Let's Sketch in 360º: Spherical Perspectives for Virtual Reality Panoramas

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Abstract

In this workshop we will learn how to draw a 360-degree view of our environment using spherical perspective, and how to visualize these drawings as immersive panoramas by uploading them to virtual reality platforms that provide an interactive visualization of a 3D reconstruction of the original scene. We shall show how to construct these drawing in a simple way, using ruler and compass constructions, facilitated by adequate gridding that takes advantage of the symmetry groups of these spherical perspectives. We will consider two spherical perspectives: equirectangular and azimuthal equidistant, with a focus on the former due to its seamless integration with visualization software readily available on social networks. We will stress the relationship between these panoramas and the notion of spherical anamorphosis.

Immersive Anamorphoses and Spherical Perspectives

The purpose of this workshop is to teach a simple method for drawing 360-degree perspectives such as those of Figures 3, 5, and 7, using only pencil on tracing paper and drawing from either observation or imagination. These drawings can then be uploaded to online platforms and visualized as 3D virtual reality panoramas (Figure 6). We can avoid performing the implicit ruler and compass constructions by drawing over equirectangular perspective grids of geodesics and making use of perspective symmetries [4]. This makes for a quicker and simpler drawing method that can be quickly taught and applied, and is also more convenient for *plein air* drawing without cumbersome apparatus. Following the workshop, after conference hours, participants who want to apply their acquired knowledge are invited to accompany me in outdoor drawing around Stockholm, for some informal panoramic urban sketching.

This workshop is based on a set of activities that I first drew up for a module of the doctoral program in Digital Media Arts (DMAD) at Universidade Aberta (UAb, Portugal). The purpose of this module was the "cardboarding" [3] (rudimentarization into "analog" form) of certain digital processes, as a way of exposing their mathematical basis to art students who might not have the foundational knowledge to understand it otherwise. For this purpose, I sought subjects whose geometry could be adequately explained within the tradition of formal drawing that connects Euclid to Monge, but also explorable in the form of freehand sketches, and especially within the practice of *urban sketching* [13], the activity popular among artists and especially architects, of drawing urban landscapes on location. The notion of *anamorphosis* proved most useful for this purpose, as it relates virtual reality panoramas with spherical perspectives. Recall that anamorphosis is a geometrical construction that, under controlled circumstances, creates the optical illusion of a 3-dimensional object on a 2-dimensional surface. It has a long history, that arguably predates that of perspective itself, and has been long studied both for the purpose of optical games [11] and as a practical auxiliary to architecture [12]. It is based on the principle of radial occlusion: for a monocular observer, two points are indistinguishable if they lie on the same ray emanating from the observer's eye. This principle implies that two objects look the same from a point O if they subtend the same visual cone from O, and allows 3D objects to be mimicked by 2D drawings.

Virtual Reality Panoramas, now commonly used in 360-degree interactive photography and video display, provide an interesting connection between curvilinear perspective and anamorphosis. Far from totally novel, VR panoramas are rather the natural development – in electronic and miniaturized form – of the 19th century painted panoramas and cycloramas [9]. 360-degree cameras work by gluing photos onto a virtual sphere, creating a spherical anamorphosis. This spherical picture is then flattened onto a single chart using one among several known cartographic options straight out of the geographer's cookbook – the result, a plane picture that stores all the information of the sphere, is called a spherical perspective. This spherical perspective is in turn interactively visualized as a virtual reality panorama, that is, as a set of plane anamorphosis, whose projection planes change interactively with the user's motion of a mouse or of a visualizing device with directional sensors (VR helmet or smartphone). Through this visualization software the user experiences some notion of seeing the original 3D object, thus going full circle in the anamorphic process.

What interests us here is that in the intermediate stage of this process the visual data is stored in a flat picture: the spherical perspective. These perspectives have been explored by artists such as Gérard Michel [10] and David Anderson [1] in online forums such as the Flickr panorama group [7], for the creation of both imaginary scenes and observational drawings that can be visualized as virtual panoramas.

