Folding Curlicue and Exploring Its Mathematical Properties

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Abstract

This workshop will introduce participants to a model of kinetic origami called a curlicue which can be used in mathematical lesson. By exploring relationship between the length of paper strip and number of curlicue layers participants can learn basic trigonometry. The workshop will have three parts. The first part will be introduction to the benefits of using origami in mathematical lessons. The second part will be analyzing mathematical properties of paper folded curlicue, while the third part would be folding curlicue models. This kind of lessons can be applied in regular mathematical lessons and enhance teaching and learning process.

Introduction

These three parts workshop provides an example how can mathematical lessons based on regular curriculum be enriched and expanded with creative activities. Creativity and art in education should be a priority, since there is a need in creative and scientifically educated workforce [4], [7], [9]. This concept is still not fully integrated in the teaching process, since there are many obstacles such as lack of resources or teachers' trainings [13], [17]. We have chosen origami as an illustration how can artistic elements and folding process of curlicues be used to learn high school trigonometry in innovative way. This kind of activity can be applied by an individual teacher in regular school condition.

Origami and Mathematical Education

The first part of workshop (20 minutes) would be dedicated to the application of origami in mathematical lessons. Origami activities of creating a model by following the procedure and examination of the modification have a great potential in the process of teaching mathematics [21], [15], [20]. What is more, the combination of hands-on and minds-on activities increases interest in mathematics, especially in geometry and spatial relations [14]. In our case, we have chosen trigonometry to illustrate an example how mathematical concepts can be demonstrated trough creative origami.

Trigonometry is the basic part of high school curriculums worldwide [16], [10]. This important concept is part of many advanced mathematical courses, but also architecture, physics or engineering relay on trigonometry [20]. Understanding of trigonometry needs the ability of students to coordinate among different representations and objects, such as abstract, visual or concrete. Also, students need to know how to formulate that in algebraic expression [2], [19]. Teaching trigonometry usually begins with the definition of the ratio of right angle triangle sides. Regardless the simple introduction, educational research classified trigonometry as difficult for students. In our workshop, we use origami in order to enhance understanding of trigonometry. That is an innovative approach, since majority of studies in the field are based on the investigation of the technological and interactive environment usage in the trigonometry and teaching and learning [6], [10], [11], [14].

Mathematical Properties of Paper Folded Curlicue

For the purpose of the second part of the workshop (40 minutes), we have used the models of the curlicues that are presented in the book “Curlicue, kinetic origami” by Assia Brill [5]. This book was used as the main resource, besides multiple tutorial videos on the Internet. Trough hands-on activities we would present folding process of a simple strip square curlicue. This unique paper folded model looks like

kaleidoscopic star patterns. Simple folds of paper strips generate astonishing spiral form with changeable shapes while moving. Participants would have possibility to fold curlicues with the help of video instructions prepared by high school students of Petro Kuzmjak School where the workshop was also applied during the mathematical lessons. Curlicues folded by students are in Figure 1.
The starting point of folding would be trapezoidal paper strips cut by participants which would provide different folding outcomes since everyone would start from a different strip size. The first valley fold determines rotational orientation of the curlicue. Figure 2 shows starting folding steps of curlicue that rotate clockwise. Valley and mountain folds are alternating till the end of the paper strip when the curlicue needs to be locked in order to be able to rotate. It can be done by mountain folds or twist fix method [22]. This part of the workshop would end with exploring different forms of curlicue obtained by the participants. Participants should notice that number of layers depends on the length of the paper strip which would be an introduction the third part of the workshop.

