tunnels have recently allowed the development of competitive forms, known as wind-tunnel indoor skydiving or body flight, in which team members create vertical formations, such as shown in Figure 5(b). In the competitive athletic form teams must create a series of such formations as quickly and competently as possible. In more artistic forms the competitors perform a series of dance-like maneuvers in the wind tunnel, often accompanied by music [9].

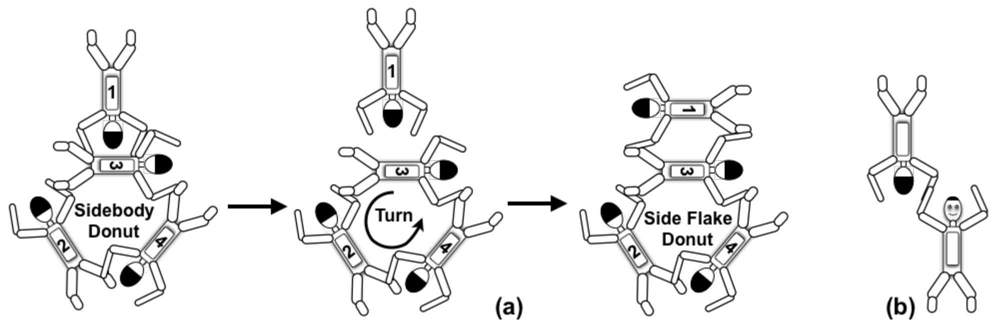


Figure 5: *(a) Formation skydiving pattern, (b) Vertical Formation Skydivng pattern.*

In this respect formation and wind-tunnel skydiving are much like competitive diving, gymnastics, and synchronized swimming, all of which award points for artistry and/or adherence to mandatory forms. Synchronized swimming (recently renamed "artistic swimming") combines dance, gymnastics and swimming and is an extremely challenging sport, requiring flexibility, strength, artistry, as well as superb breath control. Synchronized swimming teams often create many formations with clear central radial symmetry or with translation symmetry. However, because the formations involve competitors in vertical heads-up, upside down, and horizontal positions - both above and below water - three-dimensional symmetries come into play. Competitive diving often involves complex sequences of turns usually around the body's longitudinal and lateral axes. . In [3] we noted connections between dive takeoff and water entry positions and the Klein four group. Gymnasts often add turns known as cartwheels around the anteroposterior axis to the spins or pirouettes and somersaults around the other two primary axes. Gymnastic movement forms like figure skating, aerial dance and acrobatic partnering may also employ three dimensional symmetries.

Specific examples of symmetries in dance may be glimpsed in [6]. We see two dancers first performing part of a duet with a hoop in which their bodies sweep out the inner and top surface of a torus; their bodies are essentially mirror images through a curved surface. From 0:11 to 0:20 they dance in reflection symmetry through an imaginary plane down the middle of the stage. From 0:17 to 0:19 each dancer uses one arm to perform a swirling motion which takes two circles to resolve, and is perhaps best explained as an embodiment of quaternion motion. At 0:20 they switch seamlessly to translation symmetry and execute two turns or pirouettes. The first turn is called an inside or en dedans turn in back attitude. Immediately following they perform an outside or en dehors turn also in back attitude. In both turns the arms are parallel to the back attitude leg, as in Figure 2(h), and as the dancers turn this projects an image of twirling parallel circles. The turns are followed by a quick arabesque in which the arms are parallel to the right leg extended in back, and then a number of gestures with a strong sense of line, giving a sense of translation or rotary translation symmetry between the arms and legs.

Three-Dimensional Symmetry Switching

In [3,5] we showed how choreography can allow dancers to move smoothly from one symmetric position to another without moving through positions of asymmetry. This allows sequences exhibiting all seven of the frieze symmetries and all 17 of the wallpaper symmetries to be so portrayed. In [7] we showed how similar transitions can be applied to dances in which dancers represent two distinct subsets: men and women, two-colors of costumes, etc. In three dimensions there are 32 point groups which leave the origin fixed. There are 230 symmetry groups for spatial configurations that have translational symmetry in three independent directions. Here we address the simpler task of how one relatively small point group of three-dimensional symmetries may be sequenced without breaking symmetry.