It is on the creation of these pictures that this workshop will focus. Although photographic spherical perspectives are usually computed pixel-by-pixel using a computer's brute force, it is known that some spherical perspectives can be adequately drawn directly from observation, and rendered with elementary tools such as ruler and compass with reasonable effort by a human artist. Barre and Flocon [8] have shown how to draw the frontal half of the azimuthal equidistant spherical perspective with ruler and compass. Araújo [6] has shown how to extend Barre and Flocon's method to the full sphere and how to draw the equirectangular spherical perpective using a ruler, compass, and a protractor [4]. In this workshop we will be focus on drawing these spherical perspectives by hand, using the techniques described in these papers, adapted to the practical needs of the workshop by using printed grids and tracing paper instead of actual ruler and compass constructions. We expect these exercises will clarify both the geometric structure underlying virtual panoramas, and their relationship with anamorphosis. We also hope they will stimulate the participants to go out and draw their surroundings! We will focus mostly on the equirectangular rather than the azimuthal equidistant perspective, due to its ease of use with current VR software.

Description of Workshop Activities

The workshop will be split into a sequence of practical and theoretical activities. The number of participants should not exceed twenty. The workshop will be probably more enjoyable to people with moderate to good drawing skills, but these are not a requirement.

I will provide the necessary materials. These will consist of drawing paper, printed equirectangular grids, tracing paper, graphite pencils and erasers.

The main aspect of the workshop is the drawing practice, so all the theoretical activities will be very brief, and directed towards the needs of that practice. Hence, geometric propositions will be used without proof, relying on the references provided in this paper for further inquiry. We expect to accomplish four main drawings: A box seen from the inside (our workshop room), an inclined plane (ramps, stairs), a set of uniform grids (tiled floors, divisions and multiplications of rectangular walls), and, on top of those uniform grids, a loose panoramic sketch of our surroundings.

Theoretical Activity 1: I will briefly explain the general concept of anamorphosis, following the procedure described in [2], using the concept of a generalized version of the Dürer machine, seen as a machine for building anamorphoses over any surface - possibly curved, possibly not connected (Figure 1).

In this way, anamorphosis is defined as an equivalence relation between 3D objects (closed subsets of $\boldsymbol{R}^3$), where two objects are equivalent if and only if they define the same cone with vertex on the viewpoint O (the *visual cone*). As a particular case, one of the objects may be a compact subset of a 2D surface (a drawing) – hence anamorphosis allows for the construction of 2D simulacra of 3D objects, and this is the feature we will be exploring.

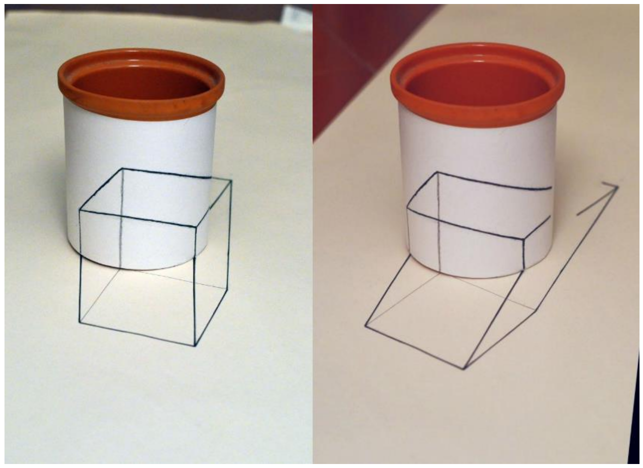

Figure 1: *Anamorphosis of a wireframe cube onto the union of a cylinder and a plane. There is a single point O from which the drawing will look like the original object (left); it will look "deformed" from any other point of view (right). Drawing by the author.*

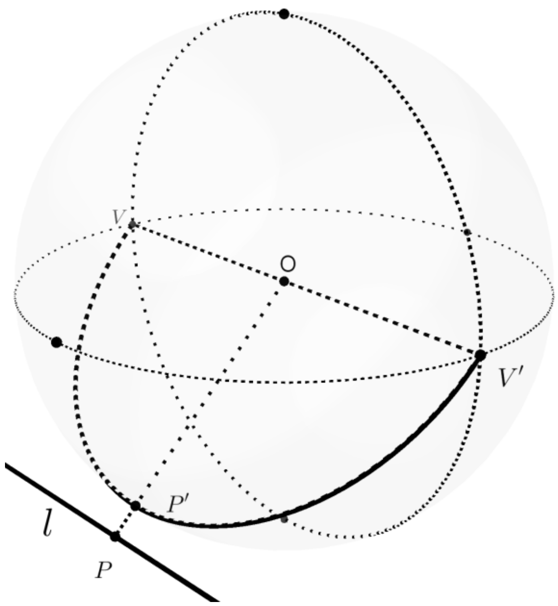

Figure 2: *Anamorphic image VP´P on a sphere of a line l with vanishing points V, V´. The vanishing points are obtained by translating the line to the center O and intersecting it with the sphere. V and P´ define the line image uniquely.*

The equivalence of points on the same ray from O implies that the unit sphere around O is the natural manifold that represents all visual data. Hence spherical anamorphosis is the most natural anamorphosis, and also the simplest, in the sense that all lines project equally into meridians and all have exactly two vanishing points. I will point out that the vanishing points and a further projected point (Figure 2) characterize a line projection uniquely, and this will be made clear by the exercises in the first practical activity.