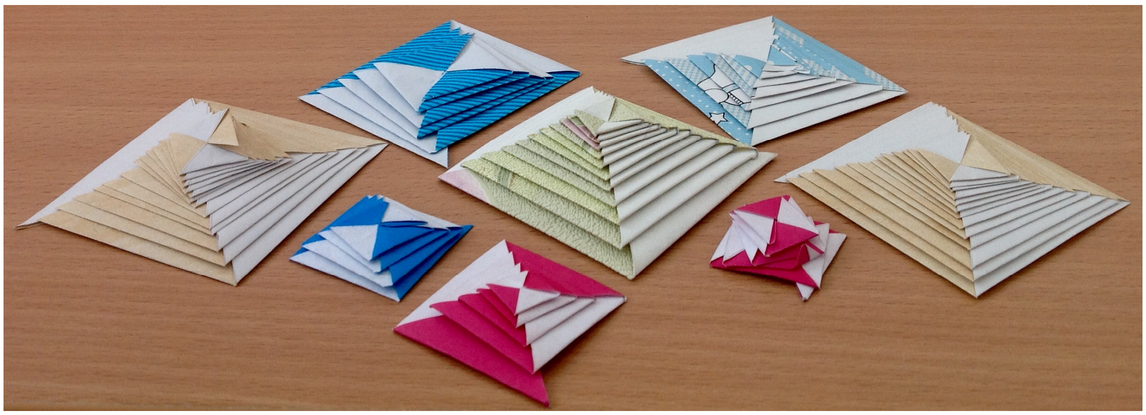

Figure 1: *Examples of curlicues*

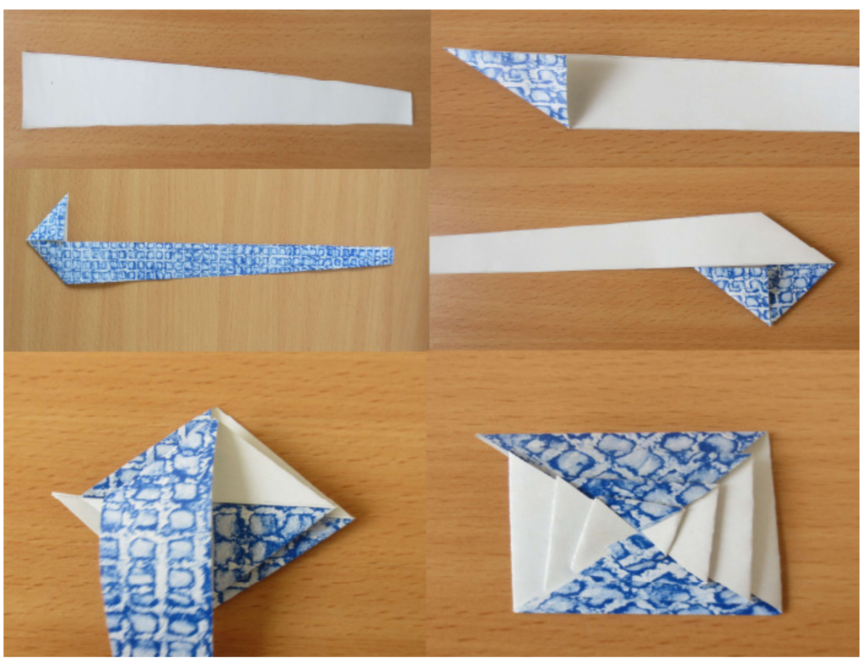

Figure 2: *Basics steps for folding curlicue*

The underlying mathematics in the folding curlicue. The third part of the workshop (30 minutes) would be dedicated to the revealing how the number of curlicue's layers depends on the paper strip length. Exploration of mathematical properties of curlicue would illustrate underlying mathematics and trigonometry in this origami model [1]. In Figure 3 we can see the sketch of the paper strip that is used to fold a curlicue. It is in the form of a trapezoid because it has a pair of parallel sides. The *p* and *q* are the bases and the length (L) is the altitude.