Figure 6 shows the eight symmetries of the group $Z_2 \times Z_2 \times Z_2$, which is the set of symmetries of a rectangular box which has distinct values for its width, length, and height. The eight symmetries are 180° rotations around the *x*, *y*, and *z* axes (R_x, R_y, and R_z), reflections through the the *xy*-, *xz*-, and *yz*-planes (M_z, M_y, and M_x, respectively), central inversion (S, a 180° rotary reflection), and the identity I.

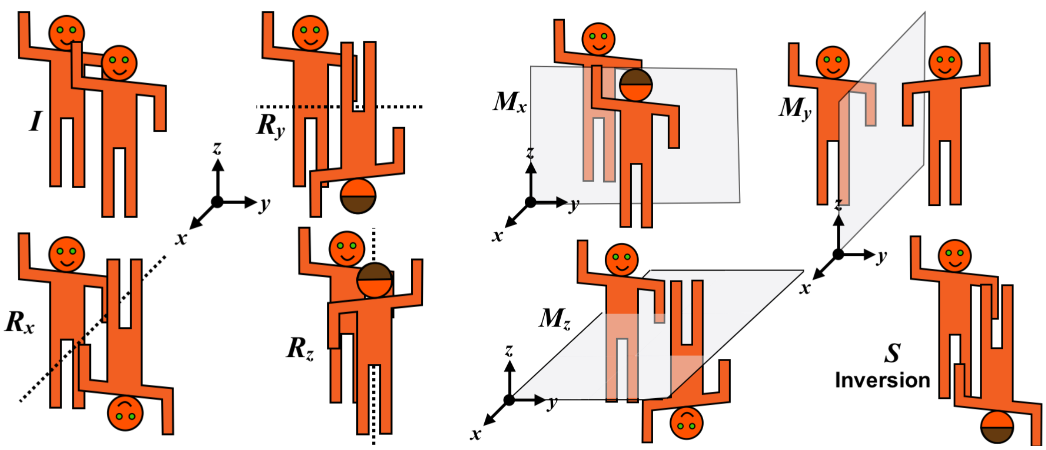
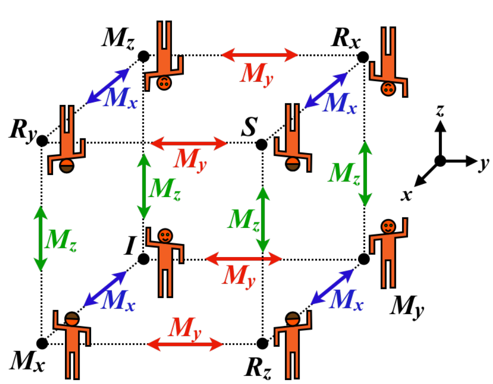


Figure 6: *Eight basic $Z_2 \times Z_2 \times Z_2$ symmetry patterns in three dimensions between two dancers.*

Figure 7(a) shows the Cayley color graph generated by the three reflections M_x, M_y, and M_z, and the group table showing how the symmetries compose. The symmetry label at each vertex relates the lone dancer at that vertex to the dancer at vertex I. The edge labels in a Cayley color graph are the group element which when composed with one of the edge's vertex labels results in the other. Note that in the group table, the rotations and identity (the "proper rotations") comprise a subgroup isomorphic to the Klein four group.



		Proper				Improper			
	$(Z_2)^3$	I	R_x	R_y	R_z	M_x	M_y	M_z	S
Proper	I	I	R_x	R_y	R_z	M_x	M_y	M_z	S
	R_x	R_x	I	R_z	R_y	S	M_z	M_y	M_x
	R_y	R_y	R_z	I	R_x	M_z	S	M_x	M_y
	R_z	R_z	R_y	R_x	I	M_y	M_x	S	M_z
Improper	M_x	M_x	S	M_z	M_y	I	R_z	R_y	R_x
	M_y	M_y	M_z	S	M_x	R_z	I	R_x	R_y
	M_z	M_z	M_y	M_x	S	R_y	R_x	I	R_z
	S	S	M_x	M_y	M_z	R_x	R_y	R_z	I

Figure 7: *Cayley color graph and group table for $Z_2 \times Z_2 \times Z_2$ discrete three-dimensional symmetries.*

Figure 8(a) shows how two dancers may move between positions exhibiting distinct planar symmetries, in this case from reflection to 180° rotation, by moving smoothly through an intermediate position of bilateral symmetry in each dancer's body. Figure 8(b) shows that the color graph for $Z_2 \times Z_2 \times Z_2$ in Figure 7 collapses to the Klein four subgroup when the dancers position themselves in bilateral symmetry. This is utilized in Figure 8(c) by the color graph arrows labeled "bilateral shift," all four of which are included in a Hamiltonian cycle that demonstrates one possible symmetry switching sequence through $Z_2 \times Z_2 \times Z_2$. Each vertex shows a pair of dancers which exhibits the symmery pattern for that vertex.