Practical Activity 1: Asking the participants to imagine a referential, we will spot lines in our environment (using features inside the room, or outside the window) and characterize them in terms of measured points and vanishing points. It will be shown how to estimate angles in the horizontal coordinate system of altitude and azimuth (respectively latitude and longitude in the anamorphic sphere) using one's hands or a pencil. Spatial lines in the room will be characterized in terms of these angles.

Figure 3: *Cubic room in equirectangular (left) and azimuthal equidistant (right) spherical perspectives. Drawing by the author. VR panorama available at the author's web page [5].*

Theoretical Activity 2: I will show on a computer what Virtual Panoramas look like and how they present themselves to user interaction. Then the spherical perspective data that generate them will be shown, using two perspectives: equirectangular and azimuthal equidistant (Figure 3). The flattening maps from the sphere to the plane will be briefly described in intuitive terms. Participants will be asked (with guidance) to compare them qualitatively with regard to their vanishing points, to whether there is any conservation of lengths along any lines, and to whether the flattening map from the sphere to the perspective is or is not an isomorphism/homeomorphism/isometry.

Practical Activity 2: I will show how one can plot the equirectangular projections of horizontals and verticals without a computer using simple diagrams of Descriptive Geometry, as described in [4], and how these can be used to plot any geodesic. Then a printed grid of geodesics will be provided to the participants, to aid in the subsequent drawings (Figure 4). We will do a brief test run of the grid, plotting verticals and horizontals from the observations registered in the first practical activity.

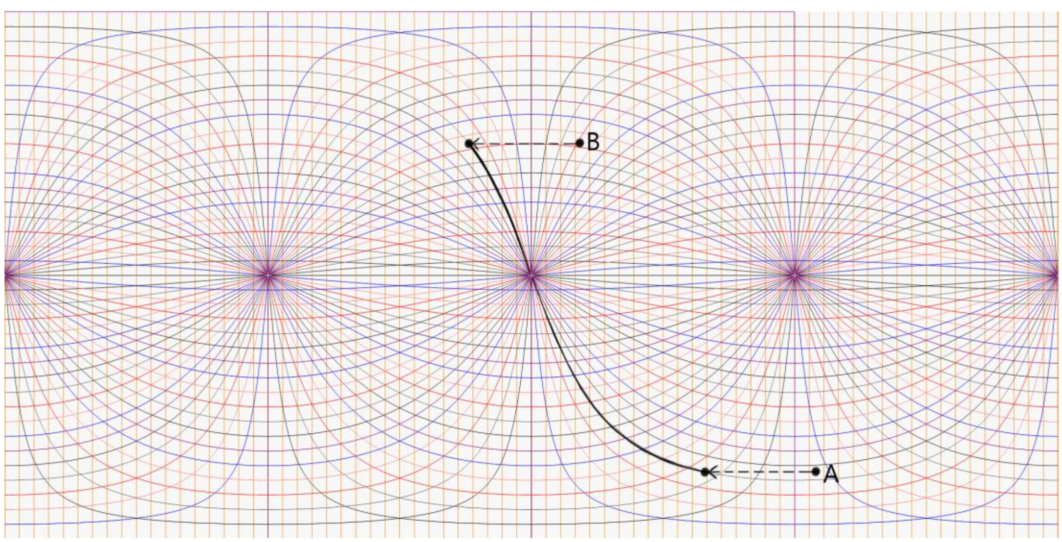

Figure 4: *Equirectangular grid of geodesics (5-degree separation between grid lines). Given two points A and B, the image of the line that joins them can be found through horizontal translation of the grid.*

Theoretical Activity 3: We will discuss symmetry groups of equirectangular perspective and their uses in drawing. The group of rotations around the zenith-nadir axis of the sphere is transformed by the flattening into a group of horizontal translations on the perspective drawing. This allows us to find the projection of any line (or line segment) AB by horizontal translation of the grid of geodesics (Figure 4). We will show how to take advantage of this in practical observational drawing, through the use of grids and tracing paper: making a sleeve by folding a sheet of A3 tracing paper, and fitting the A4 grid within it, the image of the line that joins two points A and B is determined by simply sliding the grid along the fold of the tracing paper until the fitting geodesic curve is found. This speeds up all subsequent constructions.

