Geometry and Origami to Make Dynamic Street Art

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Abstract

We would like to share our experience, proposing a workshop organized in a series of activities involving, in a funny way, mathematical topics: from synthetic geometry, useful for kindergarten and primary school students, to GeoGebra investigations, useful for high school and undergraduate students.

Introduction

Since 2013, a team of researchers, composed by mathematics and architects of Politecnico di Torino interested in visualizations of geometric shapes, decided to study and use origami in their research and didactic activities. To this purpose, origami is a very clever tool because it allows to visualize and make tangible geometric elements. We have been exploring in many directions: we have been using origami in our classes to teach Mathematics and Drawing and training teachers from kindergarten to high school to this aim, we have been organizing educational projects and dissemination events, we have been studying origami models of architectural shapes from a mathematical point of view (see [1], [9]).

In this paper we briefly describe the research project realized during Spring 2017, in which we use origami in cooperation with our students to learn Mathematics, to produce artistic results and to connect people. The aim of the research has been to use Mathematics to draw geometrical elements appearing in a selection of wall paintings and to make dynamic some of them in it, by using well known origami or specifically designing them. The use of origami in street art is not new (see, for example, [6], [7]), and origami and Mathematics have a deep relationship (see [4], [5]), but our idea is to combine origami, street art and Mathematics.

Some of the authors, Sara De Grandis, Margherita Truffa and the student Silvia Decortes developed materials of this paper as part of their degrees in Architecture, and Engineering students Silvia Fiore and Marco Torredimare were then involved later, when the research was presented in public lectures.

The Research

Our research, based on different disciplinary and methodological approaches typical of polytechnic culture, seeks to pose a question: how many examples and what types of "Geometry" can you recognize around you? The initial aim of this project was to design origami art models for exhibitions and educational events, involving the study of geometrical shapes. The chosen location was Variante Bunker, a cultural center situated in an urban regenerated context in Turin, characterized by the presence of many mural artworks. In particular, we focused on a wall painting containing colorful worms (see Fig. 1).

In order to allow us to explore such a painting from a mathematical point of view, the team built a graphic model starting from digital photographs using geometric approximations. By measuring two points on the wall, we obtained a graphical representation of what belongs to the plane, in order to become

orthographic projections at a given scale and consequently be directly measurable and suitable to be investigated geometrically. This part was done at RilDocLab of DAD with O. Bucolo and D. Miron.

Figure 1: *Street art realized by Mattia Lullini. POLIMADE (Politecnico Math Architecture, Design Education) and students at the European Researcher's Night in Turin on September 29, 2017*

Following the idea of dissemination of Maths, we used this study in many educational and public events where it received a wonderful welcome. We started at the artwork mural location, and we continued during the European Researcher's Night, where people folded a 25-meter long worm! Finally, our worm was also invited at the Royal Residence "La Venaria Reale" to celebrate its tenth opening birthday.

We worked on the geometrical reading of the wall painting and, in order to design something tangible and interactive, our students started working approximating the shape of the worm using GeoGebra (see [8]).

Figure 2: ***(a)*** *Construction of circumference passing through 3 chosen points;* ***(b)*** *Circumferences obtained by repeating the previous process;* ***(c)*** *Polynomial Regression approximating the worm*

Figure 2(a) shows the construction of a circumference, passing through 3 points, with an approximating line separating two rings of the worm: GeoGebra allows the construction of the geometry drawing by using segment axes and line intersection. In Figure 2(b) the presence of many circumferences describes how different radiuses highlight the sectional changes in the worm body. In Figure 2(c) we computed the polynomial regression of degree 4, whose control points are the circumferences' centers.

Then, we started from a simple worm origami modular model obtained by alternating right and left modules; the great potential of this model is its flexibility, uncommon property for modular origami.