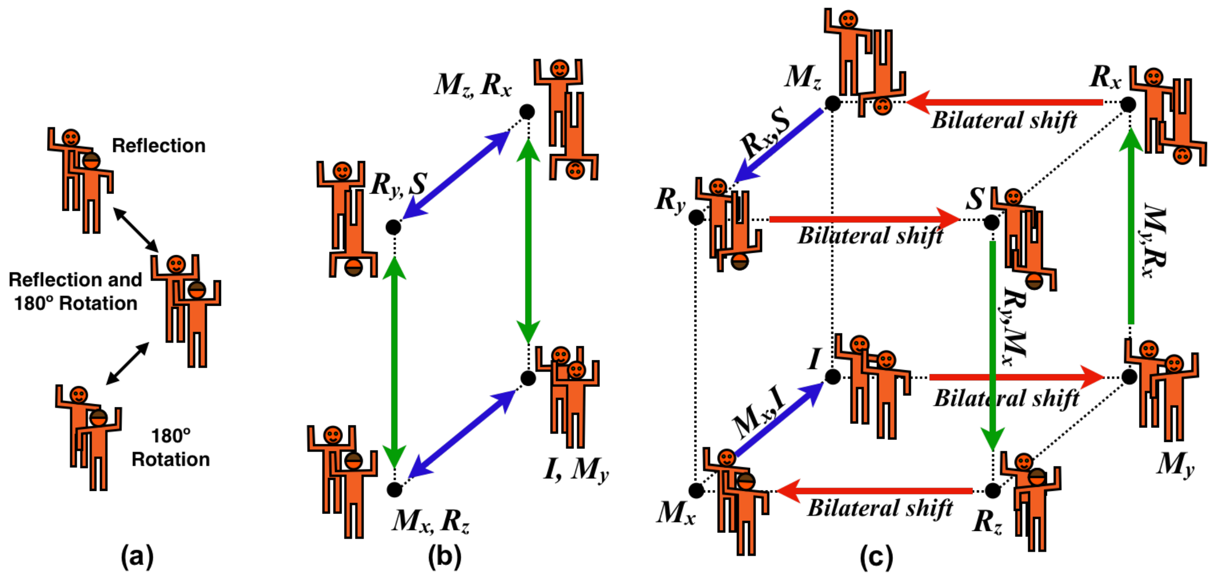


Figure 8: *(a) Bilateral shift. (b) The $Z_2 \times Z_2 \times Z_2$ subgroup when dancers adopt positions of bilateral symmetry. (c) A Hamiltonian cycle through the eight partner $Z_2 \times Z_2 \times Z_2$ symmetries.*

Figure 9 shows how the symmetry switchings in Figure 8(c) may be accomplished. All four arrows in the direction of the y-axis are simple bilateral shifts, as in Figure 8(a), while the other four switches are with the dancers remaining in bilateral symmetry in their own bodies. The switch from M_y to R_x is labeled "M_y, R_x," indicating those are the symmetries the dancers follow during the switching process, as detailed in Figure 9 (a)-(d). In Figures 9 (a), (b), and (c) the figures rotate 90° in opposite directions maintaining the mirror reflection M_y between them. In Figures 9(c), (d). and (e) the figures rotate together 90° around a line parallel to the x-axis while maintaining the 180° rotation R_x between them. Similar transitions are shown in Figure 9 for the other non-bilateral shifts. For ease of constructing the diagrams the figures are allowed to overlap, see Figure 9 (f)-(j) which is equivalent to Figure 9 (a)-(e). In Figure 9 (k)-(m) the dancers rotate around the z-axis away from each other for 90°, remaining in R_x symmetry with respect to each other, then rotate in S symmetry back toward each other. In Figure 9 (n)-(p) the dancers rotate toward each other while in R_y, then together back to vertical while in M_x. In Figure 9 (s)-(u), they rotate to the side while in M_x, then together back to the front in I, meaning in translation symmetry.