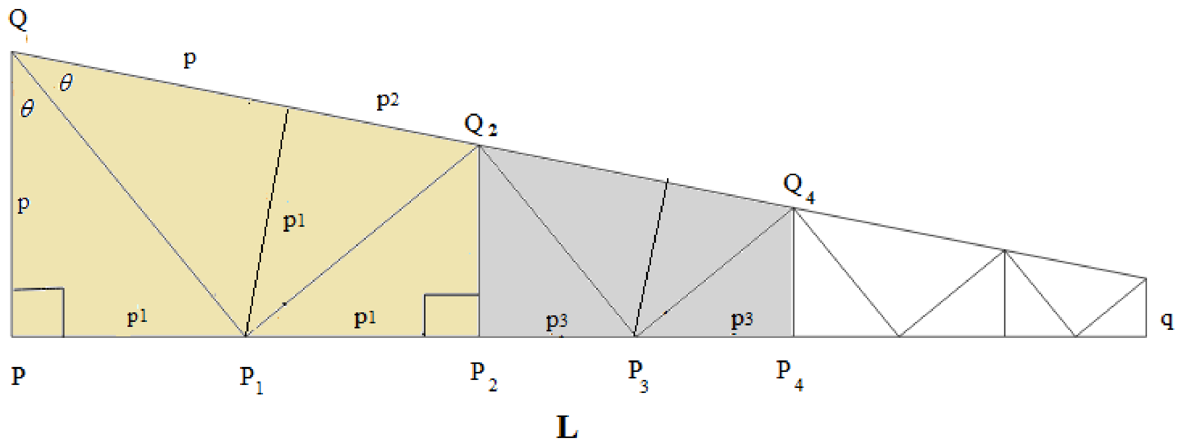

Figure 3: *Notation of the paper strip of folded curlicue*

It can be noticed that $tg\,2\theta = \dfrac{L}{p-q}$, where θ ($2\theta<90°$) is a half of the angle on the trapezoid base *p*. Following the notation from the Figure 3 we have $tg\,\theta = \dfrac{p_1}{p}, tg\,\theta = \dfrac{p_2}{p_1}, \ldots, tg\,\theta = \dfrac{p_n}{p_{n-1}}$ which mean that $p_n = p_{n=1} \cdot tg\,\theta$ or $p_n = p \cdot tg^n\theta$. Let us note some of curlicue's units such as QPP_2Q_2 and $Q_2P_2P_4Q_4$. For one curlicue layer (m) we need two units (c), which can be written as *c=2m*.

If we observe the length of the curlicue in one layer we have $L = p_1 + p_1 + p_3 + p_3 = 2(p_1 + p_3) = 2(p \cdot tg\,\theta + p \cdot tg^3\theta) = 2p \cdot tg\,\theta(1 + tg^2\theta)$ or in general $L = 2p \cdot tg\,\theta(1 + tg^2\theta + \ldots + tg^{2(c-1)}\theta)$. By simplifying expression as a sum of geometrical sequence, we get formula that describes the relationship between the length of the strip and the number of the layers: $L = 2p \cdot tg\,\theta \dfrac{1-(tg^2\theta)^c}{1-tg^2\theta} = 2p \cdot tg\,\theta \dfrac{1-tg^m\theta}{1-tg^2\theta}$ [1].

Summary and Conclusions

This workshop describes how to use hands-on activities and paper folding of curlicues in order to revise trigonometry and other mathematical concepts in the high school education. Connection of this kind of activities helps in developing interdisciplinary and unconventional ways of thinking [18]. Artistic component in the STEM disciplines increase engagement and develop imagination [8]. The workshop promotes synergy between different approaches of learning, where one topic is examined from two perspectives: origami and mathematics. Participants will have an opportunity witness how mathematics and art, in our case, origami can contribute to the formal high school education.

References

[1] N. Aggarwal. "A Mathematical Relation to Make the Right Curlicue." 2015. https://nancyaggarwal.mit.edu/sites/default/files/documents/CurlicueFormula.pdf

[2] N. Blackett and D. O. Tall. “Gender and the Versatile Learning of Trigonometry Using Computer Software.” *Proceedings of the 15th Conference of the International Group for the Psychology of Mathematics Education*, vol.1, 1991, pp. 144–151.

[3] A. Boruga. “Origami Art as a Means of Faciliatating Learning.” *Procedia - Social and Behavioral Sciences,* vol. 11, 2011, pp. 32-36.

[4] G. Boy. “From STEM to STEAM: Toward a Human-Centered Education, Creativity & Learning Thinking.” European Conference on Cognitive Ergonomics, ECCE 2013, 2013.

[5] A. Brill. *Curlicue: Kinetic Origami*. Creative Space, 2013.