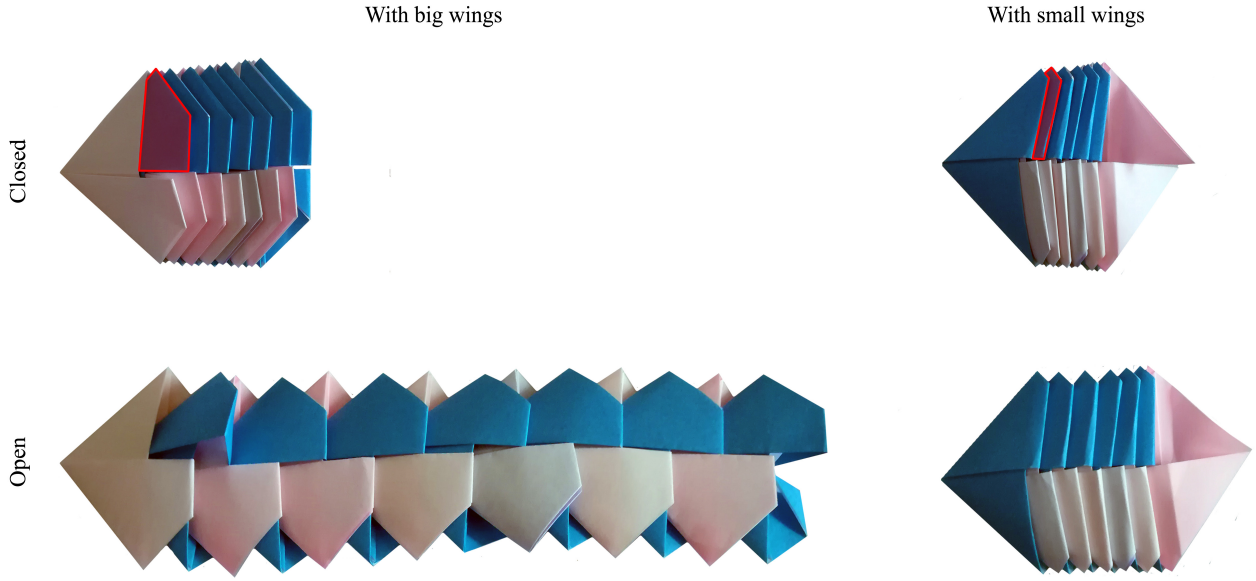

Figure 3: M*odular origami and its flexibility: how to match the single elements folding wings*

This property allowed us to introduce the definition of curvature, when we use it in didactical activities (see Activity 4 in the next section). Our team designed variations of this classical model, taking care of sectional changes of the mural artwork, then producing both two- and three-dimensional models.

Students had to think about gradual changes of the dimension of the modules in order to make an approximation of the original figure as accurate as possible. Problems they solved were related in particular on how to assemble them.

	2D		3D	
MODULS	DIMENSION l/A [cm/cm²]	QUANTITY	DIMENSION l/A [cm/cm²]	QUANTITY
1	-	-	l=29,7 A=882	0
2	-	-	l=25 A=625	0
3	l=14 A=196	5	l=21 A=441	2
4	l=12,7 A=161,29	17	l=18 A=324	18
5.1	l=11,3 A=127,69	31	l=14,8 A=219,04	37
5.2	l=10 A=100	13		
6	l=8,5 A= 72,25	44	l=12,5 A=156,25	31
7.1	l=7 A= 49	23	l=10,5 A=110,25	35
7.2	l=5,7 cm A= 32,49 cm²	32		

Figure 4: *Table containing the calculations to accommodate the different changes in section of the worm, and the actual realization with origami in 2D and 3D, placed over the mural artwork*

To obtain the desired effect, they created 7 different modules for the 3D-version and 7 additional modules for the 2D-version (see table in Fig. 4). At the end they folded 420 modules in total, respectively 255 for the 3D, and 165 for the 2D, that took 20 hours of work and more than 9 m^2 of recycled paper overall.

Combined work between the geometric representation of the image and the tangible paper model led to specific considerations on the representation language, on the interpretation and on the approximation of a non-strictly defined geometry as an artistic expression. The mutual interaction of the disciplined mathematical approach and the creative handmade artwork guided and refined the design path of the origami installation between Mathematics and Art. This has thus proved to be an excellent teaching and communicative tool at different levels. This experience allowed the preparation and launch of the Workshop, already tested, presented here following.

Workshop Activities

In this Workshop, with the participation of Sara De Grandis, Silvia Fiore, Marco Torredimare and Margherita Truffa, we propose some activities involving different mathematical and visual topics, accessible to students from kindergarten to university. At the beginning, we present an overview of the work we have done in our previous experiences, and then we select some of these activities depending on the interest of participants.

The strength of these activities is that the philosophy can be applied to other artistic media and can be adapted to any school level. The origami technique is well suited to other mathematical lessons (see, for example [2]). The powerful tool of modular origami in Mathematics classroom was described in [3] and [10].

Material and Facility

We use origami papers and A4 (21x29.7 cm) sheets of paper, that we share with the participants and we hang onto the wall as posters. For activity 5, it would be better to have an electronic screen connected to our pc.

Activity 1: Observe to Read

A first step of observation of the mural then leads to identify the simple forms to begin the study of physical modeling.