Conclusion

We might ask whether such symmetric transitions are actually used in choreography [5]. The issue is complicated because symmetries appear and disappear quickly in many dance forms, and may only exist partially. For example, in a back attitude, such as depicted in Figure 2(h), the arms are in bilateral symmetry, but the legs are not. Here we have taken a preliminary look at how three-dimensional symmetries appear in dance and may be utilized in choreography to maintain symmetry. Transitions similar to the $Z_2 \times Z_2 \times Z_2$ example above may be constructed for other patterns of dancers in space.

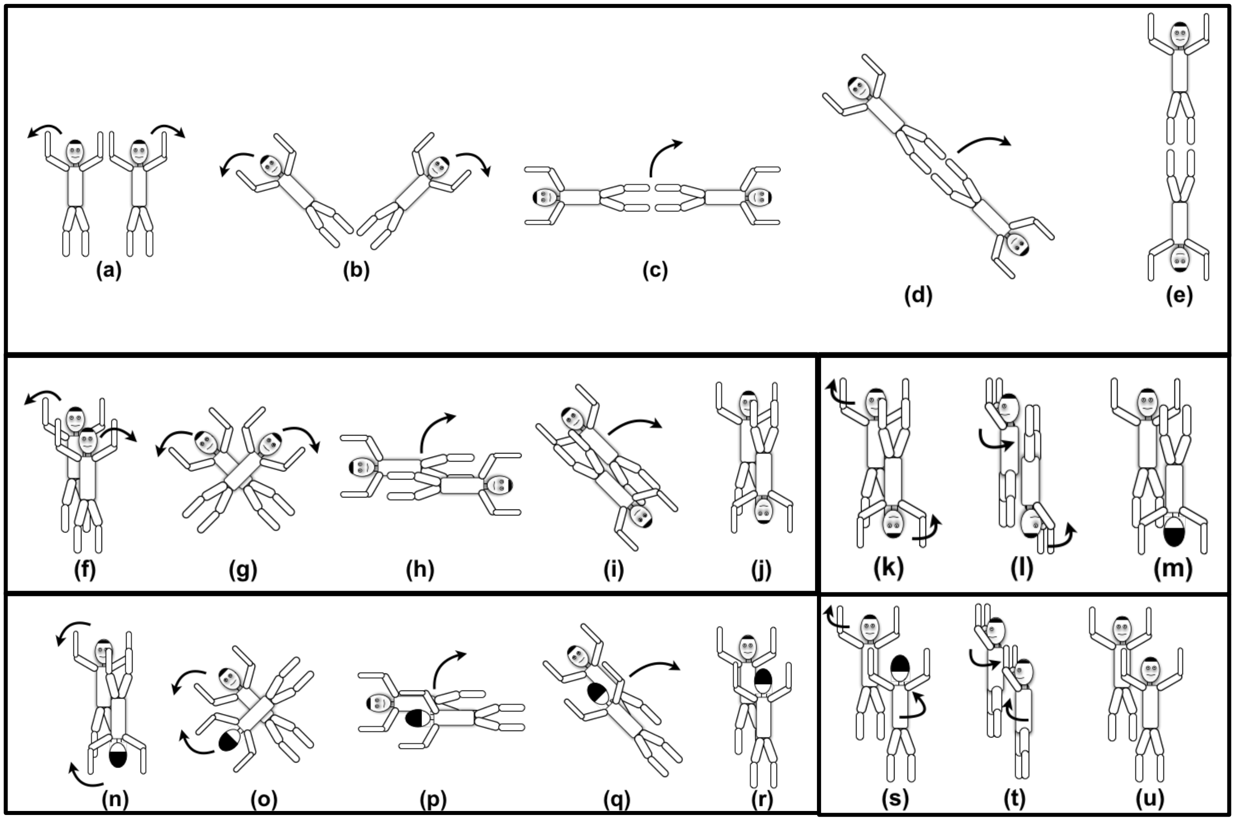


Figure 9: *(a)-(e) and (f)-(j) M_y to R_x, using M_y and R_x. (k)-(m) M_z to R_y using R_x and S. (n)-(r) S to R_z using R_y and M_x. (s)-(u) M_x to I, using M_x and I.*

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