Action Modular Origami

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Abstract

Action modular origami combines modular and action origami (folding paper). *Modular origami* involves joining several identically-folded units, without cutting or gluing, into a structure such as a polyhedron. It is well-suited for learning mathematics and the assembly is a kind of puzzle. An appealing type of origami is *action origami* which moves, spins or changes shape. I discuss the art, craft, engineering and mathematics of action modular origami.

Introduction

Origami (folding paper) is geometry in action and has been used for mathematics education for many years [12]. A particularly appealing type of origami is *action origami* which moves, spins or changes shape.

Modular origami usually involves joining, without gluing or cutting, several identically-folded units into a structure like a polyhedron or other kind of pattern. It is well-suited for learning mathematics as it can encourage learners to improve folding sequences and the assembly is a kind of puzzle [15][7, p. 20–29].

Figure 1: *Magic Star – Spiral Version*

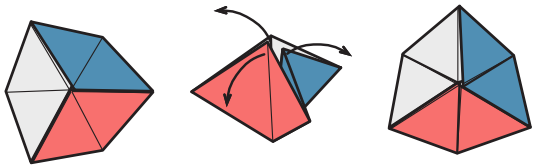

Figure 2: *Trihexaflexagon*

Motivation. I have run and co-run many workshops and sessions for children and adults using origami for learning mathematics. Origami models that are particularly popular are action models like Robert Neale's *Pinwheel–Ring–Pinwheel* [7, p. 20] and Dave Mitchell's *Ad Infinitum* [7, p. 21]. The following questions for the *Magic Star* (Figure 1) show the kind of mathematics that can be learned. *What shapes can you make the hole into and what are their mathematical names? What shapes can't you make? Why? What kinds of symmetry can you identify? What's special about the shape of the units that make them slide? How could you generalise this model to other shapes and symmetries?* Learners make the models by direct instruction or by reverse-engineering i.e. take existing models apart and work out how to make them.

Some of the first action origami that I made were the classic flexagons (Figure 2) [1] but the need for cutting and taping meant I didn't make them as often as other models like the eight-piece *Carousel* (creator unknown) and the *Magic Star*. Over the years I have looked for, and created, more action modular origami.

Here are some reasons to make action models by only folding without cutting and gluing:

- cutting instruments like knives and scissors may be restricted in some learning environments
- gluing can be messy and takes time to dry—assembly by folding alone is usually quicker
- some types of glue and sticky tape can degrade over time
- mistakes in cutting and gluing are hard to reverse, but folds are usually easy to undo

- glued models are sometimes weaker than models assembled without glue, and also harder to repair

Some folders believe that origami must be folded from a single square. However, even some of the most technical folders sometimes use more than one sheet. The benefits of using more than one sheet include:

- being able to use more than two colours
- economy and efficiency: you can usually make a larger model using several rectangles of paper than one from a single large rectangle. For example, the cubes of *Pseudo-Fluxicube* (Figure 4) are made from six oblong cards: make two folds on each card so that it has a central square, then assemble. If you use a single rectangle with the same area as the six cards, any cube you make by folding alone will be smaller than the modular cube. When the six cards are merged into a single sheet, most of the raw edges are no longer free to move independently. In a modular construction, the layers of paper are more likely to be evenly distributed across the model.

Action Modular Origami

Almost all of the action modular origami presented here are previously unpublished and novel. Animations of all action modular origami can be viewed at foldworks.net/action-modular-origami. They combine modular and action origami in five groups.

Sliders are flat models made to expand and contract, forming different shapes. They are inspired by Robert Neale's *Pinwheel–Ring–Pinwheel* [7, p. 20] and exchanges with Francis Ow. Practical experience shows that using a small even number of units usually makes better sliders than a large number, or an odd number. Even a small variation can make a dramatic visual difference (Figure 1).

Flexagons and Rotating Rings are models that fold to expose previously hidden faces. Rotating rings can be turned inside out in special ways (Figure 3). These build on Arthur Stones's *Trihexaflexagon* (Figure 2) and Dave Mitchell's *Ad Infinitum* [7, p. 21].

Magic Wallet Series models (Figure 4) use the two-way hinge from a famous magic trick to make flexagons, rotating rings and other action models. Wells describes the wallet and the rotating ring of six cubes [14, p. 218–219]. The models are made from used rail tickets and strips cut from A4 paper.