Activity 2: Geometrical Shape

In this activity we teach how to fold one of the modules of the worm, starting from a square sheet of paper. During the folding process we ask people to recognize some geometric polygons that naturally appear: triangles, hexagons, trapezoids. By this tangible method, everyone will be able to find and count the axis of symmetry, for each figure. Teachers can repeat and adjust this activity, based on the choice of origami models.

We continue the workshop by teaching how to fold the second module, drawing attention to the fact that we need a right and left module to create the worm, and how the two components fit together (see Fig. 5). In order to make the worm "grow", it is necessary to fold other couples of modules.

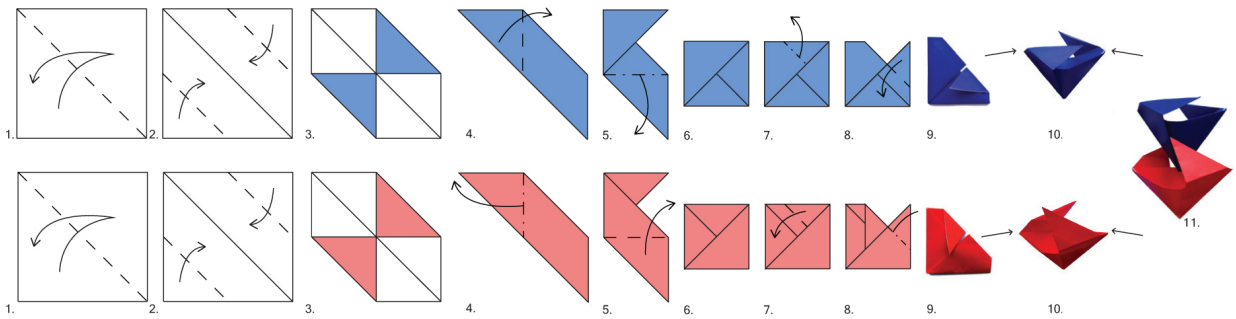

Figure 5: *The right and left modules and assembling diagrams (pictures taken from a gadget designed by trainee students)*

The activity can be done by groups and in different ways (in series or in parallel), thus also working on the aspects of collaboration.

Activity 3: Colour Coding

We ask people to set the colors for right and left modules, to obtain a particular worm decoration; e.g. to get axial coloring symmetry or chessboards distribution (see Figures 6(a) and 6(b), respectively).

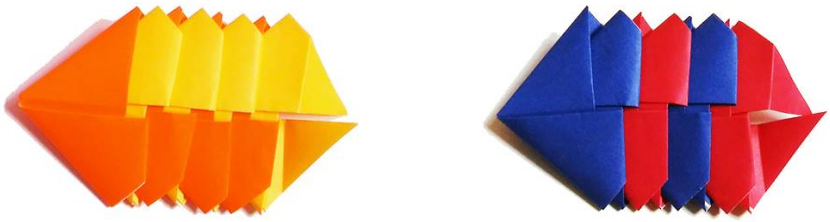

Figure 6: *Examples of "color coding".*
Are you able to deduce the sequence of right and left modules to obtain these coding?

Activity 4: Exploring Curvature

As we said before, one of the strength of this model is its flexibility which is not usual in modular origami. This allow us teachers to introduce the concept of curvature and to give the tangible shape. As a first example, we construct a polycentric curve (see Figure 7): a piecewise curve differentiable in each point, obtained by joining circumference arcs. In this case, in each point the curvature is $C = 1/r$, where r is the radius of the circumference containing the given arc. By modelling the worm, curving and stretching it, students can "touch" the curvature concept.