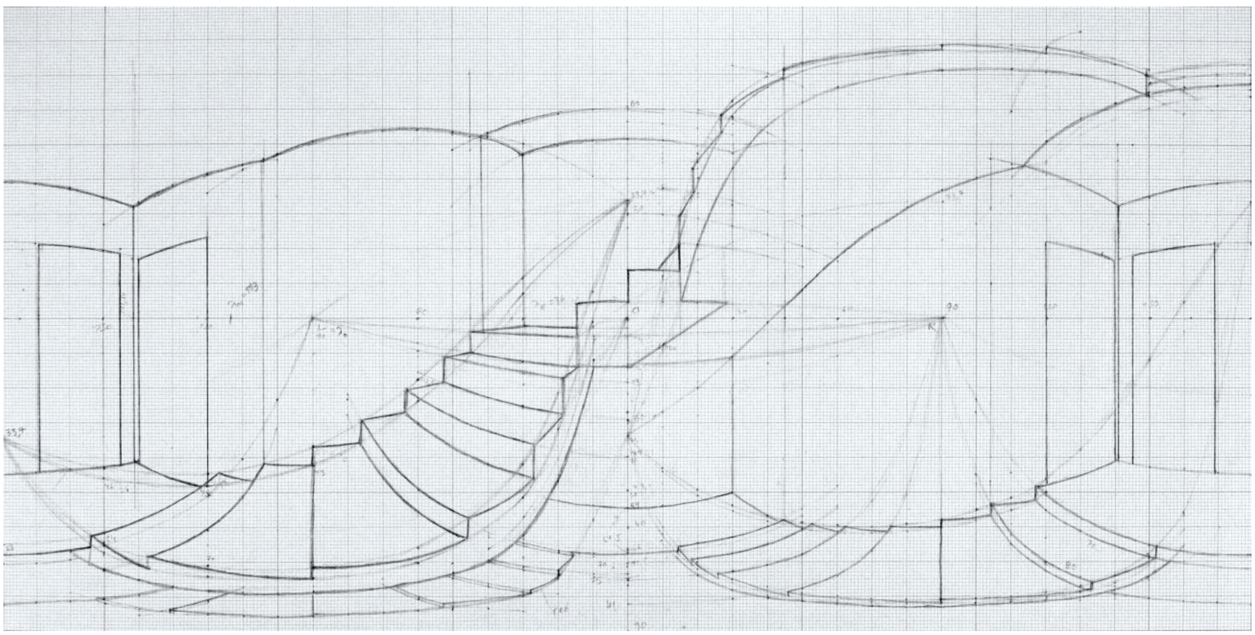

Figure 5: *A building's stairwell, drawn from observation – equirectangular drawing by the author. The corresponding virtual reality panorama can be seen at the author's web page [5].*

Figure 6: *A frame of the interactive view of the panorama built from Figure 5 (available at author's web page [5]). The stairs look straight in the interactive panorama, and one can look freely up and down the stairwell.*