Spinners and Wheels are models that you blow to spin, or turn on a flat surface. The spinners are mostly skeletal polyhedra (octahedron and cuboctahedron, Figure 5). Their action was an extra effect inspired by Kasahara's presentation of Robert Neale's *Skeletal Octahedron* [5, p. 28]. The waterwheels are inspired by works by Mette Pedersen and Paolo Bascetta, which fit together using friction. I wanted to make waterwheels that stayed together even when vigorously blown around an axle.

3D Shapeshifters smoothly transform from one 3D shape to another. Some of these are based on works by Buckminster Fuller (*Jitterbug*)[3], Petrie-Coxeter (*Honeycomb*) and Pedersen (*Collapsoid*)[14, p. 34, 105]. Others are well-known hinged poyhedra dissections [5, p. 233, 377] [16]. I discovered *Radioactive Ball* (Figure 6) myself: it uses circular paper to conveniently generate the required 60°geometry and efficiently use paper in a way not possible from a square or rectangle.

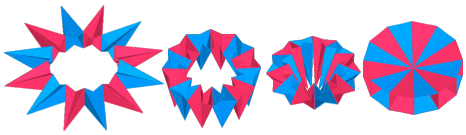

Figure 3: *12-piece Carousel*

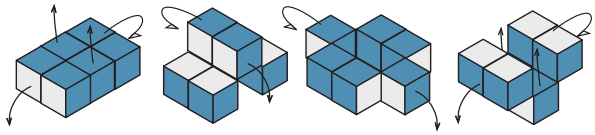

Figure 4: *Pseudo-Fluxicube*

Figure 5: *Beak Unit (4, 6 and 12 units joined)*

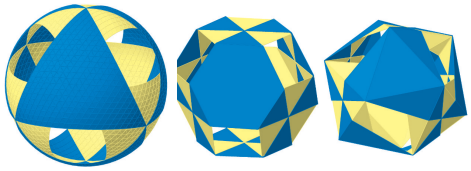

Figure 6: *Radioactive Ball*

The Art, Craft, Engineering and Mathematics of Action Modular Origami

The art of action modular origami is explicit in the appearance of the finished origami e.g. choice of paper, colour or pattern, size, etc. There's also the aesthetics of the movement e.g. the unexpected 'pop' in the middle of *Carousel's* cycle (Figure 3). Making the models requires a degree of craft and skill e.g. folding accuracy, minimising unwanted creases and assembling without crushing or ripping the paper.

The mathematics is clear in the need to use geometry (angles, lines, planes, polygons, angle bisection, etc.) to make the units the right shape that fit together correctly. Other mathematical skills can be used like generalising e.g. if this works for a square, what about a triangle or octagon, say?

There is an implicit art in choosing what to make, how to make it, and creating a folding sequence:

Choosing what to make. Some models are 'made to order' e.g. the rotating ring of six cubes was fairly straightforward once I found satisfactory ways to make the magic wallet hinge and the cubes.

Some are discovered by *serendipity* e.g. I made the *Pseudo-Fluxicube* (Figure 4) by incorrectly joining the cubes in the rotating ring of six cubes; others by *pseudo-serendipity* [10] e.g. I made an octagram ring and folded it up to make it smaller and more portable, but discovered it turned inside out when I opened it up, hence the *Rotating Octagram Ring*. Sometimes I could not make a satisfactory modular origami version of what I wanted e.g. a single sheet version of the Schatz invertible cube with slits and tape was always better than my origami attempts (in form and flexing action).

How it should be made. Like the best of mathematics, elegance in origami is a subjective criterion, but simplicity, economy and inevitability are factors. Good origami should not be obvious and be surprising (in a pleasant way)—although some of the best origami has a 'why didn't I think of that?' quality.

Practical matters include optimising the number of layers of folded paper: too many and it's hard to fold, too few in the wrong place and the structure may be weak. The assembly should be secure but relatively straightforward to make. Proportions and angles should be convenient to make.

Creating a satisfying folding sequence. This can be a compromise between accuracy and brevity e.g. folding two layers of paper is quicker, but less accurate, than creasing each layer separately. Is there a way of making a 'climactic' sequence so that the result is a (pleasant) surprise? The diagrams have their own aesthetics in terms of content (being thorough but not verbose) and appearance (line weights, shading, rendering of 3D steps and paper layers, page layout, etc).