[6] S. S. Choi-Koh. “Effect of a Graphing Calculator on a 10th Grade Student’s Study of Trigonometry.” *The Journal of Educational Reserearch*, vol. 82, no.6, 2003, pp. 359-369.

[7] A. M. Connor, S. Karmokar and C. Whittington. “From STEM to STEAM: Strategies for Enhancing Engineering & Eechnology Education.” *International Journal of Engineering Pedagogy*, vol.5, no.2, 2015, pp. 37–47.

[8] M. Cunningham. (2014). “From STEM to STEAM: The Potential for Arts to Facilitate Innovation, Literacy, and Participative Democracy.” *The Impact Blog, London School of Economics and Political Science*, 2015. http://blogs.lse.ac.uk/impactofsocialsciences/2014/03/14/stem-to-steam-creative-innovation/.

[9] A. Feldman.“STEAM Rising: Why We Need to Put the Arts into STEM Education.” 2015. http://www.slate.com/ articles/technology/ future_tense/2015/06/steam_vs_stem_why_we_ need_to_put_the_arts_into_stem_education.html.

[10] I. Kepceoglu and I. Yavuz. “Teaching a Concept with GeoGebra: Periodicity of Trigonometric Functions.” *Educational Research and Reviews*, vol.11, no.8, 2016, pp. 573-581.

[11] B. Kissane and M. Kemp. “Teaching and Learning Trigonometry with Technology.”*14th Asian Technology Conference in Mathematics, Beijing Normal University, Beijing, China,* 2009.

[12] J. M. Lee and Y. J. Shin.“An Analysis of Elementary School Teachers' Difficulties in the STEAM Class.” *Journal of Korean Elementary Science Education*, vol.33, no.3, 2014, pp. 588-596.

[13] E. Mafi and F. H. Lotfi. “Efficacy of Computer Software on Trigonometry.” *Applied Mathematical Sciences*, vol.6, no.5, 2012, pp.229-236.

[14] D. Meyer and J. Meyer.“ Teaching Mathematical Thinking through Origami.” *Bridges: Mathematical Connections in Art, Music, and Science; Conference Proceedings,* Winfield, Kansas, USA, July 30.-Aug. 1, 1999, pp. 191-204. http://archive.bridgesmathart.org/1999/bridges1999-191.pdf

[15] N. Orhun. “Students’ Mistakes And Misconceptions On Teaching Of Trigonometry.” 2015, http://www.math.unipa.it/~grim/AOrhun.PDF

[16] H. Park, S. Byun, J. Sim, Y. Baek and J. Jeong. “A Study on the Current Status of STEAM Education.” *Journal of the Korean Association for Science Education*, vol.36, no.4, 2016, pp. 669-679.

[17] E.W. Robelen. “Building STEAM: Blending the Arts with STEM subjects.” *Education Week*, 2015. http://www.bmfenterprises.com/aeparts/wp-content/uploads/2012/02/Ed-Week-STEM-to-STEAM.pdf

[18] J. A. Ross, C. D. Bruce and T. M. Sibbald (2011). “Sequencing Computer-assisted Learning of Transformations of Trigonometric Functions.” *Teaching Mathematical Application*, vol. 30, 2011, pp. 120-137.

[19] A. Russell. “From Rabbit Ears to Origami Flowers: Triangle Centers and the Concept of Function.” *Proceedings of Bridges 2017: Mathematics, Art, Music, Architecture, Education, Culture*, Waterloo, Canada, July 27-31, 2017, pp.533-538. http://archive.bridgesmathart.org/2017/bridges2017-533.pdf

[20] K.Weber. “Students' Understanding of Trigonometric Function.” *Mathematical Education Research Journal*, vol.17, no.3, 2005, pp. 91-112.

[21] B. Wenciker and P. Flynn. “Modular Origami in the Mathematics Classroom.” *Bridges: Mathematical Connections in Art, Music, and Science, Conference proceedings,* Winfield, Kansas, USA, July 30.-Aug. 1, 2004, pp. 293-296. http://archive.bridgesmathart.org/2004/bridges2004-293.html

[22] https://www.youtube.com/watch?v=LZ0Ik0zFUL4