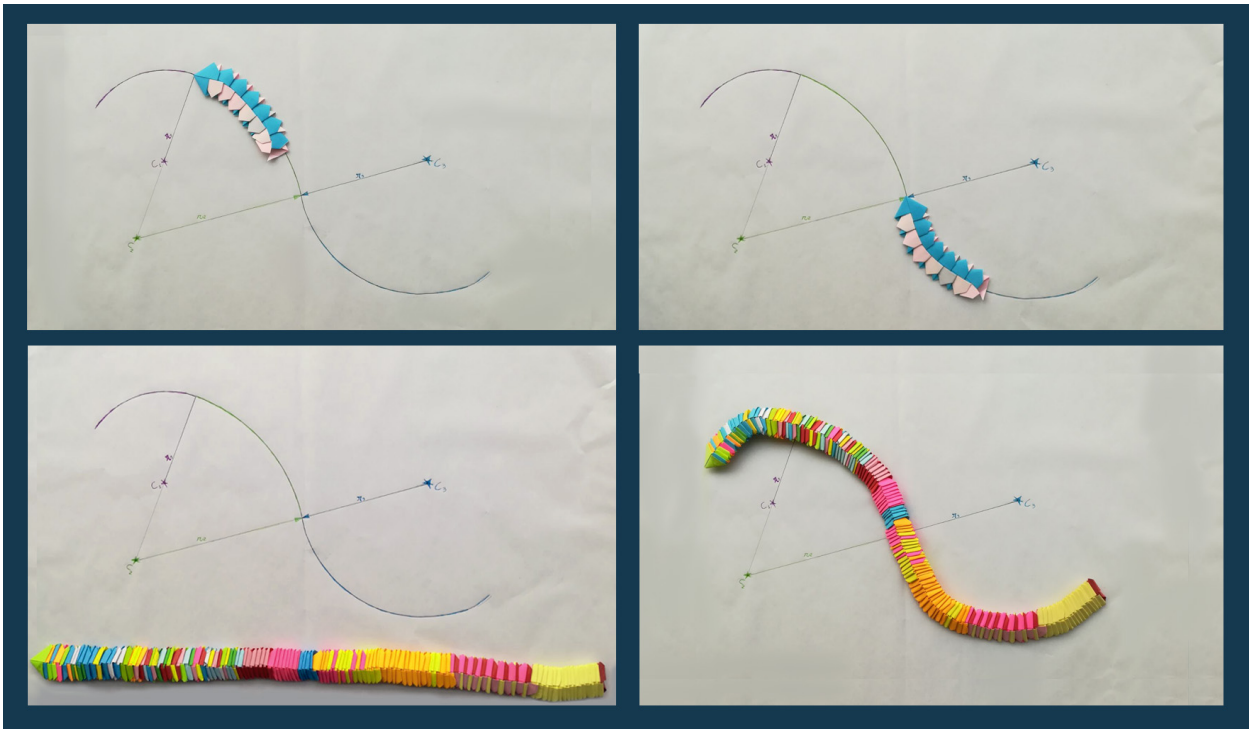

Figure 7: *Examples of polycentric curve and Worm on it*

Activity 5: Exploring Wall Paintings Using GeoGebra

In this activity we show some examples of investigation we can do by using mural artworks, starting from the "worm", but also proposing some other artistic realization.

The power of this software allows us to produce both geometric and analytic descriptions. In this way this game can be adapted to students from middle-school to university.

In the case of the worm for example, we described some GeoGebra investigation in Figure 3 (the firsts two steps have been adapted for middle schools too), but we can also use transformations of graphs of functions to describe some artistic lines in the wall painting. The chosen functions depend on the students' mathematical background: from polynomial functions to transcendental ones. We can also use the integral calculus to estimate the colored parts.

(a) (b)

Figure 8: ***(a)*** *Approximation of sinusoidal function;* ***(b)*** *Regression of polynomial function of degree 3*

In Figure 8(a), starting from the parametric functions F(x) = $a\sin(bx + c) + d$, our students determined the real parameters a, b, c and d in such a way that the graph of the function approximates the body. Figure 8(b) shows an example of regression of a polynomial function of degree 3, that approximates better the worm's body, compared to the curve shown in Figure 2.

Activity 6: Looking Inside Another Mural

We propose some other wall painting and we choose with the participants some elements, in order to make them dynamic through origami. To realize the subjects, we can use well known origami models or we can design new models at the moment.

Figure 9: *Example of mural artworks (Corso Castelfidardo, Torino), and Abacus folded and created by Pasquale D'Auria*