Art-with-a-capital-A? Origami is an art when defined as a 'skilled activity', but Kenneway proposed that 'an object in which neither the matter nor the form can be perceived cannot be described as an art work.'[6] For example, viewers of origami may question the nature of the matter ('is it folded from a square?', 'is it one-piece?', etc) or the nature of the form e.g. 'is it cut? is it glued?'. '[I]t is by perceiving the formal arrangement of matter that the…viewer…has his or her aesthetic experience.'

Inadvertently supporting Kenneway's argument is Robert Lang, 'self-described "leading master of the art" of origami' [2]: a crease pattern (CP) 'can be a "proof certificate": an indication that a folded object really is what it claims to be' [13] i.e. folding on the lines of a CP will make a particular origami shape without cutting, gluing, etc. Jackson wrote of 'so-called "origami artists"…whose work is not only technically sophisticated but also visually beautiful.' [4, p. 31] This echoes Kenneway's comment that 'it is the rhythm

of fold lines and balance of shapes *apparent on the surface* which are admired in such a case.'

McArthur claimed origami has 'developed beyond the realm of craft and hobby... into a highly sophisticated art form... A significant number of origami folders and professional artists whose work is commissioned by private collectors and sought after by museums—and not only craft museums but institutions such as the Museum of Modern Art in New York.' [8, p. 8] This seems to fulfil Perry's definition that art needs validation from 'peers, serious critics and collectors, dealers and then the public'; then 'at the top of the tree... the curators [giving] perhaps the greatest accolade [for] a work of art... museum quality.' [9]

'[W]hy would someone want anything they're doing to be considered art? ... Maybe they just want a good excuse to do something. You know there's a lot of, "I fancy doing that. Let's call it art."... one of the strongest reasons... is economic because there's an awful lot of money—£43 billion last year—sloshing through the art market, so that's quite a nice incentive to call what you do art.' [9] There's also greater intellectual property protection for artistic works than for designs [13].

The art of action modular origami. Origami may be more like music than fine art [11]: musicians improvise or interpret a score and create a performance for an audience. 'If there is an art in origami then it exists within the sequence of folds [transforming] a sheet of paper into a model... A proper perception of origami can only be achieved by doing it.' [6] Action modular origami aims to be pleasing to fold, assemble and use (play).

References

[1] J. Ball. *Johnny Ball's Think Box*. Puffin, 1982. p. 42–50.

[2] D. Duray. *Beneath the Fold: The Twisted Tale of Origami v. Sarah Morris*, 2013. observer.com/2013/05/beneath-the-fold-the-twisted-tale-of-origami-v-sarah-morris/.

[3] A. Edmondson. *A Fuller Explanation: the Synergetic Geometry of R. Buckminster Fuller*. Design Science Collection. Birkhauser, 1987. p. 160.

[4] P. Jackson and J. Thomas. *On Paper: New Paper Art*. Merrell, 2001.

[5] K. Kasahara. *Origami Omnibus: Paper-Folding for Everybody*. Japan Publications, 1988.

[6] E. Kenneway. *Complete Origami*. Ebury, 1987. p. 18.

[7] T. Lam and S. Pope. *Learning Mathematics with Origami*. Association of Teachers of Mathematics, 2016.

[8] M. McArthur and R. Lang. *Folding Paper: The Infinite Possibilities of Origami*. Tuttle, 2013.

[9] G. Perry. *The Reith Lectures*, 2013. bbc.co.uk/programmes/b03969vt.

[10] R. Roberts. *Serendipity: Accidental Discoveries in Science*. Wiley, 1989. p. x.

[11] S. Sternberg. *Sculptural Origami*. Dover Publications, 2011. p. vii.

[12] T. Sundara Row. *Geometric Exercises in Paper Folding*. Dover Publications, 1966. First published 1893.

[13] C. Weaver. *Robert J. Lang On Origami, Sarah Morris Lawsuit*, 2011. hyperallergic.com/25741/lang-art-origami-science/.

[14] D. Wells. *The Penguin Dictionary of Curious and Interesting Geometry*. Penguin, 1991.

[15] B. Wenciker and P. Flynn. "Modular Origami in the Mathematics Classroom." *Bridges Conference Proceedings,* Winfield, Kansas, Jul. 30–Aug. 1, 2004, pp. 293–296. http://archive.bridgesmathart.org/2004/bridges2004-293.html.

[16] T. Yenn. *Orikata*. British Origami Society, 1987. p. 23.