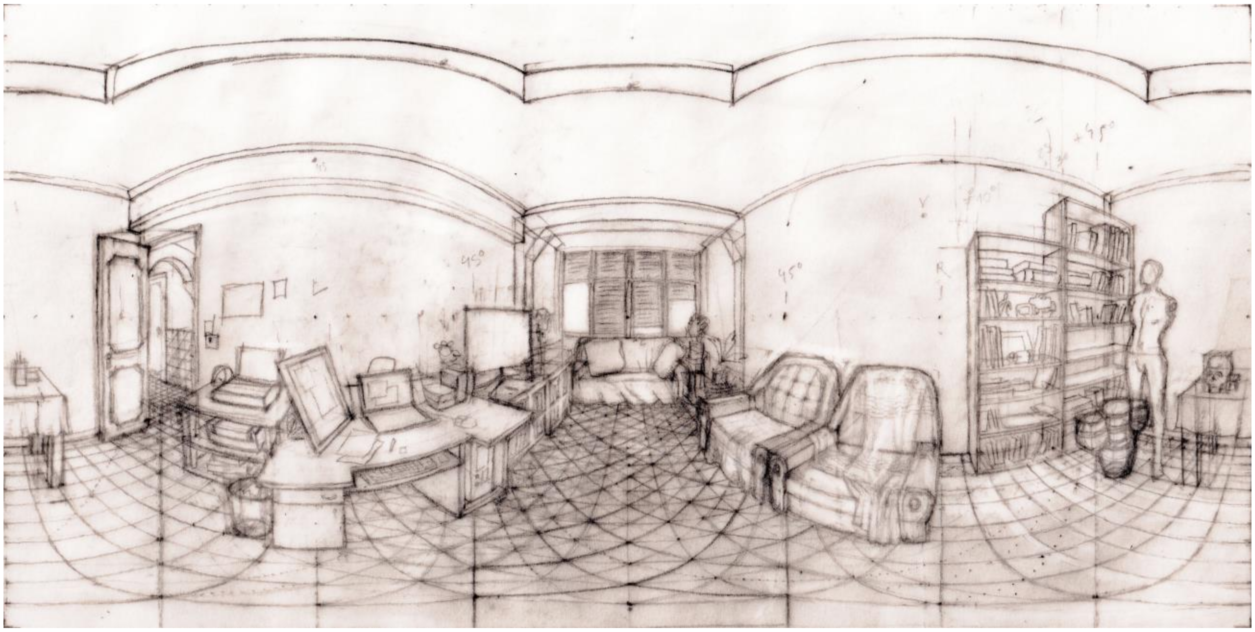

Figure 7: *Sketch of a room. Graphite on A4 tracing paper. The uniform tiling on the floor was sketched through the use of the printed grid of verticals and horizontals combined with the translational symmetries of the equirectangular projection. Objects were boxed in with a small number of measurements and much guesswork. Drawing by the author. VR panorama at author's web page [5].*

Practical Activity 3: Given two arbitrary points, it will be shown how to obtain the line projection that joins them in equirectangular perspective using translations of a grid along a sheet of tracing paper.

Practical Activity 4: We are now ready to draw! Using what we learned about measuring from observation and perspective drawing, we will draw a box of horizontal and vertical edges, seen from the inside – namely, the room where we will be working (obtaining a simple version of Figure 3).

Practical Activity 5: We will draw a ramp (either a real one in the environment or an imaginary one with specified measures). We will determine its vanishing points, measure necessary additional points, and trace the ramp through the use of the grid and its translational symmetries, obtaining a simpler version of Figures 5 and 6 (just the ramps, not the steps, as these would take too much time).

Practical Activity 6: Anything can be drawn once we know how to draw uniform grids. The classical example is a tiled floor. We will draw a tiled floor and a tiled vertical wall – essential to draw floor divisions of a tall building, for instance. We will show how to do multiplication and division of a rectangle. Further, we will show how scenes with finite rotational symmetry around the vertical axis can be drawn as simple tilings of the plane.

Practical Activity 7: We will use our uniform tiles as guides to draw objects around us by "guesstimation" (Figure 7). Then we shall discuss how to use what we learned to make natural, spontaneous sketches in urban settings ("urban sketching").

Practical Activity 8: I will show how the perspective drawings can be uploaded to popular visualization platforms (facebook, flickr) to be seen as VR panoramas. We will not actually do it with all the day's works, due to time constraints, but the participants will take their drawings home and do this at their leisure.

Theoretical Activity 4 (optional): If time allows, I will show how to build azimuthal equidistant grids (as described in [6]) and how to use their own set of rotational symmetries by a process analogous to the one described for the equirectangular case. We will speak of the tradeoffs between the two types of spherical perspectives. For instance, in the case of the stairwell picture of Figure 5, it is hard to draw more than a few flights of stairs up and down, as the equirectangular deformations quickly become extreme in the vertical axis. It is much more natural to use the azimuthal equidistant perspective in this case (Figure 8), but we pay for it with large deformations to the back of the observer. The artist must choose his perspective both according to his artistic intent and to the features of the environment.