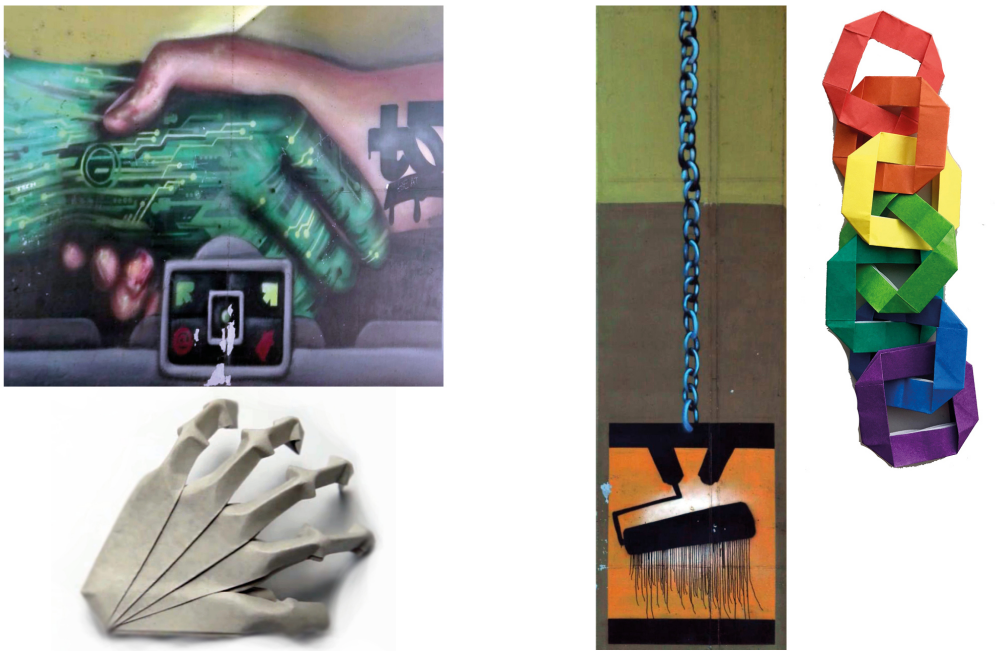

Figure 10: *Examples of mural artworks, from Corso Castelfidardo in Torino, and their respective origami models: Chain folded and created by Laura Kruskal, Hand folded and created by Jeremy Shafer*

Activity 7: Looking Around

Moreover, we invite people to look all around them, inside the room, in the building or in the garden, to discover some mural artworks or any general pictures that they can make dynamic through origami or can read under a geometrical point of view. This last activity is intended to stimulate creativity by designing shapes.

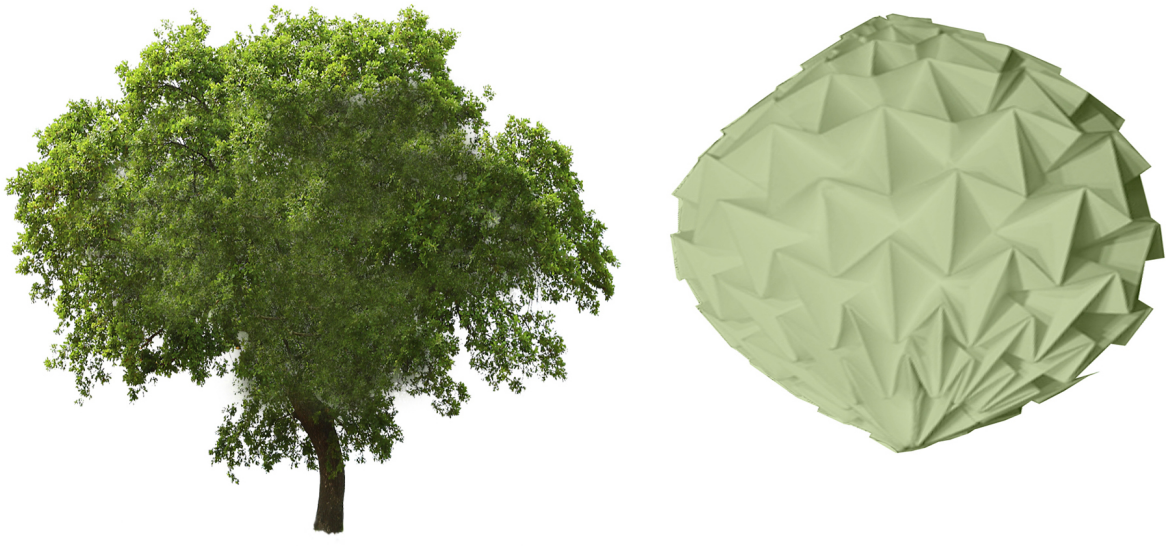

Figure 11: *Example of a general tree shape and its respective origami model: Magic Ball created by Yuri Shumakov and folded by Marco Torredimare*

Conclusions

We tested these activities in a few classrooms, with a specific mathematical aim, and in many outreach events, all of them with very good feedback. The fact that one can choose different origami models as animals, flowers, stars, etc, allows the educators to choose the difficulty of such models, of the mathematical topics involving people and students of different age and with different skills.

This activity encourages us to observe what surrounds us, by recognizing simple shapes and using a tangible mode of interaction to increase our understanding.

Starting from an art form, our workshop became an educational tool: it has made dynamic not only the artistic performance, but has triggered a long-term educational path, leading to the repetition and the reiteration of the method, conveying to a new way to look around by Math, daily Math!

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