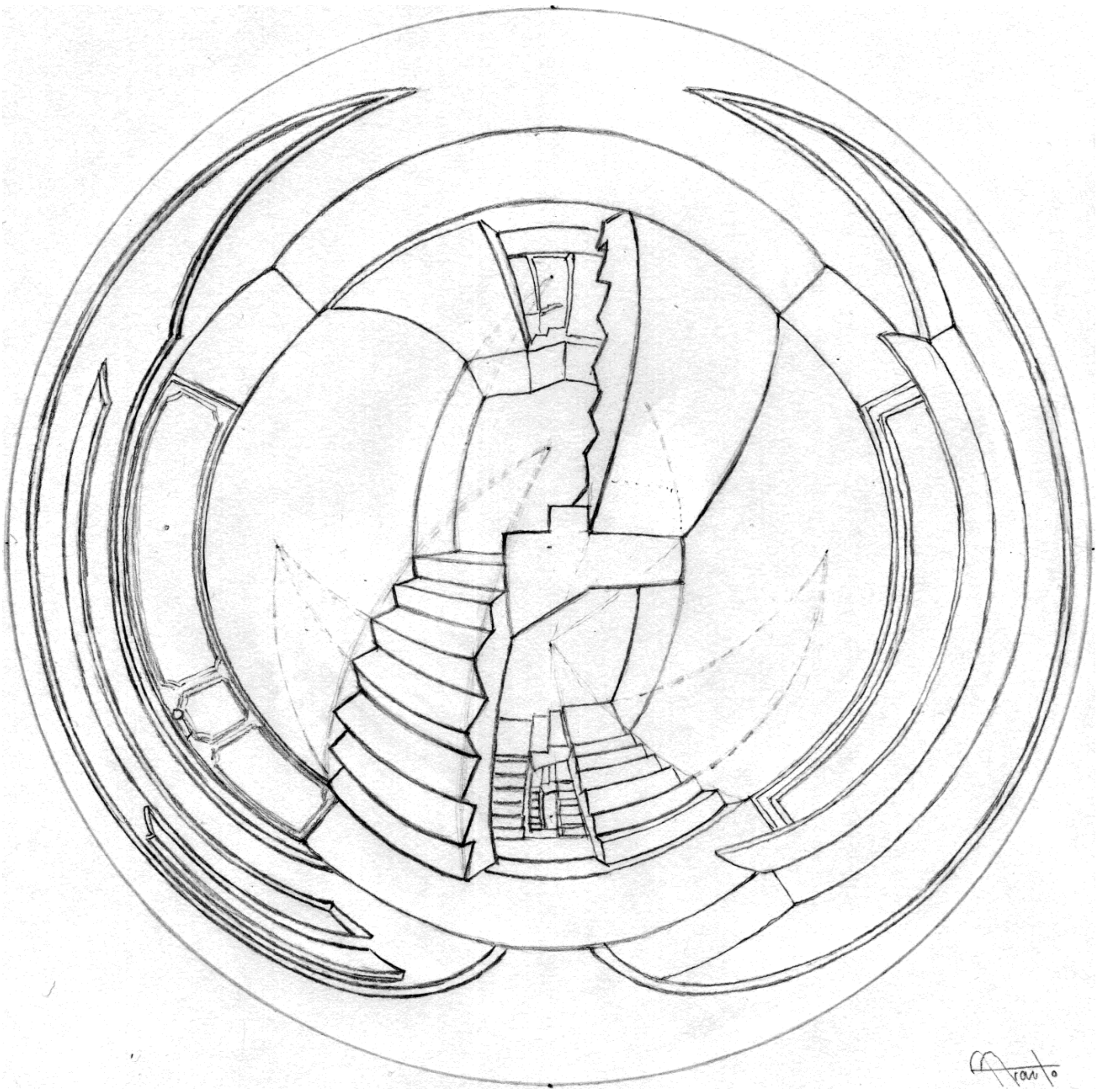

Figure 8: *Construction in azimuthal equidistant perspective of the same stairwell as in Figure 5. It is easier in this perspective to picture further flights of stairs going to the zenith and nadir, that get too compressed in the equirectangular case [6]. On the other hand, the doors in the back view get very compressed in this perspective. Drawing by the author.*

Summary and Conclusions

Spherical Perspectives are practical tools for hand drawing immersive environments, both from observation and from imagination. Two recent papers by the present author have shown how to completely solve the azimuthal equidistant [6] and equirectangular [4] spherical perspectives using ruler and compass constructions. In this workshop we demonstrate the fundamental methods for producing equirectangular spherical perspective drawings and clarify the connection of spherical perspective with both the classical practice of anamorphosis and current VR visualization technologies. We also demonstrate and make use of advanced equirectangular gridding techniques that allow the draughtsman to work outdoors in a practical manner using simple, portable tools. Finally, we briefly compare the equirectangular to the azimuthal equidistant perspective.
Further materials and notes will be made available at the author's web page [5], including printable equirectangular grids and the VR panoramas rendered from the drawings in this paper.

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